NEUTROSOPHIC STATISTICAL APPROACH TO THE CAPITAL ASSET PRICING MODEL

by

Keshani Nisansala Udawatta Kankanamge

M.Sc., University of Colombo Sri Lanka, 2015
B.Sc., University of Colombo Sri Lanka, 2011

THESIS SUBMITED IN PARTIAL FULFILMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE
IN
MATHEMATICS

UNIVERSITY OF NORTHERN BRITISH COLUMBIA

October 2024

Abstract

Neutrosophic Statistics is an emerging trend in the field of statistics, designed to address the challenges of uncertainty. Given that the stock market is inherently uncertain, this study aims to apply a Neutrosophic statistical approach to the Capital Asset Pricing Models.

The neutrosophic methodologies, techniques and calculations are consistently used through out the study to address the significant criticisms of the existing model such as unrealistic assumptions, misleading beta and single valued risk and return. The study findings effectively capture these criticisms up to certain extent and offer customized neutrosophic models that investors can use based on their specific investment needs. Further these results may be able to explain the relationship between risk and return which many of the CAPM studies struggle to justify in literature. The models' flexibility and ability to capture the indeterminacy may enhance the quality of the results. Additionally, this approach enhances the reliability and accuracy of financial modeling in decision making. Not only finance, but also these sophisticated neutrosophic methodologies can be employed in most of the real-world scenarios with full of indeterminacy.

Netflix is a leading company in the streaming industry, and it has high trading volume which reflects the market sentiment. Netflix stock prices, S&P500 market index, NASDAQ market index, Gross Domestic Product (GDP) and Consumer Price Index (CPI) from 2009-2023 have been used to conduct 5-year and 15-year period analysis using the proposed models in this study. The primary goal is to develop one-factor, two-factor, and three-factor Neutrosophic models. The focus is on calculating appropriate neutrosophic beta values to develop different Neutrosophic Models and then calculate the neutrosophic expected return across four different scenarios.

TABLE OF CONTENT

List of Figures

List of Tables

Glossary

AAPL: ticker symbol for Apple Inc
AMZN: ticker symbol for Amazon Inc
APT: Arbitrage Pricing Theory
CAPM: Capital Asset Pricing Model
CM: classical mean
CPI: Consumer Price Index
GDP: Gross Domestic Product
I: Indeterminacy
LL: lower limit of the return
MA: Moving Average
MAD: Mean Average Deviation
NAPT: Neutrosophic Arbitrage Pricing Theory
NASDAQ: National Association of Securities Dealers Automated Quotations
NCAPM: Neutrosophic Capital Asset Pricing Model
NFLX: ticker symbol for Netflix Inc
NLS: Neutrosophic Least Square
NM: neutrosophic mean
NR: Neutrosophic Return
NS: Neutrosophic Statistics
$NE(R_i)$: Neutrosophic Expected Return
$N\beta$: Neutrosophic Beta
$N\beta_1$: Neutrosophic Beta 1
$N\beta_2$: Neutrosophic Beta 2
$N\beta_3$: Neutrosophic Beta 3
OLS: Ordinary Least Square
S&P500: Standard & Poor's 500

UL: upper limit of the return

Acknowledgement

My heartfelt gratitude goes to my supervisor, Dr. Pranesh Kumar, for his unwavering encouragement, support, and guidance throughout my research journey. His expertise in statistics and enthusiasm for the field of Neutrosophic statistics were significant sources of inspiration, driving me to explore and contribute to this innovative area.

I am also greatly thankful to my committee members, Dr. Edward Dobrowolski and Dr. Hossein Kazemian, for their invaluable insights, constructive feedback, and continuous support. Their guidance was instrumental in refining my work and guaranteeing its academic rigor.

Lastly, I would like to extend my appreciation to the Faculty of Mathematics and Statistics as well as the faculty of business and Economics at the University of Northern British Columbia (UNBC) for providing the resources and environment necessary to carry out this research.

Thank you all for your contributions to this thesis on the Neutrosophic Statistical Approach to the Capital Asset Pricing model (CAPM).

Chapter 01

1.0 Introduction

Everything around us in the world is full of indeterminacy and these indeterminacies may occur due to imprecise, incomplete, or unknown data values and/or size of data set. Neutrosophic statistics offers a framework to handle these indeterminacies and uncertainties effectively. Weather forecasting, epidemiological modeling, financial risk management, climate change projections, supply chain management and natural disaster preparedness are some examples of indeterminacies in real-world scenarios where neutrosophic statistics could be applied.

Among all these fields, this research is focused to address the indeterminacy in the stock market which is inherently uncertain. Neutrosophic concepts can be used to address the uncertainty and indeterminacy in the stock market (Abdelfattah et al., 2024). By capturing these complexities this study aims to develop more sophisticated version of the often-used financial economic model called Capital Asset Pricing Model

Basically, this research directs to investigate a new way of understanding of the Capital Asset Pricing Model (CAPM) using Neutrosophic logic and Neutrosophic Statistics. CAPM is a widely used model in financial decision-making. In financial decision-making, indeterminacy refers to the inherent uncertainty and ambiguity surrounding financial variables, outcomes, and different market conditions. These uncertainties such as market volatility, market risk, unrealistic assumptions in financial modeling, unpredictable regulatory environment, corporate decision-making, globalization and geopolitical risks due to currency fluctuations, set challenges for investors, financial analysts, and decision-makers. Since Neutrosophic statistics provide an effective means of handling all types of indeterminacies, this study helps to reduce the indeterminacy associate with financial market. (Smarandache,2022)

CAPM is a controversial financial decision-making method, primarily due to its unrealistic assumptions. Time to time the researchers and financial analysts strive to address the limitations of the CAPM framework, by modifying or developing independent variables that are perceived to be more relevant and not adequately addressed by CAPM. For illustration extended CAP models were introduced in different periods such as Intertemporal CAPM, Fama French three factor model, Fama French five factor model and Arbitrage pricing theory. This research can be introduced as a novel approach to the classical CAPM by addressing indeterminacies in stock market and limitations of the model. By applying neutrosophic statistical theories to CAPM, analysts can more effectively address the uncertainties and ambiguities inherent in financial markets, allowing more robust and accurate inference in complex and uncertain financial environments. (Smarandache,2022)

The stock market can be explained as one of the vital components of the global economy but highly unpredictable and volatile. Indeterminacy increases the complexity of factors influencing trend fluctuations in the stock market, making it challenging for investors and analysts to predict and interpret market movements accurately. By addressing the indeterminacy, decision makers can better assess the risks and uncertainties associated with their portfolios. Further this proposed model with neutrosophic logic may help investors to identify their risk boundaries. The stock market which is characterised by a collection of neutrosophic random variables, presents a complex challenge for investors, financial analysts, and researchers.

In this research study, we develop the neutrosophic CAP models and estimate their parameters. Further for illustration, we use NASDAQ market Index, S&P500 market index, NFLX stock prices, AMZN stock prices, AAPL stock prices, risk-free rates, GDP and CPI from 2009-2023. MS Excel and python in Jupyter notebook are used for the calculations and graphing. In this analysis, historical data sets in Yahoo finance are used to calculate Neutrosophic stock price, Neutrosophic return, Neutrosophic beta, Neutrosophic expected returns and other considerable factors.

1.1.Introduction to Neutrosophic Statistics

Neutrosophic logic was first introduced by Florentin Smarandache in the 1990s as a generalization of fuzzy logic and intuitionistic logic. It allows for dealing with incomplete, imprecise, and uncertain information. Neutrosophic statistics can be applied in various statistical analyses and modeling tasks, including descriptive statistics, data analysis, hypothesis testing, regression analysis, time series analysis and decision making under uncertainty.

According to Smarandache (51), a random neutrosophic sample of size n from a classical or neutrosophic population is a sample of n individuals such that at least one of them has some indeterminacy. He explains this nicely through below example.

"Consider a random sample of 1,000 homes, in a city of over one million inhabitants, in order to investigate how many houses have at least a laptop. One finds out that 600 houses have at least one laptop, 300 houses don't have any laptop, while 100 houses have each of them a single laptop, but not working. Some of these 100 house owners tried to have their laptop

fixed, others said their laptops' hard drives have crashed and it is little chance to fix them. Therefore indeterminacy. We have a simple random neutrosophic sample of size 1000."

Neutrosophic statistics extend beyond classical statistics by allowing the representation of indeterminacy through sets, intervals, or approximate values, acknowledging the ambiguous, vague, imprecise, incomplete, or unknown nature of data. This flexibility in handling uncertainty is valuable when dealing with real-world scenarios where exact values may not be available or feasible to obtain.

We can replace any indeterminate parameter a by a_N in neutrosophic statistics. a_N can be imprecise, unsure, and even completely unknown. Further a_N can be a neighbour or an interval that includes a and it can be any set, which approximates a. In worst scenario a_N could be unknown and in best scenario a_N could be equals to a.

In traditional statistics, sample sizes are often precise numerical values, like 10, 50, or 100. However, in real life, most of the time we deal with approximations instead of exact numbers. Hence instead of using exact number, it is possible to deal with sets/intervals. In neutrosophic statistics, sample sizes can be sets or intervals, such as [5, 10] or [90, 100], representing uncertain or imprecise information about the sample size. The sample size is [90, 100] in neutrosophic signifies that the analyst isn't entirely certain whether these individuals fully belong, partially belong, or do not belong to the population under study. There are several examples of imprecise data in day-to-day life such as flock of migratory birds, trees in a jungle and fish in a river. So, there are many of such cases when the sample size may not be exactly known. Practically in the real world it is not possible to exactly estimate a sample or population size of most of the events (Smarandache,2022).

In essence, neutrosophic samples and neutrosophic statistics provide a way to handle and represent indeterminacy in data analysis by allowing for the use of sets, intervals, or approximations instead of relying solely on precise numerical values.

Hence Neutrosophic Statistics can be considered as a generalization of the classical statistics and deals with set values instead of crisp values. As an illustration the table 1.1.1 represents crisp numbers and neutrosophic numbers separately.

Crisp Number	Neutrosophic Number
2	[0,5]
0.5	[0.1,0.6]
-3	[-5,0]
8	[0,10]

Table 1.1.1: Crisp Numbers and Neutrosophic Numbers.

The neutrosophic statistical methods can be used to interpret and organize the neutrosophic data to discover underlying patterns. Specially, neutrosophic statistics can be used to analyze data sets with uncertain or contradictory information. By representing data as neutrosophic numbers, we can account for the inherent uncertainties in the data and make more reliable statistical conclusions.

A neutrosophic number N has the form: $N = d + i$, where d is the determinate /sure part of N, i is the indeterminate /unsure part of N. For example, $a = 2 + i$, where $i \in [0, 0.3]$, is equivalent to $a \in [2, 2.3]$, so for sure $a \geq 2$ and this indicates that the determinate part of a is 2, while the indeterminate part $i \in [0, 0.3]$ means the possibility for number a to be a little bigger than 2 but smaller than or equal to 2.3. Logically a neutrosophic number can be written in different ways. Reconsider, $a = 2 + i$, with $i \in [0, 0.3]$, or $a = 1 + i_1$, with $i_1 \in [1, 1.3]$, or, in general, $a = \propto + i\propto$, with $i\propto \in [2-\propto, 2.3-\propto]$, and $\propto$ any real number. Or, in opposite way, $a = 2.3 - i_2$, with $i_2 \in [0, 0.3]$, and in general, $a = \beta - i\beta$, with $i\beta \in [\beta - 2.3, \beta - 2]$, and β any real number. (Smarandache,2014)

We can also represent data and statistical parameters using neutrosophic numbers, which consist of three components: the degree of truth (T), the degree of indeterminacy (I), and the degree of falsehood (F). These components represent the extent to which a particular value or parameter is true, uncertain, or false, respectively. (Zhang et al.,2014)

1.1.1 Neutrosophic Algebra

a) Addition: For $I \in [0, 1]$, let consider two neutrosophic numbers $N_1 = 5+2I$ and $N_2 = 5-2I$. The indeterminate part of N_1 is $2I=[0,2]$ and the indeterminate part of N_2 is $-2I=[0,-2]$. Therefore, $N_1 + N_2 = 5+2I+5-2I = 10$.

   By adding $N_1 + N_2$, the indeterminate parts cancel out as $2I-2I=0$. Determinate part is equal to 10.

b) Subtraction: For $I \in [0, 1]$, let $N_1 = 5+2I$ and $N_1 = 5-2I$. The indeterminate part of N_1 is $2I=[0,2]$ and the indeterminate part of N_2 is $-2I=[0,-2]$. Therefore, $N_1 - N_2 = 5+2I-5+2I = 4I$

   Let's take other neutrosophic example: For $I \in [2, 3]$, $N_3 = 6+4I$ and $N_4 = 4+5I$. Then, $N_3 - N_4 = 6+4I-4-5I = 2-I = 2-[2,3] = [0,-1]$.

c) Multiplication: For $I \in [0, 1]$, let $N_1 = 5+2I$ and $N_2 = 5-2I$. Then the product $N_1 \times N_2 = (5+2I) \times (5-2I) = 25-4I$ ($I^2 = I$, because indeterminacy × indeterminacy = indeterminacy)

d) Division: For N_1 =2+I and N_2=4+2I, the division N_1/N_2=(2+I)/(4+2I)=(2+I)/[2×(2+I)]=1/2=0.5.

Hence from one operation to another, neutrosophic statistics may diminishes the uncertainty. (Smarandache,2022)

Difference between Classical Mean and Neutrosophic Mean

Let S = {a, b, c, d} be a sample set of four elements, such that a = 1, b = 2, c = 3, and d = 4. In classical statistics it is assumed that all elements belong 100% to the sample, therefore S = {a(1), b(1), c(1), d(1)}.

Whence the classical mean (CM)

$$CM = \frac{(1.1 + 2.1 + 3.1 + 4.1)}{1 + 1 + 1 + 1} = 2.5$$

But, in the real world, not all elements may totally (100%) belong to the sample, for example, let's assume the neutrosophic sample be NS = {a(1.1), b(0.2), c(0.5), d(0.3)}, which means that the element a belongs to 110%(for example someone works overtime), b belongs only 20% to the sample, c belongs to 50%, and d belongs to 30%.

Whence the neutrosophic mean (NM) is

$$NM = \frac{1\,(1.1) + 2\,(0.2) + 3\,(0.5) + 4(0.3)}{1.1 + 0.2 + 0.5 + 0.3} = \frac{4.2}{2.1} = 2.$$

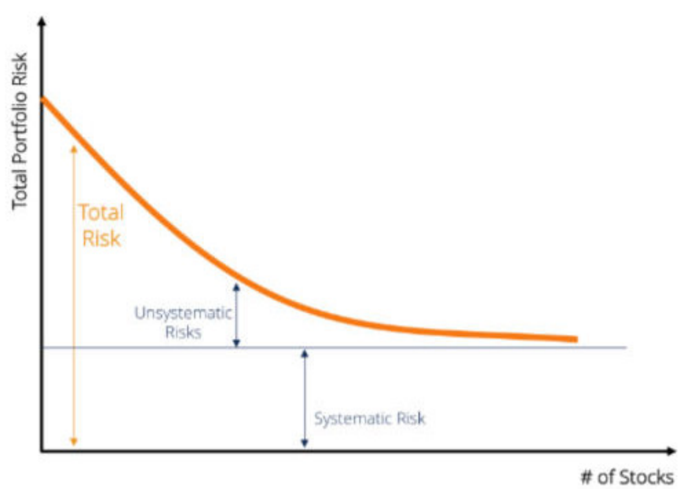
Obviously, the two mean classical and neutrosophic are different, 2.5>2.0. Therefore, in this example, CM > NM. And subsequently other statistical features such as the variance, covariance, standard deviation, probability distribution function will also be different. The neutrosophic mean consider the degree of membership of the elements so it is more accurate since it reflects the real mean. (Smarandache,2022)

1.2. Introduction to CAPM

CAPM is a mathematical model widely used in finance for several purposes, mainly to estimate the expected returns of assets and to evaluate investment opportunities based on the risk and uncertainty in the market. In CAPM, risk is quantified by the beta coefficient (β), which measures the volatility. If $\beta = 1$, or $\beta > 1$ or $\beta < 1$, it indicates that the stock price tends to move with the market, or the stock is more volatile than the market, or the stock is less volatile than the market, respectively. When $\beta = 0$, then the stock price does not correlate with the market and is independent of the market movements. The stock with negative β indicates inverse movement to the market.

Risk cannot be described in the abstract, but it depends on the factors such as investor's preference, wealth position, time horizon and so on. Basically, risk factors fall in to two categories as represented by graph 1.2.1.

Total Risk=Systematic Risk+ Idiosyncratic Risk



Graph 1.2.1: Total Portfolio Risk.

The risk, that arises from the market structure, general economic conditions and affects all market players, is called a systematic risk. Systematic risk factors such as inflation, GDP growth, interest rates, consumer confidence cannot diversify away. But the idiosyncratic risk or unsystematic risk that applies to a specific firm or industry such as success or failure of R & D, change in CEO, can be diversified away. The factor models such as CAPM focuses only on dealing with systematic risk.

For assembling a portfolio of assets that maximizes the expected return, given a specific level of risk, Markowitz in 1952 introduced a mathematical framework called Mean-Variance portfolio theory. This method was developed by adding risk free rate to open a new range of possible returns with lower levels of risk than what is possible on the efficient frontier. Together, asset pricing and portfolio theory provided a framework to specify and measure investment risk and to develop relationships between expected asset return. The CAPM builds on the work of Markowitz on diversification and modern portfolio theory. The development of the CAPM is usually credited to several different people Sharpe (1964), John Lintner (1965), Jack Treynor (1962) and Jan Mossin (1966). Markowitz, Sharpe, and Miller received the Nobel Prize in Economics in 1990 for their contributions to financial economics. CAPM is an equilibrium asset pricing model, and it was developed based on several assumptions. (Laopodis, N. T. , 2021)

The Basic Assumptions of CAPM

1. All investors are risk-averse and would take a position on the efficient frontier, focus only on portfolio return (or mean) and the related variance (risk).
2. All investors invest for the same one period, with the same investment planning.

3. Investors are able to buy or sell portions from their shares of any security or a portfolio they hold.
4. The market is frictionless such as no taxes, no inflation or no transaction costs on purchasing or selling assets.
5. All information is publicly available, and all assets are publicly held and traded on public exchanges.
6. Capital markets are in equilibrium, and all investments are fairly priced. Each investor is very small and has a limited power compared to the market and cannot affect prices.
7. Investors have similar expectations; hence they choose the same distributions for the future rates of return.
8. There is a risk-free asset, and investors can borrow and lend any amount at the risk-free rate.

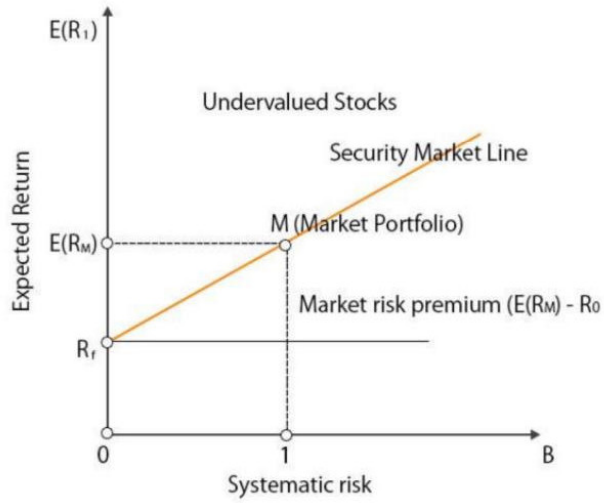
Basically, these assumptions cause all firms look the same to investors and suggest an efficient market. (Laopodis, N. T. , 2021)

In traditional CAPM, the expected return of an asset i is calculated using the below formula.

$$E(R_i) = R_f + \beta_i \left[E(R_m) - R_f\right] \tag{1.1}$$

where R_f is the risk-free rate, β_i is the asset's beta coefficient representing its sensitivity to market movements, and $E(R_m)$ is the expected return of the market portfolio. $\left[E(R_m) - R_f\right]$ is known as the risk premium/market premium.

Individual Risk Premium=Beta × Market Premium. This relationship between market beta and asset's risk premium is called the security market line (SML). Its slop indicates the risk premium while intercept indicates the risk-free return. (Laopodis, N. T. ,2021). The graph 1.2.2 illustrate the relationship between expected return and the β.



Graph 1.2.2: Expected Return and β

One of the primary uses of CAPM is to estimate the expected return of an asset. Shareholders and financial experts use CAPM to decide the appropriate expected return for individual stocks, portfolios, or entire markets. This expected return serves as a key input in various financial decisions, including asset allocation, portfolio management, and investment valuation.

CAPM is also used to estimate the cost of equity capital for companies. By applying CAPM to estimate the expected return on equity, companies can conclude the required rate of return that investors demand for holding their equity shares. This cost of equity is a crucial input in valuation models such as discounted cash flow analysis and in assessing the feasibility of investment projects.

Portfolio managers use CAPM to construct and manage investment portfolios. By considering the expected returns and systematic risks of individual assets, portfolio managers can optimize portfolio allocations to achieve desired risk-return profiles. CAPM helps investors make informed decisions about asset allocation, diversification, and risk management within their portfolios.

CAPM provides a benchmark for evaluating the performance of investment portfolios and individual assets. Portfolio managers compare the actual returns of their portfolios to the expected returns predicted by CAPM to assess whether their investment decisions have generated excess returns or underperformed the market. This performance evaluation helps investors to assess the effectiveness of their investment strategies and to identify skilled managers.

1.2.1 Introduction to Arbitrage Pricing Theory (APT)

CAPM is a one-factor model used to determine the expected return of a stock, based on its risk relative to the market. But there are several other factors, which can influence the stock, return such as economic factors, political and regulatory factors, technological changes, market dynamics and even global events. Further to this foundational one factor model, multifactor models have been developed over the period to handle the effect of different factors such as Fama-French three factor model, Fama-French five factor model and APT.

To address these macroeconomic factors, APT can be considered as the one of the sophisticated factor models. It can capture the effects of systematic risk factors. APT is a multi-factor model, which builds up with the stochastic properties of stock returns of capital assets and several macroeconomic factors including the market factor. APT was developed by Ross in 1976. He acknowledged that there are many sources of systematic risk beyond just market risk. APT can be considered as an extended CAPM which utilizes multiple regression analysis. APT is naturally aligned with multiple regression analysis.

The multiple regression equation

$$(R_i) = \alpha_i + \beta_{i1}[F_1] + \beta_{i2}[F_2] + \cdots \beta_{ik}[F_k] + \varepsilon \tag{1.2}$$

Where, R_i represents actual return of a stock, α_i is intercept term (the expected return of the asset when all factors are zero), β_{ij} is the factor loadings or sensitivities of asset , F_j is the factor values and ε is the error term which capture the idiosyncratic risk or noise.

βij coefficients can be estimated using multiple regression analysis. Regression analysis helps to understand the impact of these factor on the stock return.

Traditional APT equation can be written as below.

$$E(R_i) = \mathrm{R}_i + \beta_{i1}[F_1] + \beta_{i2}[F_2] + \cdots + \beta_{ik}[F_k] \tag{1. 3}$$

Where, β_{ik} represents the beta (or risk exposure) on the k^{th} factor and F_k is the factor risk premium for the k^{th} factor.

The factors are allowed to be correlated and are meant to simplify and reduce the amount of randomness required in an analysis. When $k = 1$, we have a single-factor model and when $k \geq 2$ we have a multi-factor model. When $k=1$ and the single factor (F_1) is the market portfolio factor, the APT implies the CAPM.

If an asset has only a unit beta risk on the second factor (β_{i2} = 1) and zero betas on all other factors, then its expected return will be:

$$E(R_i) = \mathrm{R}_i + [F_2] \tag{1.4}$$

The $E(R_i)$ has increased by an extra amount to compensate for taking on the factor risk of the second factor (F_2). This is why F_2 is called the factor risk premium for factor 2 or else extra return one earns by taking 1-unit beta risk on the factor. As an example, $E(R_M)$-R_f is market risk premium, $E(R_{CPI})$-R_f is CPI risk premium and $E(R_{GDP})$-R_f represents GDP risk premium.

The APT makes no specific claim about what factors influence returns, or how many factors are relevant. Both CAPM and APT models assume efficient markets and access to information, but APT relaxes the most of the CAPM assumption.

Overall, while CAPM and APT provides a useful framework for estimating expected returns and evaluating investment opportunities, its accuracy depends on various factors, including model assumptions, data quality, and market dynamics. Understanding these considerations is essential for effectively applying CAPM and extended CAP models in investment decision-making and financial analysis.

1.2.2. Key Differences between CAPM and APT

There are some key differences in the assumptions of the CAPM and the APT models (Laopodis, N. T., 202). In brief APT is associated with less stringent assumptions on the statistical distributions of asset returns compared to CAPM as illustrated by table 1.2.2.1.

Key difference	CAPM	APT
Number of Factors	One factor-Market index	Multiple factors
Market Efficiency Borrowing and Lending	perfectly efficient markets risk-free borrowing and lending	Does not strictly assume market efficiency but assumes no arbitrage No specific assumptions about borrowing and lending.
Investor Behavior and Expectations	homogeneous expectations and single-period horizon	Allows for heterogeneous expectations and multi period analysis
Model Flexibility	Less flexible, strictly defines the market portfolio as the risk factor.	More flexible, allows for multiple and different risk factors to be included.

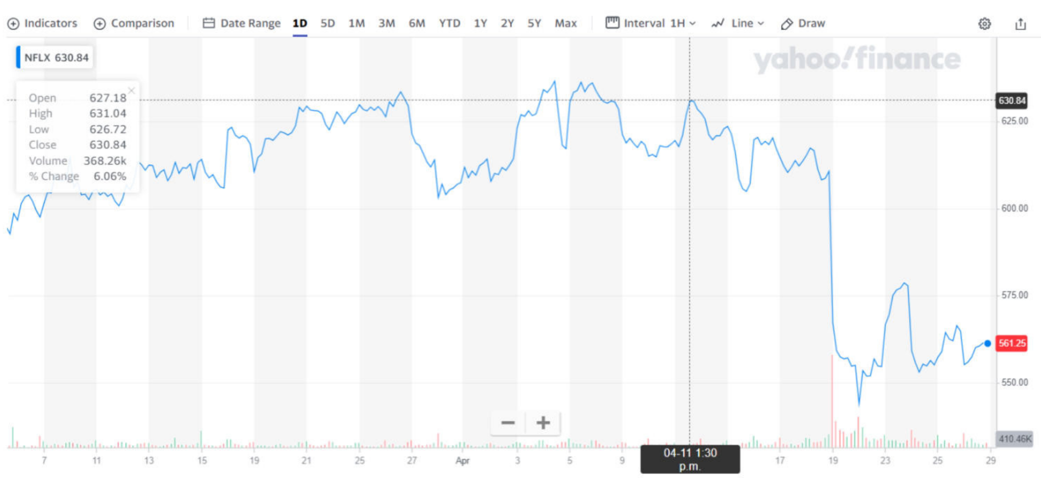
Table 1.2.2.1: Key differences in the assumptions of the CAPM and the APT models

These differences highlight the individual approaches and applicability of each model in different financial contexts. APT holds simplistic and less restrictive assumptions compared to CAPM. This makes APT more flexible and adaptable to different financial contexts. Therefore, this study also aims to consider the development of Neutrosophic APT, which can further increase the effectiveness of this study.

1.3. Relevance of Neutrosophic Statistics to CAPM and APT

CAPM is a one-factor model, and that single factor is the market meanwhile APT is an extended CAPM which can be corporate with more than one factor. The crucial point of the CAPM or APT is to determine beta which is known as the sensitivity or risk factor. Normally daily, weekly, quarterly, or annual beta can be calculated based on the stock market data. However, stock market operates with full of indeterminacies. Trend may be fluctuated due to the various macroeconomic factors contributing to the volatility and unpredictability observed in stock prices. Some of the key factors which affect the trend movements are economic indicators, monetary policies, corporate earnings, market sentiments, political events and policy changes, global economic conditions, technological innovations, psychological factors, natural disasters and external shocks, market structure and trading dynamics and so on.

All these factors can affect investor's reaction and decision making, which may lead to unpredictable trend fluctuations. This may happen in every single minute, hour, and day or at any time, making them inherently hard to predict and analyse. All these factors are affected by indeterminacy, and it adds another complex layer to the factors, which are affecting the trend in stock market. As an illustration below graph 1.3.1 shows the hourly trend fluctuations of the Netflix data set during a particular day.



Graph 1.3.1: Hourly Historical Data Set-Netflix.

Data revisions, measurement errors, unforeseen events, investor's sentiment, interconnectedness and interdependence among economies, irrational market behaviour due to the factor such as fear, greed, and herd mentality can lead to create uncertainty and indeterminacy in stock market data to exhibit unpredictable trends. This unpredictable and indeterminate nature of stock market highly affect the volatility of the stocks. Therefore, understanding and addressing indeterminacy is crucial for effectively navigate the uncertainties of the stock market and determining beta to calculate the expected return of factor models to investors to make the informed decisions on their investments.

Recently, neutrosophic statistics is widely used as a powerful tool in decision making to address the situation with indeterminacy (imprecise, unsure, and even completely unknown values). Neutrosophic statistics have emerged as a compelling research frontier since around 2013, attracting significant attention from researchers across the globe (Bolos et al., 2019). Hence to navigate these unpredictable factors in stock market, neutrosophic analysis is the ideal approach.

The primary goals of this study are to develop and apply Neutrosophic statistics to address the uncertainty involved in stock market data, represent the data as neutrosophic numbers, develop a neutrosophic CAP model that account for uncertainty, extend the CAPM to capture more than one factors (address APT model), and use forecasting techniques to make predictions while considering uncertainty and ambiguity. This Neutrosophic approach will enable a more comprehensive and sophisticated analysis of CAPM and extended CAPM, improving the accuracy and reliability of decision making in complex economic environments.

1.4.Motivation

Neutrosophic statistics have emerged as an expanding area of research that has significant attention from scholars worldwide. Smarandache (51) proposed a Neutrosophic logic to deal with indeterminate and inconsistent information, simultaneously generalizing the concepts such as classical, fuzzy, interval-valued and intuitionistic fuzzy sets which can address study of data bases, medical diagnosis, image processing and the decision making.

Stock markets are complex and unpredictable frameworks impacted by many interrelated financial, political, and internal factors. Stock market decisions are a significant challenge to the investors as understanding the portfolio diversification, how much to invest in stock markets and when to make the decision are extremely complex tasks. To navigate this complexity, it will be a great advantage for the investors to give a neutrosophic approach for the market analysis.

The motivation for this analysis stems from the desire to explore the criticisms, discover recent advancements in CAPM and apply Neutrosophic Statistical techniques and methodologies to make optimal strategies for investment decisions while addressing the indeterminacy of the stock market and the limitations of CAPM. The financial markets and their dynamic nature have always fascinated and motivated me to study deeper into understanding the complexities of financial modeling and analysis. This exploration will serve as a driving force for uncovering innovative Mathematical and Statistical approaches to asset pricing and risk assessment in modern financial markets.

1.5. Main Objectives

Primary objectives of this study can be categorized as below.

- Apply neutrosophic statistics technique to calculate neutrosophic market returns to capture the indeterminacy of the stock market.
- Apply neutrosophic statistics technique to calculate neutrosophic beta which represent the unpredictable nature of the stock market.
- Develop a single factor Neutrosophic model-Neutrosophic CAPM (NCAPM) by incorporating neutrosophic statistics, which is more sophisticated in understanding the risks compared to traditional models.
- Extend the NCAPM to capture the effect of macroeconomic factors which can provide a more comprehensive risk assessment.
    - Develop 2-factor Neutrosophic models (NAPT) which can address the risk boundaries.
    - Develop a 3-factor Neutrosophic model (NAPT) to further optimize the risk boundaries.
- Use these neutrosophic models to estimate neutrosophic beta and neutrosophic expected return which can guide investors to make informed investment decisions.
- Verify the model Accuracy.
- Comparative analysis among neutrosophic models

Chapter 02

Literature review

2.0 Literature review for CAPM and APT models

The CAPM was introduced in the early 1960s by William Sharpe, Jack Treynor, John Lintner, and Jan Mossin (Laopodis, N. T., 2021). It was considered as a significant milestone in modern finance. This model introduced the initial structured approach to linking the expected return on an investment with its associated risk.

Even though CAPM is a widely used financial model it was criticized due to some reasons (Laubscher, 2002). Mainly CAPM is based on several assumptions that may not hold true in the real world. One assumption of the CAPM is that the market is frictionless such as no taxes, no inflation, or no transaction cost. But in reality, we cannot completely avoid these kind of cost. Also, the model assume that all the investors are risk avers. But different investors have different expectations and different risk preference levels. (Laubscher, 2002). The assumptions, that short selling is not restricted and unrestricted risk-free borrowing and lending, are also unrealistic (Fama & French, 2004). Furthermore, other unrealistic assumption is that the share returns are normally distributed, but in reality, portfolio returns are asymmetrically distributed. Hence beta is viewed as an incomplete risk measure (Ward, 2000). However, these assumptions are often violated by behavioral biases, market inefficiencies, and institutional constraints that influence the genuine behavior and performance of investors and assets.

Beta coefficient is one of the key inputs of CAPM and the volatility of a stock is measured by beta, which estimates how its returns or those of a portfolio will fluctuate concerning movements in the market portfolio (Moyer, 2001; Jones, 1998 as cited by Laubscher in 2002). Beta is calculated using the historical returns of a particular stock against the returns of a market index, such as the S&P 500 or NASDAQ. However, beta estimation can be problematic for several reasons. One reason is that the historical data may not be representative of the future conditions and risks of the asset.

On the other hand, the beta value may change over time and across different market segments, making it unstable and unreliable. Beta is specific to the data set being used to calculate. If we use a different data set, or different time frame, the beta value will most likely be different. In different financial website, they may give different betas for the same stock (Dybek, 2024; Yahoo!, 2024). Not knowing how the numbers have been arrived at, it is unclear to use which one, if any of them is accurate, resulting in uncertainty relating to the choice of beta value to use. The regression line is used to estimate the beta and the line of best fit that minimises the residuals. It is not a perfect fit but be the best guess using a specific data set. As the errors are ever present, most of the time, the asset will not move in line with regression estimate. This can be seen in the graph, which also highlights some of the outlying moves. This is a particular risk for popular hedge fund strategies such as long, short and equity market neutral positions (Fama & French, 1992).

Another criticism of CAPM is that it does not capture all the factors that affect share returns. CAPM is a one factor model which capture the market factor (Fama & French, 2004). Historically, several multivariable CAPM models have been introduced by different researchers in different time periods at different locations all around the world, which assume that risk is influenced by multiple factors including market returns, indeterminacy, incompleteness, and several other factors (Assagaf, Aminullah, 2015). While multivariable models represent a positive advancement in financial theory, they still have shortcomings when applied to decision-making processes. The development of asset pricing models has been marked by several significant contributions, each aimed at better explaining the cross-section of stock returns.

The intertemporal capital asset pricing model -ICAPM (Merton, R. C. ,1973), Fama and French's three factor model (1992), Fama and French's extended three factor model (1993), Carhart four- factor model (1997), Fama and French five factor model (2015) and Arbitrage pricing theory are some of the extended CAPM. Basically, such factor models have more than one beta (Laopodis, N. T., 2021).

The ICAPM is introduced by Nobel Robert Merton in 1973.This is a consumption-based capital asset pricing model. It extends the traditional CAPM by accounting for investors' desire to hedge against market uncertainties and construct dynamic portfolios over time. Instead of one beta in CAPM, ICAPM employs multiple beta coefficients (Merton, R. C.,1973) Especially investors use ICAPM in economic downturns. Even though this model stands as a significant progression in financial modeling, it also has some limitations. This is recognised as a complex model to understand and implement. ICAPM also holds some unrealistic assumptions and challenge of accurately estimating beta coefficients over several time periods (Fama and French, 2004).

Alternative asset pricing model is a valuable report to get a clear understanding of different CAP models because it has carefully discussed the theories behind the different CAP models (Graham and Stephen 2020). The reviews and evaluations are based on past literature, which expands over 100 years in academia, financial economics research and consulting. Graham and Stephen (2020) have examined and reviewed several articles of useful studies on CAPM.

They successfully described the different CAPM models such as ICAPM and CCAPM (Consumption-based Capital Asset Pricing Model). Graham and Stephen (2020) observed the input variable of CAPM such as beta, alpha, market premium and risk-free rate while highlighting the conclusions of several empirical studies. Further, they outline the issues in determination of the rate of return and discussed the significant details of implementation of CAPM and the necessary adjustments based on prior beliefs and market conditions.

Fama and French's Three-Factor Model (1992) expanded the traditional CAPM by adding two additional factors to market risk. The three factors of the model are market Risk (Beta), size and the value. The size or SMB is the return difference between small-cap and large-cap stocks (i.e., SMB: small minus big). Value or HML is the return difference between high book-to-market and low book-to-market stocks (i.e., HML: high minus low). This model explains a larger portion of variations in stock returns compared to the CAPM. Strong empirical evidence supports the presence of size and value effects in stock returns. Even though this is an extended model it may still miss other significant factors, which are affecting returns. Another drawback of this model is that the factors are fixed and do not account for changing market conditions over time (Fama and French, 2004; Laopodis, N. T., 2021).

Fama and French extended their 1992 model by including changes to better capture the size and value premiums, confirming the significance of three factors in different market conditions. The model is Fama and French extended model (1993) and it enhanced methodology for calculating size and value factors. This Fama and French extended model provides consistent outperformance over the CAPM. But it still suffers from potential exclusion of other relevant factors and fixed factor structure (Fama and French, 1993; Fama & French, 2004 and Laopodis, N. T., 2021).

The Carhart Four-Factor Model (1997) is another extended factor model. Carhart extended the Fama-French three-factor model by adding a momentum factor which explain the return difference between stocks with high past returns and those with low past returns. Presence of momentum accounts for the momentum effect and properly explains anomaly in stock returns. This model delivers a more comprehensive explanation of asset returns than the three-factor model, but momentum is harder to explain using traditional financial theories. Also, adding more factors can sometimes lead to overfitting and less robustness across different data sets (Carhart, 1997).

In 2015, Fama and French added two more factors, profitability and investment, to the three-factor model and suggested a Fama and French's Five-Factor Model (2015). This model provides a more complete framework for understanding stock returns. Empirical evidence supports the relevance of profitability and investment factors. The model is more complex and harder to implement. Moreover, some factors might be correlated, and presence of multicollinearity may reduce the clarity of their individual contributions (Fama and French, 2015; Laopodis, N. T., 2021).

Arbitrage Pricing Theory (APT) was developed by Stephen Ross (1976) by extending CAPM with multiple factors which affect the returns. It doesn't specify which factors to use, or the numbers of factors need to use. Hence, APT offers a more flexible framework than the CAPM or Fama-French models. Further it has simplified model assumptions compared to restricted CAPM assumptions. specification issues and potential biases are some disadvantages of this model (Ross, 1976; Laopodis, N. T., 2021).

Each model has its own set of strengths and weaknesses. Researchers, such as Fama and French, Carhart and Ross made some efforts to capture the complexities of asset returns more accurately. However, the choice of model should always consider the specific context, data and information availability, and the objective of the analysis.

Multivariate test of financial models was the earliest journal in financial economics, which discussed new approaches of financial economics (Gibbons, 1982). Gibbons discussed about the multivariate CAPM models and tried to increase the precision of estimated risk premium parameters. He provided the theoretical superiority to commonly used procedures and demonstrate practical applications.

Nevertheless, due to practical challenges encountered in this model, investors are advised to exercise caution when using the CAPM to estimate share returns and assess investment performance (Laubscher, 2002). In the CAPM, the focus is primarily on systematic risk, which cannot be diversified. This suggests that investors should expect proportional compensation for assuming such risk. Laubscher (2002) thoroughly discussed the empirical studies of CAPM, its criticisms and concluded that CAPM is a valid model but should be cautiously applied to evaluate investment performance.

In recent literature, several studies have been conducted to determine relevance of CAPM model. Athens Stock Exchange, focusing on weekly returns of 100 listed companies, questioned the efficacy of CAPM (Michailidis et al., 2006). This study argued that higher risk did not necessarily correspond to higher returns. Similar analysis has been conducted to check the validity of CAPM using empirical evidence from Amman Stock Exchange (Alqisie and Alqurran, 2021). The main objective was to examine whether a higher/lower risk stocks yields higher/lower expected rate of return. They used 60 companies in AMANA Stock Exchange from 2010-2014. The study used the monthly closing stock prices to calculate the rate of return of each stock. They used the same methodology as Black et al. (1972) and Fama and MacBeth

(1973) to test the non-linearity. In this study, they concluded that there was no conclusive evidence in support of validity of CAPM in Amman Stock Exchange (ASE) for the period (2010 – 2014). They recommended expanding the study period and repeating the analysis by considering several other financial and marketing indicators.

In 2015, Canadian Center of Science and Education published an article “Analysis of Relevance Concept of Measurement of CAPM Return and Risk of Shares”, which emphasises the relevance of CAPM by comparing alpha and beta values of earlier research models (Assagaf, 2015). For the analysis, he used most actively traded stocks on the Indonesian Stock Exchange (BEI) weekly during the period September 2014 to November 2014. The study has concluded that despite these advancements made in alternative models, the CAPM concept remain the foremost and most widely utilized method for estimating stock returns.

A review of Capital Asset pricing model by Iqbal (2011) is another interesting article which discussed the review of foreign studies on CAPM. According to this study, ongoing debate surrounding the dominance of APT over CAPM remains unsettled. Throughout the study, he discussed that numerous researchers have identified strengths and weaknesses in both models. Several empirical studies, including those by Sharpe and Cooper (1972), Foster (1978), and Sauer and Murphy (1992), have recognised CAPM as a feasible asset-pricing model, despite criticisms from Roll (1977, 1981), Dimson (1979), Fama and French (1992, 1996), and Davis (2000) who have challenged the validity of CAPM tests (Iqbal, 2011). According to Iqbal (2011), it is evident that CAPM research has provided invaluable insights into stock return behavior across diverse global markets. CAPM is still anticipated to maintain dominance in the capital market as a method for estimating expected returns of risky securities. The CAPM holds considerable relevance in financial economics, with broad applications such as testing asset-pricing theories, assessing the cost of capital, estimating portfolio performance, and defining hedge ratios for index derivatives (Iqbal, 2011).

2.1. Literature review for Neutrosophic statistics to capture the indeterminacy associated with financial decision making.

Integrating neutrosophic methodologies to address its indeterminacies and limitations can modify CAPM. Data exactness, accuracy in expressing data and uncertainty of the closing prices are the major issues of stock market (Jha et al., 2018). Neutrosophic soft sets can be used to tackle the exact state of these data sets. Based on high, low, open and closing prices, they have developed a technique to determine adjusted closing price in stock market.

Boloș et. al. (2019) modeled the return on financial assets, the financial asset risks, and the covariance between them, with the help of triangular neutrosophic fuzzy numbers. This neutrosophic approach of three financial assets performance indicators addressed the scenario of uncertainty, the scenario of non-realization, and the scenario of indecision. All three scenarios have attached performance, non-execution, or uncertainty ratios according to the investor's professional judgment. They checked the possibility of stratification, or the clustering of the financial asset return values with the help of triangular neutrosophic fuzzy numbers.

The paper entitled "Novel Single Valued Neutrosophic Hesitant Fuzzy Time Series Model: Applications in Indonesian and Argentinian Stock Index Forecasting" by Tanuwijaya et al (2020) can be considered as an improvement of neutrosophic time series model which absorb the degree of the uncertainty using single-valued neutrosophic hesitant fuzzy set. Tanuwijaya et al. (2020) proposed a model, which capture the uncertainty of the data movement and simplified de-neutrosophication process with guaranteed forecast.

The study entitled “A Neutrosophic Forecasting Model for Time Series Based on First-Order State and Information Entropy of High-Order Fluctuation” by Guan et al. (2020) presented the concept of neutrosophic fluctuation time series (NFTS) and proposed a new financial forecasting model based on neutrosophic soft sets. This analysis discussed the first-order neutrosophic time series to describe the uncertainty and information fluctuation entropy to measure the complication of historical fluctuations.

Neutrosophic portfolios were characterized by their composition of financial assets, wherein the neutrosophic return, risk, and covariance can be quantified (Boloș et al, 2021). They investigated the concept of neutrosophic portfolios within the framework of modern portfolio theory. These portfolios offer insights into the probability of attaining the neutrosophic return at both the individual asset and portfolio levels, which experience the neutrosophic risk. They introduced two key concepts, neutrosophic covariance of financial assets and independent neutrosophic portfolios. The proposed methodology involves a three-step approach aimed at identifying the independent neutrosophic portfolio return, risk, and structure. The paper includes numerical examples at each stage of the methodology to enhance understanding. The study's findings can aid capital market investors in making informed decisions (Boloș et al., 2021).

Abdelfattah et al. (2024) introduced a stock market movement prediction model that combines social media data with historical stock price data. They addressed the importance of stock market prediction in recent years and the significant influence of social media on it, using neutrosophic logic. The results of the study revealed the superiority of the model in return and

Sharpe ratio scores, indicating effective ability for generating excess return and managing risk (Abdelfattah et al., 2024).

Recently, there is a growing trend of utilizing neutrosophic statistics to address uncertainty in the financial market in decision-making and we could find considerable number of scholarly studies in literature, which have been dedicated to study this area.

Chapter 03

3.0.Introduction to Data availability

In financial decision-making, data availability and reliability are important to conduct a comprehensive analysis. In this study identify the required data set is a challenge as there are several factors influence on the fluctuation in the stock market. Most powerful factors need to be carefully chosen according to the requirement of the analysis. stock prices, market indices (S&P500 and NASDAQ), risk-free rates, GDP, and CPI are recognised as the main data set need to be addressed carefully. the sources of these data sets, access methods, timelines and challenges associated with those data sets are discussed through out this chapter. Accurate data set is an advantage to make informed decisions.

3.1. Indeterminacy of the data set

Stock market, which is inherently indeterminate, displays the wealth of information for shareholders such as stock price, price movements, dividends, trading volumes, announcements, financial statements of the company, comparison charts and several other important statistics such as earning per share, market volatility and growth estimates.

Based on the stock market information, analyst and financial managers help investors to take the most desirable decisions on their portfolios and buying and selling opportunities. Future is inherently uncertain. Unexpected things may happen in unforeseen future. Events like economic downturns, technological disruptions, natural disasters or political instability can significantly affect on a company performance. Not only the future events but also market sentiment, limited information and market manipulations can highly impact on a company's performance.

Market sentiment is a one of the critical reasons which course indeterminacy in the stock market. Now and then, the stock market is driven by emotions and psychology. News reports, social media also can influence investor sentiment. This may cause to fluctuate prices even without any underlying change in a value of the company.

Limited information is another factor which impact the stock price. Most of the investors do not have access to all the available information that could impact a stock price. Management decisions, internal developments, and upcoming regulations might not be publicly available and that may cause to create uncertainty. The Efficient Market Hypothesis (EMH) is a theory in financial economics that suggests stock prices reflect all available information. EMH indicates that the market is efficient at incorporating all relevant news and data into stock prices, making it difficult to consistently outperform the market through security selection or market timing.

Manipulation of stock market data is another factor, which highly affects the movement of the prices. Some people do the market manipulations with the aim of personal gains. Pump and Dump is one type of manipulation. In this scheme individuals or groups of people artificially increase a stock price. They mislead other investors by spreading false information, then investor rush ups in buying those shares and price of the stock goes up. They sell their stocks once the price rises leaving other investors with overvalued shares.

Wash Trading is also a type of market manipulation, which involves buying and selling a stock back and forth to create a fake impression of trading activity and potentially manipulate the price.

Another way to manipulate the market is known as spoofing. Investors place large orders to buy or sell a stock, but they cancel them before execution. Their aim is to move the price in a desired direction before placing their actual order. Even though Regulatory groups constantly try to detect these manipulations and work to prevent them. But they remain a risk in the stock.

Apart from these kinds of uncertainties there may be several other reasons to fluctuate the stock prices while creating indeterminacy. In stock market financial data such as stock prices, bond yields and interest rates are observed daily (intra-day or even minute-by-minute (tick) basis), weekly as well as monthly. Hence even basic data recording can have errors leading to uncertainty. It is important to identify these uncertainties and manipulation in the stock market.

The financial managers and investors can use advance methodologies such as neutrosophic statistics, which can effectively address the indeterminacy to minimise the risk of their decisions. Neutrosophic statistic is an emerging trend to address uncertainty in the real-world scenarios. This approach is to apply neutrosophic logic and statistical methods to the widely used decision making models in stock market predictions.

Data sets may be downloaded from financial web sites Yahoo finance, the U.S. Bureau of Economic Analysis (BEA), U.S. Bureau of Labour Statistics (BLS), US Department of the Treasury, Stock Analysis from Web, Netflix official website and Kenneth & French data library. The analysis in this thesis is based on the stock market data sets of Netflix, Amazon, Apple, NASDAQ and S&P500 from 2009-2023 monthly basis.

3.2. Data sets

15-year period from 2009-2023

15-Year period is chosen to conduct a comprehensive analysis considering long-term perspective. This long-term understanding may help to capture the uncertainty associate with multiple economic cycles, structural changes, and long-term trends.

5-year period from 2019-2023 ,2014-2028 and 2009-2013

Short term analysis is a more sensitive analysis, and this may consider the impact of changes in economic policies, short-term market cycles, and sector-specific movements during that period. Specially the recent period 2019-2023 may consider the recent trends, market conditions, and economic factors.

Stock market data

Stock market index

- S&P500
- NASDAQ Index

A stock market index is a benchmark that tracks the performance of a segment of the stock market. It is a comparison tool, which compares the performance of different indices to understand how various market segments are faring. This helps to evaluate overall market trends and identify possible opportunities in the market.

Well-known stock market indices around the world can be named as Dow Jones Industrial Average, S&P 500 and NASDAQ Composite Index. They serve as useful tools for understand

and direct the stock market. For this study NASDAQ and S&P 500 are used as stock market index to analyse the stocks.

S&P 500 (^GSPC)

The S&P 500 (or Standard & Poor's 500) is an important stock market index that captures the performance of 500 of the largest publicly traded companies in the American economy. The companies with the largest market capitalization have a greater influence on the overall performance of this index. The S&P 500's value fluctuates daily based on the stock prices of the companies it includes. Usually positive trend of S&P500 indicate an economic growth meanwhile negative trend may signal an economic challenge. This index is diversified across different sectors including financials, healthcare, technology, industrials and many more. This diversified exposure cause to diminish the risk and provide a solid long-term return. S&P500 is a common choice among long term investors.

NASDAQ Composite Index (^IXIC)

NASDAQ Composite Index is a stock market index that includes almost all stocks listed on the NASDAQ stock exchange which includes more than 3000 companies in all over the world. This is one of the most popular market indices which heavy weighting towards technology-oriented companies such as Apple, Microsoft and Amazon. But it also includes several other companies in different sectors such as financial services, healthcare, consumer service industries, and industrials. Hence this diversity provides a wider picture of the entire market performance. This index is widely used by the investors particularly in tech related fields. Due to the economic growth and technological advancement NASDAQ composite shows a strong growth, outperforming other major indices such as Dow Jones and S&P500.Futher NASDAQ is more volatile index compared to the other indies due to high concentration of technology stocks.

Key differences between S&P500 and NASDAQ market indices

S&P500 and NASDAQ are widely followed two distinct market indices which have different characteristics. Normally NASDAQ composite is considered as a gauge of the performance in technology sector and S&P 500 is viewed as a key indicator in the economy in United State.S&P500 is more diversified index compared to NASDAQ. It represents broad spectrum of industries such as technology, healthcare, and financials, meanwhile NASDAQ heavily weighted towards technology and biotech companies. Larger companies have a more significant impact on the NASDAQ performance. But S&P 500 has a more balanced sector representation compared to the NASDAQ market index. NASDAQ is more volatile due to its' high concentration of technology and biotech firms which are more sensitive to the rapid changes in investor sentiment and market conditions. S&P500 is less volatile with its' more stable and established companies. Even though all stocks listed on the NASDAQ stock exchange are included in NASDAQ composite; to be in the list of S&P500, the companies need to fulfil some specific requirements like a lowest market capitalization and a positive earnings record.

Both indices have their own characteristics and unique objectives to fulfil various investor requirements and inclinations. Netflix is a technology-based service. But to have a wider picture, this study uses both S&P500 and NASDAQ market indices for this analysis.

Stock price

Stock price of a particular stock reflects the current price of a share. Price movements are available for users in daily, weekly and monthly basis with low price, high price, open price, closing price and adjusted closing price.

Stock: Netflix Inc. (NFLX)

This study is based on the variation of Netflix stock prices which provides entertainment services with 270 million paid memberships. Netflix has operations in approximately 190 countries. This leading streaming service provider offers TV series, documentaries, films, and games in different categories and languages. This entertainment company also provides members the ability to receive streaming content through a host of internet connected devices, including Televisions, digital video players, TV set-top boxes, and mobile devices. The company was incorporated in 1997 and is headquartered in Los Gatos, California. Mr. Wilmot Reed Hastings is the co founder and the executive chairman of this company.

NASD: NFLX

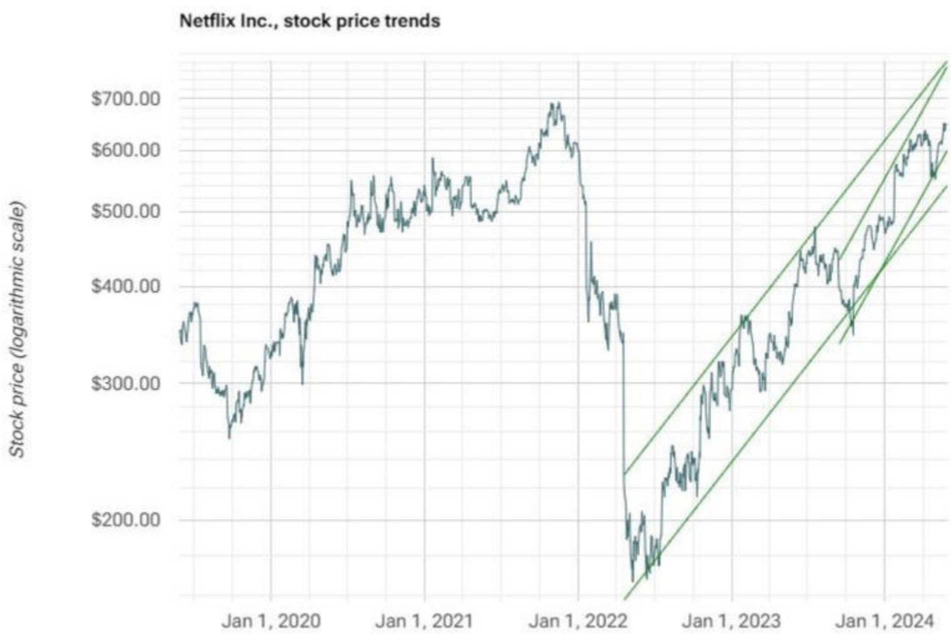
Price	$641.62	Change	-6.04
Volume	4,071,023	% Change	-0.93%
Intraday High	$648.58	52 Week High	$664.25
Intraday Low	$628.30	52 Week Low	$344.73
Today's Open	$644.01	Previous Close	$647.66

May 31, 2024 4:00 PM — *Pricing delayed by 20 minutes*

Table 3.2.1: Details of Netflix stock.

Netflix is a publicly traded technology and internet-based company listed on the NASDAQ stock exchange in the United States under the ticker symbol "NFLX”. This means that Netflix is initially market traded in the United States. But investors in other countries such as Canada can still trade Netflix stock through local brokerage accounts that have access to U.S. markets. There is no separate stock market specifically for Netflix in Canada.

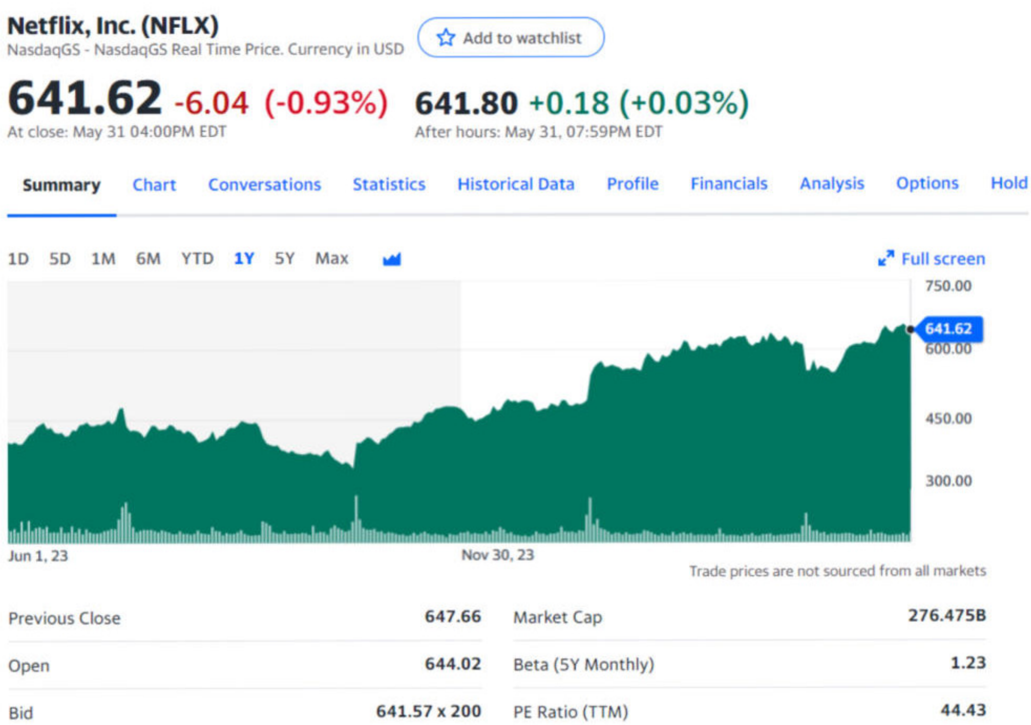
Netflix's stock price has an upward trend overall in 2024.But there are daily fluctuations as illustrated in graph 3.2.1.



Graph 3.2.1.: Netflix stock price trend.

Website: https://www.stock-analysis-on.net/NASDAQ/Company/Netflix-Inc/Financial-Statement/Income-Statement

Netflix's stock price has undergone considerable variations during the period of 15 years from 2009 to 2023.From 2009 to 2010 is a significant time period for Netflix streaming platform because the popularity of this service was risen up during that period and at the end of 2010 the share price of the Netflix was around $25-$30.But the company's decision to split DVD and streaming services caused to show a downtrend in next two years 2011 and 2012 .However at the end of 2012 the company recovered and achieved an upward trend. This uptrend continued by attracting more subscribers and at the end of the 2015 the share price was over $100.The international expansion caused to sky rocketed share price of Netflix, reaching over $300 by mid 2018.The COVID-19 pandemic was a turning point to this online service provider, and this caused to accelerate its demand. The Netflix stock price recorded its highest price during the pandemic, and it was $691.69 in November 2021.Conversely stock prices started falling down in 2022 and dropped up to $294.88 due to high competition in the market. In 2023 it recovered slightly. At the end of the period the share price is approximately $500.Currently stock prices has been fluctuating between $600 and $700 (Investing.com,2024) . According to the below graph in Yahoo finance the NFLX stock price is 641.62USD and beta is 1.23 on 31st March 2024.



Graph 3.2.2: Netflix stock price.

Stock: Amazon.com, Inc. (AMZN)

Amazon is a well-established tech company, which has a major interest in e-commerce, cloud computing, and digital streaming. The historical data set of Amazon from 2009-2023 has been used to calculate the neutrosophic expected returns with the aim of testing the proposed neutrosophic regression models.

Stock: Apple Inc. (AAPL)

Apple is a one of the biggest publicly traded companies by market capitalization. As a top technology company, Apple stock is listed on the NASDAQ exchange. It is a key component of the S&P 500. The stock is known for its solid financial performance, strong brand, and consistent innovation. Investors consider factors such as product launches, earnings reports, and market conditions when evaluating APPL stock. The historical data set of APPL from 2009-2023 has been used to calculate the neutrosophic expected returns with the aim of testing the proposed neutrosophic regression models.

- **Risk Free Rate**

The risk-free rate refers to the hypothetical rate of return on an investment with absolutely zero risk. In reality, every investment (even government bonds considered as the safest investments) have some risk, like inflation which can diminish their purchasing power. Though, there are investments that can be considered as risk-free such as government bonds. Especially short-term bonds like treasury bills issued by governments with strong credit ratings are considered as risk free securities. They are very safe. Risk-free investments typically offer low returns. Investors can buy these bonds directly from government agencies or through brokerage firms.

There are several ways to access data relevant to risk-free rates. Government treasury bill rates are available on government websites or financial data platforms. Several online financial databases provide historical data on government bond yields, allowing investors to track changes over time. These databases might require a subscription fee.

Fama-French Keneth data library is one of another easy way to download risk free rates which are updated regularly and are available freely. This data library is a worthy resource for researchers and finance professionals interested in testing asset pricing models and portfolio strategies. Data are available for the factors like market risk, risk free rate, size, and book-to-market ratio in monthly, weekly and daily basis.

Web site :(https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

Systematic Risk Factors (External factors)

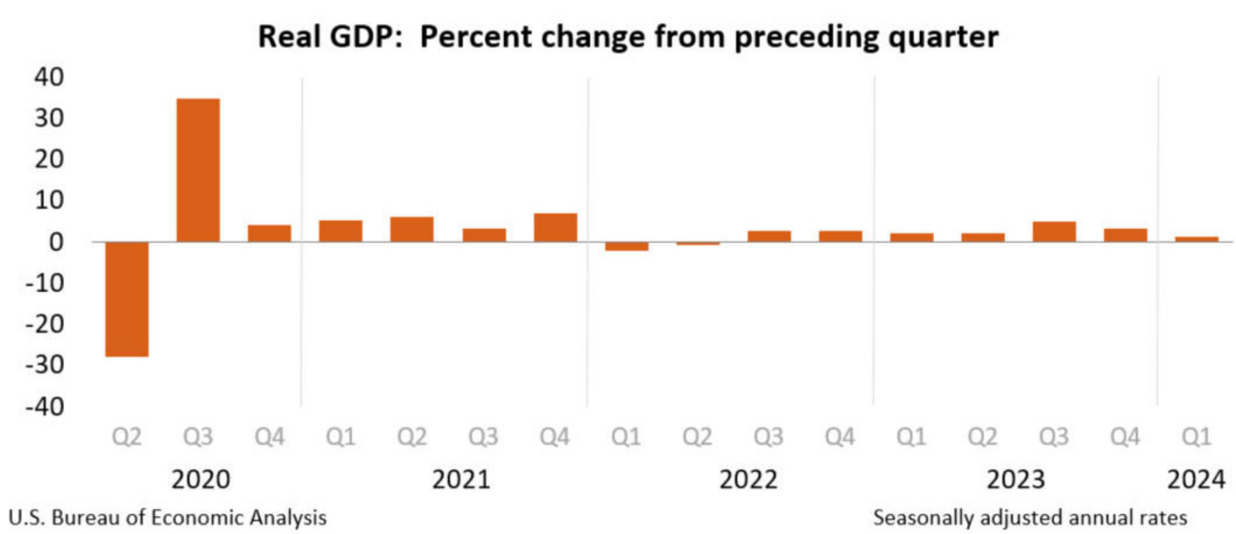
- Gross Domestic Product GDP
- Consumer Price Index CPI

Gross Domestic Product (GDP)

GDP is a significant metric used to measure the economic stability of a country. It represents the total monetary value of all final goods and services produced within a country in a specific period. GDP is calculated by summing up the components such **as** consumer and government spending, investment and net exports. GDP is a central indicator for policymakers, businesses, and investors, which helps to assess economic growth. Apart from that, GDP can be used to compare economies of different countries and make informed decisions about resource allocation and investment strategies.

Generally, GDP has a positive relationship with stock prices. When GDP increases the stock price increases accordingly and when GDP decreases normally the stock price falls. Rise in GDP indicates the expanding economy, and this leads to increase consumer spending, increased investments, and up-lift the economic confidence. In an economic boom, companies generally see higher revenues and profit. This is the reason to rise in stock prices as investors expect better future earnings. By contrast, in an economic downturn, it leads to reduce consumer spending and lower investments. In the economic uncertainty, investors avoid invest in stock market leading to a price decrease in the stock market. Hence, GDP is a crucial factor, which affect the stock market fluctuations.

USA annual GPD can be downloaded from the U.S. Bureau of Economic Analysis (BEA). One needs to register for an API key from the BEA at BEA API to download the data set.



Graph 3.2.3: GDP quarterly.

Web site: https://apps.bea.gov/api/signup/activate.html#947AB03F-061B-4141-9F4A-BBE78D43A28B

Consumer Price Index (CPI)

The CPI represents changes in prices of all goods and services purchased for consumption by urban households except income taxes and investment items such as stocks, bonds, and life insurance. CPI is the most widely used economic indicator, which is a measure of inflation and is an indicator of the effectiveness of government policy. Investors, business executives and other private citizens use this index as a guide in making economic decisions.

CPI is a key economic indicator that measures changes in the price levels, which gauge inflation. This factor can have a considerable influence on stock market performance through various direct and indirect mechanisms. Increase in CPI indicates the inflation, which can lead to lower corporate earnings. This may negatively impact stock prices. Furthermore, rising CPI causes to reduce consumer spending and this may negatively affect companies that heavily depend on consumer sales. This also has impact on their stock prices. On the other hand, rising CPI can cause to raise interest rates to combat inflation. These higher interest rates can indirectly cause to slow down the economic growth and negatively impact stock prices.

However, lower inflation can cause to improve the purchasing power of the customer. This may positively impact on their stock prices. By contrast, falling CPI may decrease the interest rates to stimulate economic growth and may indirectly boost investment and spending which can positively impact stock prices.

Further, continuous high inflation can lead to uncertainty and decrease investor confidence. This may indirectly lead to stock market volatility as stockholder seeks safer assets. Opposite to this, deflation can be able to create a stable economic environment, which potentially boost investor confidence.

Overall, the CPI is a critical factor in shaping the economic environment in which companies operate, and investors make decisions, thereby influencing stock market performance. USA annual CPI data can be downloaded from the U.S. Bureau of Labor Statistics (BLS) for CPI. First, you need to register for an API key from the BLS at BLS Public Data API Signup to download the data set.

Website: https://data.bls.gov/registrationEngine/registerkey

3.3. Charts and Graphs

This analysis is based on the data set of stock market prices. Hence Various charts and graphs which have been used in stock market are utilized in this thesis. To get a clear understanding of these complex trends and relationships, candlestick charts and trends lines are briefly explained below.

Candlestick Chart

Candlestick chart is a specific type of price chart used in finance to do technical analysis for visualizing price movements of a security over the particular period (day, week, month, etc.). Candlesticks use bars with a rectangular body and wicks (thin lines) running from both ends to represent the price range for a specific time. The body shows the opening and closing prices, and the wicks show the high and low prices, and it indicates the market behaviour whether it has a bullish or a bearish trend. Wicks extending from the top and bottom of the body symbolize the high and low prices for the period. Long upper wicks indicate prices reaching considerably higher than the closing price. Long lower line indicates prices dipping significantly lower than the opening price.

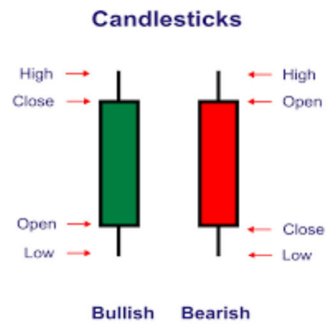


Figure 3.3.1: Bullish and Bearish Candlesticks

By analyzing the size, color, and position of a candlestick chart, financial analysts can identify potential trends, and they can use these trends to make significant trading decisions.

Graph 3.3.1: Candlestick chart illustration from NFLX stock

Trend Lines

A trend line is a line drawn through two or more price lines to connect price points, specially represent the direction of the stock price movement. If the stock price increases, we call it as an uptrend suggesting bullish market. An uptrend line can be drawn by connecting at least two higher lows; meanwhile downtrend can be drawn by connecting at least two lower highs. If it is a downtrend or the stock price is decreasing over the time, we call it as a bearish market.

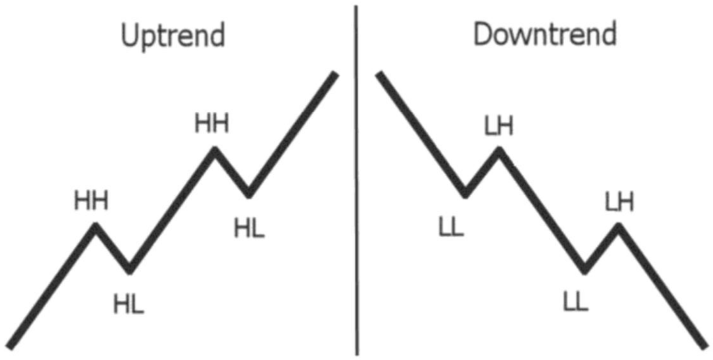
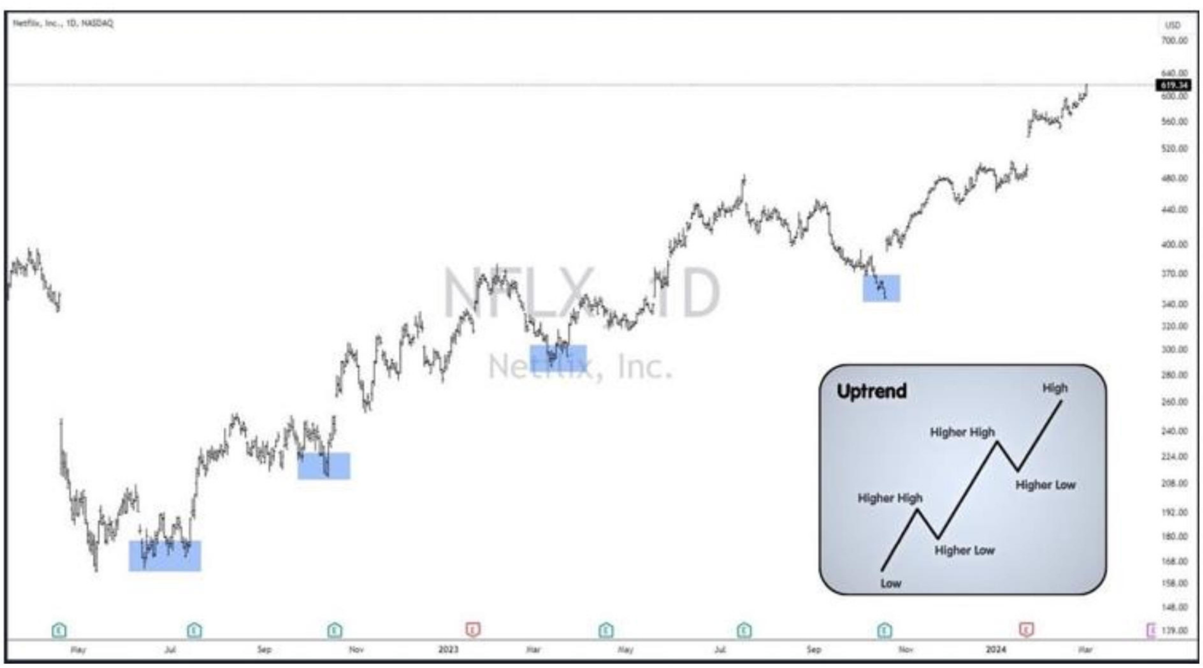


Figure 3.3.2: Uptrend and Downtrend lines



Graph 3.3.2: Uptrend illustration from NFLX one day price movement

Website: https://medium.com/@satolix.io/market-trends-uptrend-downtrend-and-range-ed989df2bc8c

Chapter 04

Research Methodologies

4.0. Introduction to Traditional CAPM and its Limitation

In finance, CAPM is a fundamental method to calculate the expected return of an asset. To understand the risk of an asset relative to the overall market, analysts calculate the asset's beta, which measures the volatility of the asset in comparison to the market. This beta can be used to estimate the expected return of CAPM. Market return is used as the systematic risk factor for this single factor CAPM.

For illustration, to calculate the market beta for NFLX, the returns of Netflix stock can be compared with the returns of a stock market index such as the S&P 500(GSPC) or NASDAQ composite index (IXIC).

Graph 4.0.1: Netflix stock price comparison with NASDAQ and S&P500.

4.0.1. Basic steps to calculate the Traditional CAPM (Stock: NFLX)

1. Collect historical data: Obtain historical prices for NFLX and a stock market index (S&P 500 or NASDAQ) for the same time period (normally collect the adjusted closing price)
2. Calculate the periodic stock returns: Compute the simple returns for Netflix stock using the following formula:

$$R_{t(NFLX)} = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{4.1}$$

   Where , $R_{t(NFLX)}$ represents NFLX return at time t,P_t is NFLX share price at time t and P_{t-1} is NFLX share price in the previous time period

3. Calculate the periodic market Return: Market return can be calculated using the following formula

$$R_{t(MARKET)} = \frac{P_{t(INDEX)} - P_{t-1(INDEX)}}{P_{t-1(INDEX)}} \tag{4. 2}$$

   Where, $R_{t(MARKET)}$ is Market return at time t, $P_{t(INDEX)}$ is the market index at period t and $P_{t-1(INDEX)}$ is the market index in the previous time-period

4. Calculate Beta: There are several different ways to calculate beta in traditional CAPM
   a. Calculate the covariance and the variance of the stock return with market return.

      Below formula can be used to calculate beta.

$$\beta_{\text{NFLX}} = \frac{COV(R_{NFLX}, R_{MARKET})}{VAR(R_{MARKET})} \tag{4.3}$$

Where R_{NFLX} is Return of Netflix stock, R_{MARKET} is Return of the market index, $\text{Cov}(R_{\text{NFLX}}, R_{\text{MARKET}})$ is covariance between Netflix stock returns and market returns and $\text{Var}(R_{\text{MARKET}})$ is the variance of market returns.

b. Conduct regression analysis to calculate beta (excel, SPSS, STATA or manual calculations can be used for this analysis). Here the dependent variable is the return of NFLX, and the independent variable is the return of market index.

$$R_{NFLX} = \alpha + \beta_{NFLX} \cdot R_{MARKET} + \varepsilon \tag{4.4}$$

Where, α is intercept of the regression line, β_{NFLX} is Slop of the regression line, and ε is error term

c. Use the slope of the least square line to calculate beta

$$\beta_{NFLX} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}} \tag{4.5}$$

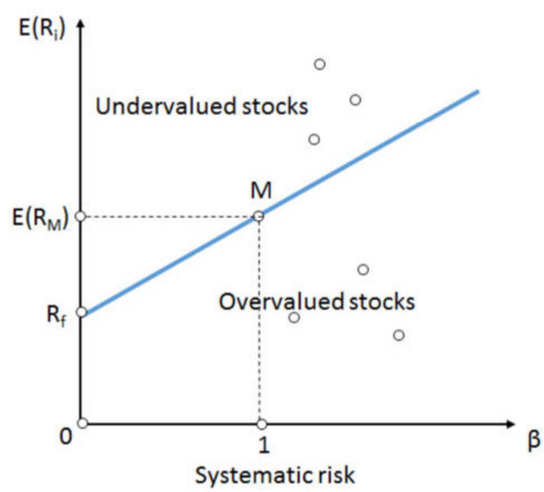
Where, Y is stock return, X is market return and n is number of observations

5. Download risk free rate for the same period (Kenth French data library or treasury-bill rates)
6. Gather all these data sets and plug them into the CAPM formula (equation 01) to calculate the expected return for Netflix.

The market beta indicates the sensitivity of stock returns to changes in the overall market returns. Therefore, beta is a valuable indicator to measure the volatility or the sensitivity of the market.

This beta is useful to measure the expected return in CAPM.CAPM can be used to make portfolio decisions. By analyzing the beta values and expected returns of different investments, CAPM helps investors to choose assets with low correlation. This helps to reduce the overall portfolio risk.

By calculating the expected return using CAPM, investor can compare it to a benchmark, whether the stock is undervalued or overvalued. Investors and decision makers can compare the CAPM derived expected return with analyst's estimates or own return expectations for the investment. Higher expected return of CAPM than required return implies that the investment might be undervalued and potentially attractive. Lower expected return than required return suggests the investment might be overvalued and less attractive. Undervalued and overvalued stocks can be illustrated as the below graph 4.0.1.1.



Graph 4.0.1.1: Under values and overvalued stocks

4.0.2. Limitation of Traditional CAPM

CAPM can be used as a starting point, but it is vital to analyze the company's fundamentals, market conditions, and other risk tolerances before making any move. CAPM is a widely used financial model, but it has some criticisms and limitations. CAPM relies on several assumptions that may not perfectly hold true in real markets (as an illustration, perfectly rational investors, all investors hold diversified portfolios).

Accurately estimating some inputs such as the market return and beta can be challenging. For illustration CAPM calculation are based on adjusted closing price of a particular day, week or month. But it may not be the most accurate measurement or the top choice for the calculations. Stock market is very complex and generates massive amounts of data at high speeds. Stock price is fluctuated in every single minute. Hence picking up the most suitable data point to measure the risk is challenging.

The use of single beta as a measure of risk is overly simplistic. It assumes that the risk of a security is fully captured by its correlation with the market, which is not always the only case. There may be several other factors, which affect the risk of a security. CAPM is a single factor model, which considers only one source of risk (market risk) measured by beta. It does not consider the dynamic nature of markets and ignores other factors that might affect the asset returns such as GDP, liquidity and inflation. Another major drawback of CAPM is that it assumes certainty around factors like expected returns and volatility (beta). But in reality, these involve some level of uncertainty and vagueness.

Further, CAPM ignores human behaviours and assumes rational behaviour and efficient markets. It does not account for psychological factors and market anomalies caused by irrational behaviour, which are highlighted by behavioural finance. The model assumes that beta is stable over time, but empirical evidence suggests that betas can change, making predictions less reliable.

Apart from that, some empirical studies have found that CAPM does not completely explain the actual returns of securities. For illustration, low beta stocks often outperform high beta stocks contradicting CAPM's predictions.

The aim of this study is to address these limitations using relatively new statistical technique of neutrosophic statistics, which capture almost all type of these indeterminacies to provide a more comprehensive analysis of risk and return.

4.1. Neutrosophic Statistical Approach to the traditional CAPM

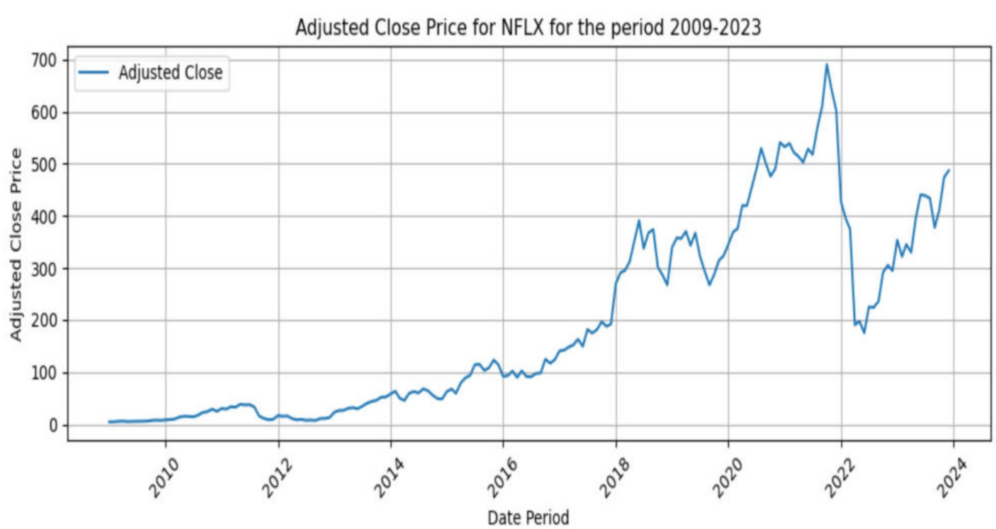
The indeterminacy of the stock market needs to be carefully addressed. Based on the risk of the traditional CAPM, analysts estimate the expected return of an investment, but with limitations. However, with the use of neutrosophic statistics, the theory of statistics which generalizes classical statistics offer a potential way to address some of these limitations.

Neutrosophic sets are a Mathematical tool, which can be used to represent circumstances with unclear, vague and inconsistent data. This study starts with converting the classical data set, which is used to calculate traditional CAPM into the neutrosophic data sets. Focus is to use neutrosophic techniques and methodologies which can help to minimise the indeterminacies. Finally calculate a neutrosophic beta value and get a neutrosophic expected return.

These neutrosophic results would include not just a single value but a neutrosophic set number or an interval. This approach could allow for a more nuanced understanding of risk and uncertainty in making decisions in stock market. It will facilitate analysts and investors to take a best investment decision based on neutrosophic expected return or neutrosophic beta.

In traditional CAPM method, adjusted closing price is used to calculate beta. The adjusted closing price reflects the value of a share after accounting for corporate actions like stock splits or cash dividends to closing price of the day. But the closing price is the last price at the end of a trading day, and it does not reveal the standard value of a share. During a day share price can be fluctuated within the high price and low price. In this study it is expected to address this indeterminacy of the price which can be considered as a very crucial input to calculate beta and expected return of asset pricing model. Share price should reveal the true economic value of a security.

The graph 4.1.1 below shows the trend of monthly-adjusted closing prices of the NFLX from 2009-2023.

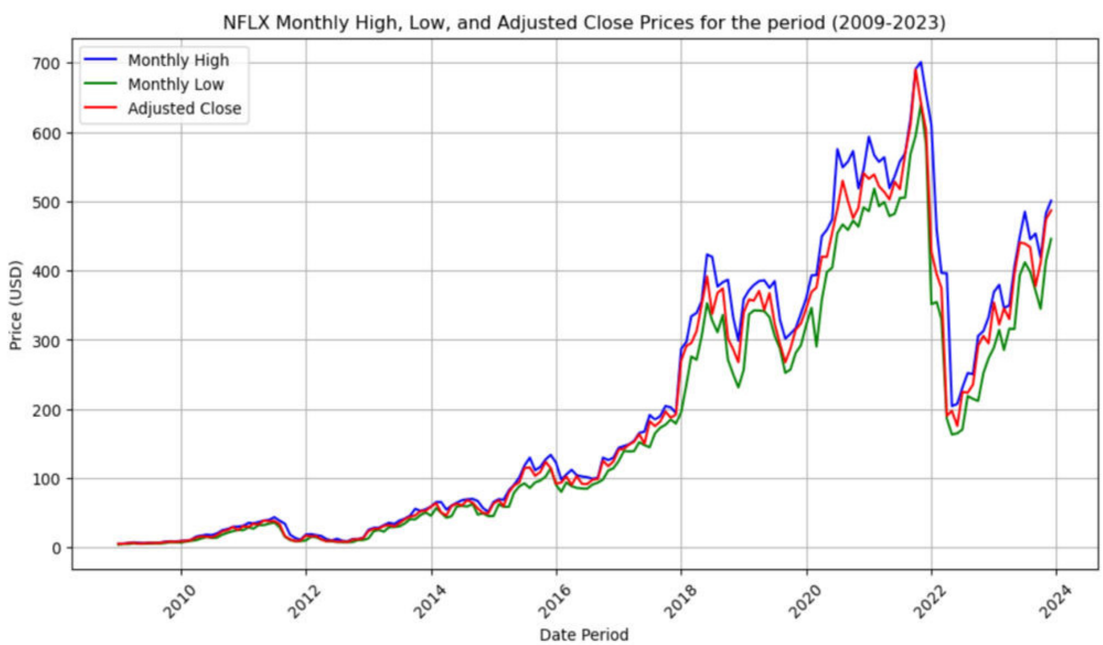


Graph 4.1.1: Monthly adjusted-closing price of the NFLX from 2009-2023

Overall NFLX depicts a significant upward trend over the entire period. From the period 2009-2013 NFLX trend is steady increase and during 2013-2015, there appears a sharp increase followed by a correction. The trend between 2015 to 2021 is mostly an uptrend with some consolidation periods and around 2021 the stock reaches to its highest price. Afterward the stock shows a sharp downtrend in. But mid 2022-2023 period can be considered as the recovery period. After mid 2022 the stock experience a gradual increase.

All the fluctuations of stock price during the period indicate the high competition of the streaming industry and all the challenges and opportunities which are faced by the company.

The graph 4.1.2. represents NFLX monthly high, low and adjusted-closing prices from 2009-2023 and the monthly price fluctuations in between low and high prices.



Graph 4.1.2: NFLX monthly High, Low and Adjusted Closing Prices from 2009-2023

The fluctuations of three prices high, low and adjusted close prices illustrate the dynamic stock market behaviours and investor sentiment towards NFLX stock. According to the graph, the adjusted-closing prices are not continuously positioned within the expected range. Sometimes it falls between the high and low prices but at other times, it aligns with either the lowest or highest price of the month. For illustration all three prices are align close to each other from 2010 to 2014 and in between 2021- 2022 adjusted-closing prices position with the lower price line of NFLX. This inconsistency can lead to an underestimation of actual volatility especially for highly volatile stocks like NFLX.

According to the graph 4.1.2, high price and low-price lines represent the two extrema of the stock price movement during the trading period. A monthly interval with low and high prices [low price, high price] captures the full range of prices that a stock can fluctuate within a month. Comparatively low and high intervals provide a more comprehensive picture of its volatility. If we consider a daily interval, it captures the whole day price range, theoretically encompassing all trading prices within that interval. This is an advantage over using a single closing price, which might not reflect the full daily price movement.

The low price and high price interval represent a range of possible "true" prices within a trading period. The interval low to high represents the indeterminacy of the price, which we use to calculate beta. The beta value could place anywhere within the range, reflecting uncertainty about the true relationship between the risk and the market.

However, neutrosophic statistics is an ongoing concept. Hence applying this new technique to the CAPM is challenging.

4.2. Basic Techniques and Methodologies

This study is based on the original data sets of S&P500, NASDAQ and NFLX. To check the effect on smoothing technique, MA of the original data set is also used with the aim of refining the reliability as well as the accuracy of the results. Further, we expect to smooth the short-term fluctuations to get a meaningful and consistent output. Altogether there are four different scenarios with the aim of capturing the limitations of CAPM and indeterminacy in the stock market. These four scenarios can be categorised as below.

Scenario 01:S&P 500 Index with original data

Scenario 02: NASDAQ Index with original data

Scenario 03:S&P 500 Index with Moving Average (MA)

Scenario 04: NASDAQ Index with MA

In the first two scenarios (Scenario 01 and Scenario 02) the original data set from yahoo finance is used as the initial data set meanwhile moving average of the original data set is used as the dataset of the second two scenarios (Scenario 03 and Scenario 04).

Scenario 01:S&P 500 Index with original data

Original NFLX monthly low and high stock prices are compared with original S&P500 market index monthly low and high prices in scenario 01.

Dependent Variable: NFLX monthly stock prices (Original data set)

Independent Variables: S&P500 market index monthly (Original data set), GDP and CPI

Scenario 02: NASDAQ Index with original data

In scenario 02, original NFLX monthly low and high stock prices are compared with original NASDAQ market index monthly low and high prices.

Dependent Variable: NFLX monthly stock prices (Original data set)

Independent Variables: NASDAQ market index monthly (Original data set), GDP and CPI

Scenario 03: S&P 500 Index with MA

In scenario 03, the MA of NFLX monthly low and high stock prices are compared with the MA of S&P500 market index monthly low and high prices.

Dependent Variable: MA of NFLX monthly stock prices

Independent Variables: MA of S&P500 market index monthly, GDP and CPI

Scenario 04: NASDAQ Index with MA

In scenario 03, the MA of NFLX monthly low and high stock prices are compared with the MA of NASDAQ market index monthly low and high prices.

Dependent Variable: MA of NFLX monthly stock prices

Independent Variables: MA of NASDAQ market index monthly, GDP and CPI

The Methodologies and techniques associated with each scenario in this thesis are explained in detailed below.

4.2.1. Time frame and Time interval

For this analysis, a wider time interval is considered just than a single trading day. A weekly or monthly interval might capture a more comprehensive price movement and reduce the influence of daily noise. Hence monthly low and high price interval [NFLX_low, NFLX_high] of NFLX from 2009 to 2023 are used for this analysis. Similarly, we have downloaded monthly low and high prices for stock market indices (S&P500 and NASDAQ) for the same period. Monthly GDP, monthly CPI and monthly risk-free rates are also downloaded for the same period. These original datasets are used for the analysis in scenario 1 and 2 while MA is used as the “smoothing technique” for other two scenarios, scenario 3 and 4.

4.2.2 Smoothing Technique

It may be noted that these two extremes, i.e., high price and low price may create unnecessary noise into the calculations. These high and low values can be influenced by temporary market variations or outliers not necessarily reflecting the underlying risk of the security. However, it is a challenge to eliminate all noise from the low and high interval. To reduce this unnecessary noise impact, a smoothing technique such as "moving averages" can be applied within the interval. This may provide a more representative picture of the trading price behavior and potentially create a more informative interval for neutrosophic calculations. Moving average smooth out the price fluctuations by considering an average price over a defined period (eg. 3 months or 6 moths moving average).

Moving Average (MA) is used as the smoothing technique for scenario 03 and scenario 4 in this analysis.

Steps to calculate MA:

i) Choose a MA Window

A larger window size smooth out more noise. For this calculation monthly stock prices are taken as the data set. Hence, window size is chosen as 3 months to smooth out the noise.

ii) Calculate the MA for monthly Highs and monthly Lows

For each month, we calculate the MA for the highs by taking the average of the high prices within 3 consecutive months to create a 3-months window. Similarly, we calculate the MA for the lows by averaging the low prices within the consecutive 3-month window. Iterate through all monthly high and low data points.

iii) Create the Smoothed Interval:

Once MA is calculated for both highs and lows, the new interval for each month will be MA low for that month and MA high for that month [MA_low,MA_high]. Hence, MA interval for NFLX stock and stock market indexes can be calculated for the whole period 2009-2023.

4.2.3 Generate a neutrosophic number a+bI

Neutrosophic analysis deals with sets/intervals of data when there is some indeterminacy in data. In such cases neutrosophic analysis can be considered as a generalization of the set or interval analysis. If sets/intervals are used in the analysis and there is no indeterminacy, then neutrosophic analysis coincides with set/interval analysis. If there is some indeterminacy, no matter if using only intervals, or using sets, then neutrosophic analysis is more appropriate (Smarandache, 2015).

The stock market, by virtue of its nature, is full of vagueness, uncertainty, ambiguity, incompleteness and contradiction. Hence in this analysis, neutrosophic techniques and methods can be utilized to capture the indeterminacy.

For this study, in addition to the standard procedure, the intervals for NFLX stock prices and stock market indices are converted into the neutrosophic form. This transformation is essential because direct use of these intervals may not reduce the indeterminacy. Direct use of intervals gives the results for interval statistics. But neutrosophic numbers can represent uncertain, imprecise, incomplete, indeterminate, contradictory, and vague information. These two approaches are fundamentally different (Smarandache, 2015).

The aim of this study is to give a neutrosophic statistical approach to the CAPM or develop a neutrosophic CAPM model. The conversion from interval to neutrosophic numbers may involve subjective decisions, expert's opinion or a priori information. The applications of neutrosophic techniques help to minimise effects of the indeterminacy.

Role of Indeterminate Component (I)

"I" is a theoretical symbol used in neutrosophic statistics to denote indeterminacy. Usually, it is not a number but a placeholder indicating that there is an element of uncertainty associated with the value. Indeterminate part of the neutrosophic number "b" combined with "I" to create the indeterminacy. If "b" is greater, then the indeterminate part "bI" has a greater influence on the overall number and it indicates higher uncertainty.

Steps to convert interval statistics to neutrosophic number a+bI

Let's take the Interval as [L, U]; where "L" represents Lower Limit and "U" represent Upper Limit. We need to convert this interval format in to the Neutrosophic number =a+bI; where "I" is the indeterminacy and assume I ∈ [-1,+1].

In this study an interval [L, U] = [U, L] in the case when we do not know which one between "L"and "U" is bigger (Smarandache, 2015).

Step 01: Determine "a"

a: this determinant part is taken as the midpoint of the interval

$$a = \frac{L + U}{2} \tag{4. 6}$$

As an example, if L is 3.28 and U is 4.43, then;

$$a = \frac{3.28 + 4.43}{2} = 3.85$$

Step 02: Determine "b"

b: This indeterminate part represents the extent of the indeterminacy or the range of uncertainty. It is taken as the half the width of the interval.

$$b = \frac{U - L}{2} \tag{4. 7}$$

As an example, if L is 3.28 and U is 4.43, then;

$$b = \frac{4.43 - 3.28}{2} = 0.58$$

Step 03: write down interval [L, U] as a neutrosophic number a+bI

$$a + bI = \frac{L + U}{2} + \frac{U - L}{2}I$$

As an example, if L is 3.28 and U is 4.43, then "a" = 3.85 and "b" = 0.58. The neutrosophic numbers can be written as below.

$$a + bI = 3.85 + 0.58I$$

Similarly, the data sets of NFLX and stock market indexes in interval format can be converted into neutrosophic data sets.

4.2.4 Neutrosophic Return (NR)

The neutrosophic data set can be used to calculate NR. This NR can be represented as a neutrosophic number in the format "P+QI".

Steps to calculate the NR

Step 01: Substitute neutrosophic values to the simple return (equation 4.1)

$$\mathrm{NR_{NFLX}} = \frac{\mathrm{NP_t} - \mathrm{NP_{t-1}}}{\mathrm{NP_{t-1}}} = \mathrm{P} + \mathrm{QI}$$

(4. 8)

Where, NP_t = Neutrosophic Share price at time t and NP_{t-1} = Neutrosophic Share price at time t-1

As an example, if $NP_0 = a_0+b_0I$ and $NP_1 = a_1+b_1I$ then the result can be denoted as;

$$\mathrm{P} + \mathrm{QI} = \frac{\mathrm{a_1} + \mathrm{b_1I} - \mathrm{a_0} + \mathrm{b_0I}}{\mathrm{a_0} + \mathrm{b_0I}}$$

Step 02: Determine "P"

Multiply both sides by (a_0+b_0I) and identify the coefficients:

$$[\mathrm{P} + \mathrm{QI}][\mathrm{a_0} + \mathrm{b_0I}] = [\mathrm{a_1} + \mathrm{b_1}] - [\mathrm{a_0} + \mathrm{b_0}]\mathrm{I}$$

$$Pa_0 + [Qa_0 + Pb_0]I + b_0QI^2 = [a_1 - a_0] + [b_1 - b_0]I$$

$$a_0P + [a_0Q + b_0P + b_0Q]I = [a_1 - a_0] + [b_1 - b_0]I$$

Where, I^2 =I (Smarandache,2014). When we form an algebraic system of equations by identifying the coefficients. Therefore;

$$a_0P = a_1 - a_0$$

$$P = \frac{a_1 - a_0}{a_0}$$

Similarly Determine "Q"

$$a_0Q + b_0P + b_0Q = b_1 + b_0$$

$$[a_0 + b_0] \times Q = b_1 - b_0 - Pb_0$$

substitute the value of "P"

$$a_0[a_0 + b_0]Q = a_0[b_1 - b_0] - [a_1 - a_0]b_0$$

$$Q = \frac{a_0[b_1 - b_0] - b_0[a_1 - a_0]}{a_0[a_0 + b_0]}$$

$$Q = \frac{a_0b_1 - a_1b_0}{a_0[a_0 + b_0]}$$

Step 04: Write down the neutrosophic return

$$P + QI = \frac{a_1 - a_0}{a_0} + \frac{a_0b_1 - a_1b_0}{a_0[a_0 + b_0]}I$$

Therefore, the neutrosophic return at any time t can be introduced as

$$P + QI = \frac{a_t - a_{t-1}}{a_{t-1}} + \frac{a_{t-1}b_t - a_tb_{t-1}}{a_{t-1}[a_{t-1} + b_{t-1}]}I \tag{4. 9}$$

Where, a_t is determinant part at time t, a_{t-1} is determinant part at time t-1, b_t is indeterminant part at time t and b_{t-1} is indeterminant part at time t-1

As an example if NP_0 = (3.85+0.58 I), NP_1 = (4.58+0.52 I) where I=[-1,+1] then the neutrosophic return should be;

$$P + QI = \frac{a_1 - a_0}{a_0} + \frac{a_0b_1 - a_1b_0}{a_0[a_0 + b_0]}I$$

$$P + QI = \frac{4.58 - 3.85}{3.85} + \frac{3.85 \times 0.52 - 4.58 \times 0.58}{3.85[3.85 + 0.587]}I = 0.19 + (-0.03)I$$

Similarly, neutrosophic NFLX stock return can be calculated as a neutrosophic number "P+QI" and neutrosophic returns for stock market index can be calculated as $P^/+Q^/I$.

4.2.5 Calculations of Factors' Return

To incorporate GDP and CPI as the factors of neutrosophic models, return on GDP and CPI need to be calculated which can be seen the percentage change in GDP and CPI over time.

Return of GDP:

$$R_{CPI} = \frac{GDP_t - GDP_{t-1}}{GDP_{t-1}}$$

(4.10)

Where R_{GDP} is return on the GDP factor, GDP_t is GDP in the period t and GDP_{t-1} is GDP in the previous month-period t-1

Return of CPI:

$$R_{CPI} = \frac{CPI_t - CPI_{t-1}}{CPI_{t-1}}$$

(4.11)

Where R_{CPI} is return on the CPI factor, CPI_t is CPI in the period t and CPI_{t-1} is CPI in the previous month-period t-1

4.2.6. Obtain Neutrosophic Beta (Nβ)

In traditional CAPM, slop of the ordinary least square line (OLSL)can be used to calculate beta (β). But neutrosophic data sets need to be used instead of crisp numbers to calculate neutrosophic beta (Nβ). As an illustration neutrosophic intervals for $\sum Y$, $\sum X_1$, $\sum Y^2$, X_1^2 and $\sum X_1Y$ need to be used to calculate the Nβ for the one factor model.

Steps to calculate Nβ for one factor model

Step 01: The neutrosophic stock returns of NFLX (P+QI) are converted in to the interval format such as Y= [LL, UL], where I ∈ [-1,+1] to calculate $\sum Y$, $\sum X_1$, $\sum Y^2$,X_1^2 $\sum X_1Y$ .Here,

LL: lower limit of the return = P-Q ;(where I=-1)

(4.11)

UL: upper limit of the return = P+Q ;(where I=+1)

(4.12)

Step 02: Similarly neutrosophic returns of stock market index $P^/+Q^/I$ are converted in to interval format such as X_1= [LL, UL] where I∈ [-1,+1]

Then these intervals have been used to calculate the values for the $\sum[Y_L,Y_U]$, $\sum[X_{1L}\ X_{1U}]$, $\sum[Y_L^2, Y_U^2]$, $\sum[X_{1L}^2 , X_{1U}^2]$ and $\sum [X_{1L}Y_L,X_{1U}Y_U]$ which are also in interval format. Then Nβ for one factor model can be calculated by substituting the sums of those values to the formula (equation 4.19). The structure of the OLS table is decided by the number of factors as below. Tables for neutrosophic calculations need to be adjusted accordingly.

Factor model	Type of data required for OLS table to calculate β
1 factor model	$\sum Y$, $\sum X_1$, $\sum Y^2$, X_1^2, $\sum X_1Y$.
2 factors model	$\sum Y$, $\sum X_1$, $\sum X_2$, $\sum Y^2$, X_1^2, $\sum X_2^2$, $\sum X_1Y$, $\sum X_2Y$, $\sum X_1 X_2$
3 factors model	$\sum Y$, $\sum X_1$, $\sum X_2$, $\sum X_3$, $\sum Y^2$, X_1^2, $\sum X_2^2$, $\sum X_3^2$, $\sum X_1Y$, $\sum X_2Y$, $\sum X_3Y$, $\sum X_1 X_2$ $\sum X_1 X_3$, $\sum X_2 X_3$, $\sum X_1 X_2 X_3$, $\sum X_1 X_2Y$, $\sum X_1 X_3Y$, $\sum X_2 X_3Y$

Table 4.2.6.1: Type of data required for OLS table to calculate β for different factor models

As an example, the table for 1 factor model can be prepared as below.

	Netflix stock return		**market return**							
NO OF PERIODS	**Y**		**X1**		**Y^2**		**$X1^2$**		**X1Y**	
	LL	**UL**	**LL**	**UL**	**LL**	**UL**	**LL**	**UL**	**LL**	**UL**
1	0.167	0.080	-0.022	-0.072	0.028	0.006	0.000	0.005	-0.004	-0.006
2	0.227	0.151	-0.007	-0.046	0.052	0.023	0.000	0.002	-0.002	-0.007
3	0.142	0.131	-0.061	-0.031	0.020	0.017	0.004	0.001	-0.009	-0.004
4	0.133	0.102	-0.011	-0.021	0.018	0.010	0.000	0.000	-0.001	-0.002
5	0.017	0.052	0.057	0.021	0.000	0.003	0.003	0.000	0.001	0.001
174	0.115	0.093	0.032	0.030	0.013	0.009	0.001	0.001	0.004	0.003
175	0.095	0.112	0.031	0.031	0.009	0.013	0.001	0.001	0.003	0.003
176	0.070	0.030	0.024	0.024	0.005	0.001	0.001	0.001	0.002	0.001
177	-0.017	0.003	0.005	0.005	0.000	0.000	0.000	0.000	0.000	0.000
178	-0.056	-0.048	-0.019	-0.017	0.003	0.002	0.000	0.000	0.001	0.001
179	0.016	0.028	0.000	-0.001	0.000	0.001	0.000	0.000	0.000	0.000
180	0.063	0.035	0.021	0.020	0.004	0.001	0.000	0.000	0.001	0.001
total	**5.794**	**5.534**	**1.751**	**1.516**	**1.819**	**1.599**	**0.108**	**0.090**	**0.169**	**0.117**
	$\sum Y$		**$\sum X1$**		**$\sum Y^2$**		**$\sum X1^2$**		**$\sum X1Y$**	

Table 4.2.6.2: NLS table for one-factor model with respect to S&P 500 market index.

Similarly, NLS tables need to be prepared for 2-factor and 3-factor models to calculate the neutrosophic beta values.

4.3. Application of Neutrosophic Methods to Calculate Neutrosophic Beta

A neutrosophic model is a model, which considers indeterminacy in data. We may not appropriately construct an accurate classical model when the data set which describe the physical world is incomplete, ambiguous, contradictory and unclear. In this case, to study these kinds of environments, we need to build an approximate model. Neutrosophic statistics helps to plot the incomplete, unclear ambiguous data and then we can design a neutrosophic regression method. The neutrosophic linear regression and the neutrosophic least squares regression are the most common such methods. In this study both methods are used to capture the indeterminacy (Smarandache, 2015).

(i) General One Factor model

Simple linear regression equation with one independent variable x_1;

$$y = \beta_0 + \beta_1 x_1 + \varepsilon \tag{4.13}$$

We can rewrite this equation as

$$\varepsilon = y - \beta_0 - \beta_1 x_1 \tag{4.14}$$

Minimize the sum of squared residuals RSS for the given sample of size N.

$$\sum \varepsilon^2 = \sum (y - \beta_0 - \beta_1 x_1)^2 \tag{4.15}$$

Then partially differentiate the RSS function with respect to β_0 and β_1. For the minimum of RSS function, we take partial derivatives equal to zero.

Then two OLS normal equation can be written as,

$$\sum y = n\beta_0 + \beta_1 \sum x_1 \tag{4.16}$$

$$\sum x_1 y = \beta_0 \sum x_1 + \beta_1 \sum {x_1}^2 \tag{4.17}$$

By solving these two equations, the slop of the simple linear regression model can be obtained as below and it is the β_1 for the one variable in classical model. (Abraham & Ledolter, 2006)

$$\beta_1 = \frac{\sum x_1 y - \frac{\sum x_1 \sum y}{n}}{\sum x_1^2 - \frac{(\sum x_1)^2}{n}} \tag{4.18}$$

But in this study neutrosophic data sets are used. Therefore, the classical formula needs to be adjusted according to the neutrosophic statistical methodologies to calculate Neutrosophic beta as $[N\beta_{iL,}\ N\beta_{iU}\ ]$.To calculate this interval of $N\beta_1$, x_1 and y need to be converted into the neutrosophic intervals.

x_1 can be represent as [$x_{1lower,}$ x_{1upper}] which is [$x_{1L,}$ x_{1U}]

y can be represent as [y_{lower},y_{upper}] which is [$y_{L,}$ y_U]

For the one factor neutrosophic model, $N\beta_1$ can be represent as $[N\beta_{1L,}N\beta_{1U}\ ]$ and with the use of equation 4.18 the new formula for this $[N\beta_{1L,}N\beta_{1U}\ ]$ can be written as below. Values of the $\sum[Y_L,Y_U]$, $\sum[X_{1L}\ X_{1U}]$, $\sum[Y_L^2,\ Y_U^2]$, $\sum[X_{1L}^2\ ,\ X_{1U}^2]$ and $\sum\ [X_{1L}Y_L,X_{1U}Y_U]$ need to be calculated using NLS table and substitute to the below formula to find out this neutrosophic beta. All the calculations need to be done according to the neutrosophic logic (Smarandache, F. ,2014).

$$([\mathrm{N\beta_{1L}}, \mathrm{N\beta_{1U}}]) = \frac{\sum[\mathrm{x_{1L}}, \mathrm{x_{1U}}][\mathrm{y_L}, \mathrm{y_U}] - \frac{(\sum[\mathrm{x_{1L}}, \mathrm{x_{1U}}]\sum[\mathrm{y_L}, \mathrm{y_U}])}{\mathrm{n}}}{\sum[\mathrm{x_{1L}}, \mathrm{x_{1U}}]^2 - \frac{(\sum[\mathrm{x_{1L}}, \mathrm{x_{1U}}])^2}{\mathrm{n}}}$$

(4.19)

As an illustration, we can use the NLS results in table 4.3.5.1 to calculate $N\beta_1$ of 1-factor model. Here we must follow the neutrosophic calculations. (Smarandache, F. ,2014 page no:79&80)

$$([N\beta_{1L}, N\beta_{1U}]) = \frac{[0.169,0.117] - \frac{([1.751,1.516] \times [5.794,5.534])}{180}}{[0.108,0.090] - \frac{([1.751,1.516])^2}{180}}$$

$$([N\beta_{1L}, N\beta_{1U}]) = \frac{[0.169,0.117] - [0.0564,0.0466]}{[0.108,0.090] - [0.017,0.0128]}$$

$$([N\beta_{1L}, N\beta_{1U}]) = \frac{[0.1225,0.0609]}{[0.0951,0.0729]}$$

$$([N\beta_{1L}, N\beta_{1U}]) = [0.6403,1.6805]$$

(ii)<u>Two factor model</u>

The OLS multiple linear regression equation with two independents variable x_1 and x_2

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$$

(4.20)

where y is dependent variable, x_1 and x_2 are independent variables, β_0 is intercept, β_1 and β_2 are coefficients of independent variables and ε is error term. Then,

$$\varepsilon = \mathrm{y} - \beta_0 - \beta_1 \mathrm{x}_1 - \beta_2 \mathrm{x}_2$$

(4.21)

Minimize the sum of squared residuals RSS for the given sample of size n.

$$\sum \varepsilon^2 = \sum (y - \beta_0 - \beta_1 x_1 - \beta_2 x_2)^2$$

(4.22)

Then partially differentiate the RSS function with respect to β_0, β_1 and β_2. For the minimum of RSS function, we set partial derivatives equal to zero.

Then three OLS normal equation can be written as;

$$\sum y = n\beta_0 + \beta_1 \sum x_1 + \beta_2 \sum x_2$$

(4.23)

$$\sum x_1 y = \beta_0 \sum x_1 + \beta_1 \sum {x_1}^2 + \beta_2 \sum x_1 x_2$$

(4.24)

$$\sum x_2 y = \beta_0 \sum x_2 + \beta_1 \sum x_1 x_2 + \beta_2 \sum {x_2}^2$$

(4.25)

The OLS normal equations constitute three linear equations with three unknowns. Solving these equations give β coefficients as below (Abraham, B., and Ledolter, J., 2006).

β_1 coefficient

$$\beta_1 = \frac{\sum x_2^2 \sum x_1 y - [\sum x_1 x_2 \sum x_2 y]}{\sum x_1^2 \sum x_2^2 - [\sum x_1 x_2]^2}$$

(4.26)

β_2 coefficient

$$\beta_2 = \frac{\sum x_1^2 \sum x_2 y - [\sum x_1 x_2 \sum x_1 y]}{\sum x_1^2 \sum x_2^2 - [\sum x_1 x_2]^2}$$

(4.27)

Where;

$$\sum x_1^2 = \sum X_1^2 - \frac{\sum X_1 \sum X_1}{N}$$

(i)

$$\sum x_2^2 = \sum X_2^2 - \frac{\sum X_2 \sum X_2}{N}$$

(ii)

$$\sum x_1 y = \sum X_1 Y - \frac{\sum X_1 \sum Y}{N} \quad \text{(iii)}$$

$$\sum x_2 y = \sum X_2 Y - \frac{\sum X_2 \sum Y}{N} \quad \text{(iv)}$$

$$\sum x_1 x_2 = \sum X_1 X_2 - \frac{\sum X_1 \sum X_2}{N} \quad \text{(v)}$$

This adjustment centers the variables around their means, and it is removing the effects of the average values of x and y variables.

Similar to the one factor model, Nβ value for 2-factor model needs to be calculated using neutrosophic logic.

Now x_1, x_2 and y need to be change into the intervals.

x_1 can be represent as [x_{1lower}, x_{1upper}] which is [x_{1L}, x_{1U}]

x_2 can be represent as [x_{2lower}, x_{2upper}] which is [x_{2L}, x_{2U}]

y can be represent as [y_{lower},y_{upper}] which is [y_L, y_U]

There are two betas for the two-factor model. Those neutrosophic betas are [$N\beta_{1L}$,$N\beta_{1U}$] and [$N\beta_{2L}$,$N\beta_{2U}$].Values of the $\sum[Y_L,Y_U]$, $\sum[X_{1L}\ X_{1U}]$, $\sum[X_{2L}\ X_{2U}]$, $\sum[Y_L^2, Y_U^2]$, $\sum[X_{1L}^2 , X_{1U}^2]$, $\sum[X_{2L}^2 , X_{2U}^2]$, $\sum [X_{1L}Y_L,X_{1U}Y_U]$, $\sum [X_{2L}Y_L,X_{2U}Y_U]$ and $\sum [X_{1L}X_{2L},X_{1U}X_{2U}]$ need to be calculated using OLS table and substitute to the below formulas to find out these two neutrosophic betas. All the calculations need to be done according to the neutrosophic logic. Therefore, with the use of equations 4.26 and 4.27, Nβs for two variables can be written as below.

$N\beta_1$ coefficient :

$$([N\beta_{1L}, N\beta_{1U}])$$

$$= \frac{\sum[x_{2L}, x_{2U}]^2 \sum[x_{1L}, x_{1U}][y_L, y_U] - (\sum[x_{1L}, x_{1U}][x_{2L}, x_{2U}] \sum[x_{2L}, x_{2U}][y_L, y_U])}{\sum[x_{1L}, x_{1U}]^2 \sum[x_{2L}, x_{2U}]^2 - (\sum[x_{1L}, x_{1U}] [x_{2L}, x_{2U}])^2}$$

(4.28)

$N\beta_2$ coefficient :

$$([N\beta_{2L}, N\beta_{2U}])$$

$$= \frac{\sum[x_{1L}, x_{1U}]^2 \sum[x_{2L}, x_{2U}][y_L, y_U] - (\sum[x_{1L}, x_{1U}][x_{2L}, x_{2U}] \sum[x_{1L}, x_{1U}][y_L, y_U])}{\sum[x_{1L}, x_{1U}]^2 \sum[x_{2L}, x_{2U}]^2 - (\sum[x_{1L}, x_{1U}] [x_{2L}, x_{2U}])^2}$$

(4.29)

Where,

$$\sum[x_{1L}, x_{1U}]^2 = \sum[X_{1L}, X_{1U}]^2 - \frac{\sum[X_{1L}, X_{1U}] \sum[X_{1L,}X_{1U}]}{N}$$ (i-a)

$$\sum[x_{2L}, x_{2U}]^2 = \sum[X_{2L}, X_{2U}]^2 - \frac{\sum[X_{2L}, X_{2U}] \sum[X_{2L,}X_{2U}]}{N}$$ (ii-a)

$$\sum[x_{1L}, x_{1U}][y_L, y_U] = \sum[X_{1L}, X_{1U}][Y_L, Y_U] - \frac{\sum[X_{1L}, X_{1U}] \sum[Y_{L,}Y_U]}{N}$$ (iii-a)

$$\sum[x_{2L}, x_{2U}][y_L, y_U] = \sum[X_{2L}, X_{2U}][Y_L, Y_U] - \frac{\sum[X_{2L}, X_{2U}] \sum[Y_{L,}Y_U]}{N}$$ (iv-a)

$$\sum[x_{1L}, x_{1U}][x_{2L}, x_{2U}] = \sum[X_{1L}, X_{1U}][X_{2L}, X_{2U}] - \frac{\sum[X_{1L}, X_{1U}] \sum[X_{2L,}X_{2U}]}{N}$$ (v-a)

This adjustment centers the variables around their means, and it is removing the effects of the average values of X and Y variables. After substituting the data all the calculations need to be done as per the neutrosophic logic ((Nagarajan et al., 2021)).

(iii)<u>Three Factor model</u>

The OLS multiple linear regression equation with three independents variables x_1, x_2 and x_3

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon$$

Where y is dependent variable, x_1 ,x_2,x_3 are independent variables ,β_0 is intercept,β_1, β_2, β_3 are the coefficients of independent variables and ε is the error term. Then

$$\varepsilon = y - \beta_0 - \beta_1 x_1 - \beta_2 x_2 - \beta_3 x_3$$

Minimize the sum of squared residuals RSS for the given sample of size N.

$$\sum \varepsilon^2 = \sum (y - \beta_0 - \beta_1 x_1 - \beta_2 x_2 - \beta_3 x_3)^2 \tag{4.30}$$

Then partially differentiate the RSS function with respect to β_0, β_1 , β_2 and β_3. For the minimum of RSS function, partial derivatives need to be equal to zero.

Then four OLS normal equations can be written as,

$$\sum y = n\beta_0 + \beta_1 \sum x_1 + \beta_2 \sum x_2 + \beta_3 \sum x_3 \tag{4.31}$$

$$\sum x_1 y = \beta_0 \sum x_1 + \beta_1 \sum {x_1}^2 + \beta_2 \sum x_1 x_2 + \beta_3 \sum x_1 x_3 \tag{4.32}$$

$$\sum x_2 y = \beta_0 \sum x_2 + \beta_1 \sum x_1 x_2 + \beta_2 \sum {x_2}^2 + \beta_3 \sum x_2 x_3 \tag{4.33}$$

$$\sum x_3 y = \beta_0 \sum x_3 + \beta_1 \sum x_1 x_3 + \beta_2 \sum x_2 x_3 + \beta_3 \sum {x_3}^2 \tag{4.34}$$

These OLS standard equations constitute four linear equations with four unknowns. β coefficients can be obtained by solving these four equations (Abraham & Ledolter, 2006; PennState:Stat 501, Lesson 05, 2023).

Simplified symbols are used to reduce the difficulty of handling the complex formulas, as mentioned below.

$$\beta_1 = \frac{S_{X_2X_2}S_{X_3X_3}S_{X_1Y} - S_{X_2X_2}S_{X_1X_3}S_{X_3Y} + S_{X_1X_2}S_{X_3X_3}S_{X_2Y} - S_{X_1X_2}S_{X_2X_3}S_{X_3Y} - S_{X_1X_1}S_{X_3X_3}S_{X_2Y} + S_{X_1}S_{X_2X_3}S_{X_2X_3Y}}{S_{X_1X_1}S_{X_2X_2}S_{X_3X_3} - [S_{X_1X_1}S_{X_2X_3}]^2 + S_{X_1}S_{X_2X_3}S_{X_1}S_{X_2X_3} - S_{X_1}S_{X_1X_2}S_{X_3X_3}S_{X_3Y}} \tag{4.35}$$

$$\beta_2 = \frac{S_{X_1X_1}S_{X_3X_3}S_{X_2Y} - S_{X_1X_1}S_{X_2X_3}S_{X_3Y} + S_{X_1X_2}S_{X_3X_3}S_{X_1Y} - S_{X_1X_2}S_{X_1X_3}S_{X_3Y} - S_{X_2X_2}S_{X_3X_3}S_{X_1Y} + S_{X_2}S_{X_1X_3}S_{X_1X_3Y}}{S_{X_1X_1}S_{X_2X_2}S_{X_3X_3} - [S_{X_1X_1}S_{X_2X_3}]^2 + S_{X_1}S_{X_2X_3}S_{X_1}S_{X_2X_3} - S_{X_1}S_{X_1X_2}S_{X_3X_3}S_{X_3Y}} \tag{4.36}$$

$$\beta_3 = \frac{S_{X_1X_1}S_{X_2X_2}S_{X_3Y} - S_{X_1X_1}S_{X_2X_3}S_{X_2Y} + S_{X_1X_3}S_{X_2X_2}S_{X_1Y} - S_{X_1X_2}S_{X_1X_3}S_{X_2Y} - S_{X_2X_2}S_{X_3X_3}S_{X_1Y} + S_{X_3}S_{X_1X_2}S_{X_1X_2Y}}{S_{X_1X_1}S_{X_2X_2}S_{X_3X_3} - [S_{X_1X_1}S_{X_2X_3}]^2 + S_{X_1}S_{X_2X_3}S_{X_1}S_{X_2X_3} - S_{X_1}S_{X_1X_2}S_{X_3X_3}S_{X_3Y}} \tag{4.37}$$

Where $S_{X_1} = \sum x_1$, $S_{X_2} = \sum x_2$, $S_{X_3} = \sum x_3$, $S_{X_1X_1} = \sum x_1^2$, $S_{X_1X_2} = \sum x_1 x_2$, $S_{X_2X_2} = \sum x_2^2$, $S_{X_1X_3} = \sum x_1 x_3$, $S_{X_2X_3} = \sum x_2 x_3$, $S_{X_3X_3} = \sum x_3^2$, $S_{X_1Y} = \sum x_1 y$, $S_{X_2Y} = \sum x_2 y$, $S_{X_3Y} = \sum x_3 y$, $S_{X_1X_2Y} = \sum x_1 x_2 y$, $S_{X_2X_3Y} = \sum x_2 x_3 y$, $S_{X_1X_3Y} = \sum x_1 x_3 y$

We can remove the effects of the average values of x and y variables using the equations (i), (ii), (iii), (iv),(v) and

$$\sum x_3^2 = \sum X_3^2 - \frac{\sum X_3 \sum X_3}{N} \qquad \text{(vi)}$$

$$\sum x_3 y = \sum X_3 Y - \frac{\sum X_3 \sum Y}{N} \qquad \text{(vii)}$$

$$\sum x_1 x_3 = \sum X_1 X_3 - \frac{\sum X_1 \sum X_3}{N} \qquad \text{(viii)}$$

$$\sum x_2 x_3 = \sum X_2 X_3 - \frac{\sum X_2 \sum X_3}{N} \qquad \text{(ix)}$$

$$\sum x_1 x_3 y = \sum X_1 X_3\, Y - \frac{\sum X_1 \sum X_3 \sum Y}{N} \qquad \text{(x)}$$

$$\sum x_2 x_3 y = \sum X_2 X_3\, Y - \frac{\sum X_2 \sum X_3 \sum Y}{N} \qquad \text{(xi)}$$

For the use of neutrosophic models, βs need to be adjusted into Nβs using neutrosophic logic.

Now x_1, x_2, x_3 and y need to be change into the intervals.

x_1 can be represent as [x_{1lower}, x_{1upper}] which is [x_{1L}, x_{1U}]

x_2 can be represent as [x_{2lower}, x_{2upper}] which is [x_{2L}, x_{2U}]

x_2 can be represent as [x_{2lower}, x_{2upper}] which is [x_{2L}, x_{2U}]

x_3 can be represent as [x_{3lower}, x_{3upper}] which is [x_{3L}, x_{3U}]

y can be represent as [y_{lower},y_{upper}] which is [y_L, y_U]

According to this analysis [$N\beta_{1L}$,$N\beta_{1U}$], [$N\beta_{2L}$,$N\beta_{2U}$] and [$N\beta_{3L}$,$N\beta_{3U}$] can be written down by adjusting the formulas for β_1 β_2 and β_3 (Smrandache, F.,2014).As an illustration $N\beta_1$ can be represent as below using the equation 4.35.

$$
\begin{aligned}
&[N\beta_{1L}, N\beta_{1U}] \\
&= \Big[\Big(S_{[X_{2L},X_{2U}][X_{2L},X_{2U}]} S_{[X_{3L},X_{3U}][X_{3L},X_{3U}]} S_{[X_{1L},X_{1U}][Y_L,Y_U]}\Big) \\
&- \big(S_{[X_{2L},X_{2U}][X_{2L},X_{2U}]} S_{[X_{1L},X_{1U}][X_{3L},X_{3U}]} S_{[X_{3L},X_{3U}][Y_L,Y_U]}\big) \\
&+ \Big(S_{[X_{1L},X_{1U}][X_{2L},X_{2U}]} S_{[X_{3L},X_{3U}][X_{3L},X_{3U}]} S_{[X_{2L},X_{2U}][Y_L,Y_U]}\Big) \\
&- \Big(S_{[X_{1L},X_{1U}][X_{2L},X_{2U}]} S_{[X_{2L},X_{2U}][X_{3L},X_{3U}]} S_{[X_{3L},X_{3U}][Y_L,Y_U]}\Big) \\
&- \big(S_{[X_{1L},X_{1U}][X_{1L},X_{1U}]} S_{[X_{3L},X_{3U}][X_{3L},X_{3U}]} S_{[X_{2L},X_{2U}][Y_L,Y_U]}\big) \\
&+ \Big(S_{[X_{1L},X_{1U}]} S_{[X_{2L},X_{2U}][X_{3L},X_{3U}]} S_{[X_{2L},X_{2U}][X_{3L},X_{3U}][Y_L,Y_U]}\ \Big)\Big] \\
&/ \Big[\big(S_{[X_{1L},X_{1U}][X_{1L},X_{1U}]} S_{[X_{2L},X_{2U}][X_{2L},X_{2U}]} S_{[X_{3L},X_{3U}][X_{3L},X_{3U}]}\big) - \Big(S_{[X_{1L},X_{1U}[X_{1L},X_{1U}} S_{[X_{2L},X_{2U}][X_{3L},X_{3U}]}\Big)^2 \\
&+ \Big(S_{[X_{1L},X_{1U}]} S_{[X_{2L},X_{2U}][X_{3L},X_{3U}]} S_{[X_{1L},X_{1U}]} S_{[X_{2L},X_{2U}][X_{3L},X_{3U}]}\Big) \\
&- \Big(S_{[X_{1L},X_{1U}} S_{[X_{1L},X_{1U}][X_{2L}X_{2U}]} S_{[X_{3L},X_{3U}][X_{3L},X_{3U}]} S_{[X_{3L},X_{3U}][Y_L,Y_U]}\Big)\Big]
\end{aligned}
$$

(4.38)

Similarly, the formulas for Nβs can be obtained for $[N\beta_{2L,} N\beta_{2U}]$ and $[N\beta_{3L,} N\beta_{3U}]$ by applying neutrosophic techniques to the equations 4.36 and 4.37 respectively. All the calculations need to follow neutrosophic logic.

Below adjustment can be done to center the variables around their means, and it remove the effect of the average values of x's and y variables.

$$\sum [x_{3L}, x_{3U}]^2 = \sum [X_{3L}, X_{3U}]^2 - \frac{\sum[X_{3L}, X_{3U}] \sum[X_{3L,} X_{3U}]}{N}$$

(vi-a)

$$\sum [x_{3L}, x_{3U}][y_L, y_U] = \sum [X_{3L}, X_{3U}][Y_L, Y_U] - \frac{\sum[X_{3L}, X_{3U}] \sum[Y_{L,} Y_U]}{N}$$

(vii-a)

$$\sum [x_{1L}, x_{1U}][x_{3L}, x_{3U}] = \sum [X_{1L}, X_{1U}][X_{3L}, X_{3U}] - \frac{\sum[X_{1L}, X_{1U}] \sum[X_{3L,} X_{3U}]}{N}$$

(vii-a)

$$\sum [x_{2L}, x_{2U}][x_{3L}, x_{3U}] = \sum [X_{2L}, X_{2U}][X_{3L}, X_{3U}] - \frac{\sum[X_{2L}, X_{2U}] \sum[X_{3L,} X_{3U}]}{N}$$

(viii-a)

$$\sum[x_{1L}, x_{1U}][x_{3L}, x_{3U}][y_L, y_U] = \sum[X_{1L}, X_{1U}][X_{3L}, X_{3U}]\,[y_L, y_U] - \frac{\sum[X_{1L}, X_{1U}]\sum[X_{3L,}X_{3U}]\sum[y_L, y_U]}{N}$$

(x-a)

$$\sum[x_{2L}, x_{2U}][x_{3L}, x_{3U}][y_L, y_U] = \sum[X_{2L}, X_{2U}][X_{3L}, X_{3U}][y_L, y_U] - \frac{\sum[X_{2L}, X_{2U}]\sum[X_{3L,}X_{3U}]\sum[y_L, y_U]}{N}$$

(xi-a)

4.4 Multi-Factor Models

Multi-factor models can be developed to capture the sensitivity of the stock market by addressing the factors, which affect for the trend. In this study, we consider Market Index, CPI and GDP as the factors, which affect for the share price of the NFLX stock. Based on general CAPM and APT models, neutrosophic one-factor, two-factor and three-factor models are developed to observe the relationships and impacts of each variable on the NFLX stock.

4.4.1Neutrosophic Capital Asset Pricing Model (NCAPM)

The main objective of this study is to develop a neutrosophic statistical approach to the CAPM to address the uncertainty in the market. This analysis starts by converting stock prices and stock market indices into neutrosophic numbers. Traditional methods often fail to capture the indeterminacy of the market due to the complex interplay of several factors such as economic indicators, unpredictable investor's behaviors, political impacts and market sentiment. Hence this attempt is to capture these indeterminacies by applying neutrosophic methods to build a better model which can address this complexity. Application of neutrosophic methods into this study of stock market offers a strong approach to address the unpredictable nature of the stock market. This method develops the ability to model and understand the complicated financial data, leading to more informed and consistent investment strategies. The new model can be used to calculate the range of neutrosophic expected return of a stock and this interval is useful to get a better understanding of the uncertainty of the stock market. Furthermore, this interval captures the unpredictable stock market behaviours and fluctuations while offering a lower limit and upper limit for these fluctuations. These two limits give a control over the risk. Hence investors may be aware of the worst situation as well as the best-case scenario.

4.4.2 Neutrosophic Arbitrage Pricing Theory (NAPT)

APT is widely used in practice as a tool for estimating expected asset returns and their covariance matrix. If market participants can identify the factors, which actually affect asset returns, they can use APT to accurately estimate the expected asset returns. APT allows investors to form a better portfolio with simplified assumptions. There is an intellectual arms race to find the best portfolio strategies to outperform competitors. Hence this study aims to propose extended NCAPM to capture the risk associated with macroeconomic factors like CPI and GDP. with the purpose of offering greater flexibility on capturing the volatility or risk of a stock. Accordingly, we focus to develop 2-factor and 3-factor neutrosophic APT models. These new models can be introduced as extended NCAPM or NAPT models.

4.4.3. Development of advanced Neutrosophic Statistical Factor Models

Four different advanced Neutrosophic Statistical Factor Models have been developed in this study. Independent variables can be considered as Market Index, GDP and CPI.

Model 01: one factor model

Factor 01: Market Index (S&P500 /NASDAQ)

This is the NCAPM which explains the relationship between the neutrosophic expected return of a stock and neutrosophic expected return of the market index.

$$NE(R_{iL,}R_{iU}) \; = \; R_f + \; [N\beta_{1L,}N\beta_{1U}][NE(R_{ML\,,}R_{MU}) \; - \; R_f] \tag{4.39}$$

Model 02: Two factors

Factor 01: Market index (S&P500 /NASDAQ)

Factor 02: GDP

This is an extended NCAPM (or NAPT), which explains the relationship between the neutrosophic expected return of a stock with neutrosophic expected return of market index and GDP.

$$NE\left(R_{iL,}R_{iU}\right) = \; R_f + \; [N\beta_{1L,}N\beta_{1U}][NE(R_{ML\,,}R_{MU}) \; - \; R_f] + [N\beta_{2L,}N\beta_{2U}]\,[E(R_{GDP}) \; - \; R_f] \tag{4.40}$$

Model 03: Two factors

Factor 01: Market index (S&P500 /NASDAQ)

Factor 02: CPI

This is an extended NCAPM (or NAPT), which explains the relationship between the neutrosophic expected return of a stock with neutrosophic expected return of market index and CPI.

$$NE\left(R_{iL,}R_{iU}\right) = \; R_f + \; [N\beta_{1L,}N\beta_{1U}][NE(R_{ML\,,}R_{MU}) \; - \; R_f] + [N\beta_{3L,}N\beta_{3U}]\,[E(R_{CPI}) \; - \; R_f] \tag{4.41}$$

Model 04: Three factors

Factor 01: Market index (S&P500 /NASDAQ)

Factor 02: GDP

Factor 03: CPI

This is an extended NCAPM (or NAPT), which explains the relationship between the neutrosophic expected return of a stock with neutrosophic expected return of market index, GDP and CPI.

$$NE(R_{iL,}R_{iU}) = R_f + [N\beta_{1L,}N\beta_{1U}][NE(R_{ML\,,}R_{MU}) - R_f] + [N\beta_{2L,}N\beta_{2U}][E(R_{GDP}) - R_f] + [N\beta_{3L,}N\beta_{3U}][E(R_{CPI}) - R_f] \tag{4.42}$$

Where, $NE(R_{iL,}R_{iU})$ is neutrosophic expected return of the stock,$NE(R_{ML\,,}R_{MU})$ is neutrosophic expected return of the market,$NE[(R_{ML\,,}R_{MU})$ -R_f] is neutrosophic market risk premium, [E(R_{GDP}) -R_f] is GDP risk premium, [E(R_{CPI})-R_f] is CPI risk premium, $[N\beta_{1L,}N\beta_{1U}]$ is neutrosophic beta for market returns,$[N\beta_{2L,}N\beta_{2U}]$ is neutrosophic beta for GDP and $[N\beta_{3L,}N\beta_{3U}]$ is neutrosophic beta for CPI.

All these four models are analysed in each of four scenarios as illustrated by table 4.4.3.1. (S&P 500 Index with original data, NASDAQ Index with original data, S&P 500 Index with MA and NASDAQ Index with MA)

Model	Factors
Model 01	Market Index
Model 02	Market Index and GDP
Model 03	Market Index and CPI
Model 04	Market Index, GDP and CPI

Table 4.4.3.1: Different Factor models

In this study, these different neutrosophic factor models are used to calculate neutrosophic expected returns of NFLX stocks based on different factors, which are affecting stock market behaviour. By integrating these models, investors can improve their strategies, carefully manage risks, and make wise investment decisions to enhance their market portfolios. All these model calculations follow neutrosophic methods and techniques.

4.5. Neutrosophic Correlation Coefficient

The classical correlation coefficient is a crisp number between [-1, 1]. The neutrosophic correlation coefficient is a subset of the interval [-1, 1].

- If the subset of the neutrosophic correlation coefficient is in the positive side of the interval [-1,1], the neutrosophic variables x and y have a neutrosophic positive correlation.
- If the subset of the neutrosophic correlation coefficient is in the negative side of the interval [-1, 1], they have a neutrosophic negative correlation (Smarandache, 2015).

In this study correlation coefficients are calculated to check the collinearity of below pairs; between Market Index and GDP/between Market Index and CPI/between GDP and CPI

4.6. Statistical Hypothesis testing technique

<u>T-statistic</u>

Hypothesis testing technique is used for one factor model and t-statistic has been used as the hypothesis test to measure the significance of the regression coefficient in this simple linear model. This technique is used to test the hypothesis related to the linear relationship between the dependent variable and the independent variable. Since the regression (slope) coefficient follows the t-distribution under the assumptions of the linear model, T-test is used in this analysis to determine if the predictor variable is statistically significant in the model. A statistically significant variable has a strong relationship with the dependent variable and contributes significantly to the accuracy of the model. T-statistic can be written as;

$$T = \frac{\hat{\beta}_1}{S \Big/ (S_{X_1X_1})^{\frac{1}{2}}} \tag{4.43}$$

where, S^2 is $\sum \frac{\varepsilon^2}{n-2}$ = standard error of the estimate, $\hat{\beta}_1$ is the estimated regression coefficient for the predictor variable, $\sum \varepsilon^2$ is sum of squared errors, n is number of observations and n-2 is Degree of freedom.

The significance of relationship between response and predictor variable can be determined by the hypothesis testing.

Null Hypothesis (H_0): The null hypothesis states that there is no relationship between the predictor variable and the response variable. Thus, in terms of the regression coefficient, the null hypothesis is β_1=0.

Alternative Hypothesis (H_a): The alternative hypothesis contradicts the null hypothesis. This suggests that there is relationship between the independent and the dependent variable. It implies that the regression coefficient $\beta_1 \neq 0$.

The observed t-statistic is compared to a critical value from the t-distribution at a given significance level. If the absolute value of the observed t-statistic > critical value, reject the null hypothesis at the chosen significance level. We conclude that there is evidence about the relationship between the independent variable and the dependent variable. If the absolute value of the t-statistic < critical value, do not reject the null hypothesis. We conclude that there is no evidence that the fitted linear model is significant.

4.6.1.T- statistic for the neutrosophic models

In this study t-statistic is used only for the one factor neutrosophic model (equation 4.39) because calculate t-statistic for the neutrosophic models which has more than one factors are more complicated. Hence the equation 4.43 can be rewritten to calculate the t statistics for the one-factor neutrosophic model as below.

$$(T_{LL}, T_{UL}) = \frac{[\hat{\beta}_{1L}, \hat{\beta}_{1U}]}{[S_{LL}, S_{UL}] \Big/ \left(S_{X_{1L,1U}, X_{1L,1U}}\right)^{\frac{1}{2}}} \tag{4.44}$$

According to this study the Neutrosophic Null Hypothesis (NH_0) and the Neutrosophic Alternative Hypothesis (NH_a) are two possible conclusions which is very similarly to the classical statistics (Smarandache, F. ,2014).

Null hypothesis; NH_0: $N\beta_1 \in [N\beta_{1L}, N\beta_{1U}]$

Alternative hypothesis; NH_a: $N\beta_1 < N\beta_{1L}$, or NH_a: $N\beta_1 > N\beta_{1U}$, or NH_a: $N\beta_1 \notin [N\beta_{1L}, N\beta_{1U}]$

Level of Significance α_N of a neutrosophic study not necessarily a crisp number as in classical statistics and it may be an interval. For this neutrosophic study we will assume the set of asymptotic significance level, [0.95 ,0.99], which implies the set α_N= [0.01 ,0.05]. (Villafuerte et al., 2020).

The decision criterion is rejected NH_0 if,

$$\mathrm{Min}\{[\mathrm{t} - \mathrm{statistic}] - [\mathrm{ctitical\ t\ value(T)}]\} > \mathrm{Max}\{\mathrm{T}(1 - \alpha_\mathrm{N})\}.$$

This means, if the minimum value of difference between the test statistic and the critical value($\mathrm{Min}\{[\mathrm{t} - \mathrm{statistic}] - [\mathrm{ctitical\ t\ value(T)}]\}$) is greater than the maximum critical value($\mathrm{Max}\{\mathrm{T}(1 - \alpha_\mathrm{N})\}$), then there is a clear evidence against the null hypothesis to reject it.

4.7. Comparison of the models using Mean Average Deviation (MAD)

To compare the different models, we calculate the MAD as a major of model accuracy. The smaller MAD value implies that the model is more accurate and better. MAD value can be calculated by the below formula ([48],[53]):

$$\mathrm{MAD} = \sum \left| \frac{\mathrm{R_i} - \mathrm{E(R_i)}}{\mathrm{n}} \right| \tag{4.45}$$

Hence below formula has been used to calculate the MAD for this neutrosophic study.

$$\mathrm{NMAD} = \sum \left| \frac{[\mathrm{R_{iL}}, \mathrm{R_{iU}}] - [\mathrm{NE}(\mathrm{R_{iL}}, \mathrm{R_{iu}})]}{\mathrm{n}} \right| \tag{4.46}$$

Chapter 05

Results and Discussion

5.0. Introduction to Results and Discussion

The behaviour of a particular stock in the stock market depends upon several known and unknown factors and is full of uncertainty. The Neutrosophic approaches provide advanced techniques to control the indeterminacy and uncertainty in the stock market. Based on these models, investors and financial analysts can improve risk assessment and their decision-making in complex financial markets. In this chapter, we show the results for the proposed neutrosophic models by considering the NFLX, AMZN and APPL stock returns.

5.1. NFLX monthly β and E(Ri)

From the classical CAPM analysis, the results of beta and expected return for NFLX stock are given below in Table 5.1.1. and Table 5.1.2. for the S&P 500 and NASDAQ indices respectively for the periods of 15-years and 5-years.

SP&500	15-years	5-years
β	1.199	1.512
E(Ri)	12.20%	16.66%

Table 5.1.1 β and E(Ri) of NFLX with respect to S&P500.

NASDAQ	15-years	5-years
β	1.360	1.613
E(Ri)	19.30%	22.70%

Table 5.2. β and E(Ri) of NFLX with respect to NASDAQ.

5.1.1 The result and discussion for the Monthly beta of NFLX stocks with S&P 500 Index

Using 15 years of historical data from 2009-2023, monthly beta of NFLX stock with respect to S&P500 market index is 1.199. This indicates that if the market price goes up by 1-unit, the NFLX stock price is expected to go up by 1.199-units, and if the market price goes down by 1-unit, the NFLX stock price is expected to go down by 1.199-units. Further, E(Ri) is 12.20% using classical CAPM. With respect to S&P500 index, NFLX shows a moderate risk and balance return for 15-year period. But for 5-year period from 2019-2023, monthly beta 1.512 indicates higher volatility, and the E(Ri) is 16.66% which is comparatively higher than the 15-year period. This indicates that if the market increase/decrease by 1-unit, the NFLX stock price will increase/decrease by 1.512-units respectively.

5.1.2 The result and discussion for the Monthly beta of NFLX stocks with NASDAQ Index

According to the NASDAQ index, monthly beta for the NFLX stock for 15-year period is calculated as 1.360, indicating that it is 36% more volatile than the overall market NASDAQ index. If the market price goes up by 1-unit, the NFLX stock price is expected to go up by 1.36-units. This indicates a high risk and higher return of NFLX stocks with respect to NASDAQ market index. With the use of 5 years historical data the beta and the E(Ri) are calculated as 1.613 and 22.70% respectively which also shows higher sensitivity to market changes.

Varying level of economic exposure, investor sentiment, variations among risk profiles of the two indices and two different time periods may affect the results. Conversely, two different time periods indicate the disproportionate effect of fluctuation of interest rates, geopolitical events, financial crises, and industry disruptions over different periods. These outliers can skew beta results and different beta results in different E(Ri). Moreover, a shorter timeframe such as 5-years might capture more short-term volatility compared to 15-year period.

It is further noted that the results for 5-year monthly beta for NFLX with respect to S&P 500 in December 2023 is recorded as 1. 221.The results for 5 year-monthly beta for NFLX in July 2024 with respect to NASDAQ is recorded as 1.27 in yahoo finance website.[Source: https://www.zacks.com/stock/chart/NFLX/fundamental/beta;https://ca.finance.yahoo.com/quote/NFLX/;https://www.stock-analysis-on.net/NASDAQ/Company/Netflix-Inc/DCF/CAPM]

5.2. Results and discussions for the Nβ1 and NE(Ri) values based on neutrosophic calculations

The results obtained using neutrosophic calculation can be presents as neutrosophic beta ($N\beta_1$) and neutrosophic expected return (NE(Ri))**.**

Firstly, the results of the original historical data sets with the neutrosophic methods are analysed to explore and recognise its inherent characteristics and trends.Secondly, the results with MA are examined with the expectation of advancement in the overall analysis.

Results for each of the four models under four different scenarios are discussed below. As an illustration according to the results, neutrosophic beta 1($N\beta_1$) and NE(Ri) are represent as an intervals such as [$N\beta_{1L}$,$N\beta_{1U}$] and [NE(R_{iL}), NE(R_{iU})] respectively where; $N\beta_{1L}$ is the lower limit of neutrosophic beta $N\beta_{1L}$ is the upper limit of neutrosophic beta, NE(R_{iL}) is the lower limit of neutrosophic expected return and NE(R_{iU}) is the upper limit of neutrosophic expected return.

5.2.1. Results and Discussion based on Scenarios (15-year period).

This study is mainly conducted under four scenarios and the results are categorised accordingly. The results for $N\beta_1$ and NE(Ri) for each model are given in the tables below.

Scenario 01:S&P 500 Index with original data

MODEL	$N\beta_{1L}$	$N\beta_{1U}$	$NE(R_{iL})$	$NE(R_{iU})$
MODEL 1	0.794	1.687	9.87%	21.79%
MODEL 2	0.780	1.692	10.09%	21.56%
MODEL 3	0.822	1.755	8.40%	24.53%
MODEL 4	1.019	1.453	13.44%	18.40%

Table 5.2.1.1 Scenario 01:S&P 500 Index with original data.

Model 1:

Under the first scenario, in model 1, $N\beta_1$ value is within the interval [0.794, 1.687]. The interval of NE(Ri) is [9.87%, 21.79%]. Thus, this beta and expected return show a wide range, indicating significant uncertainty in potential returns. Accordingly, for one-unit increase/decrease in S&P500 market index, NFLX stock can be increased/decreased by 0.794 to 1.687-units. This indicates that the stock's volatility could be anywhere from less volatile to significantly more volatile. Accordingly, the results from NCAPM reveal that the investor can expect a 9.87% to 21.79% return.

Model 2

Model 2 is the one of the two-factor model of this study. In this model $N\beta_1$ takes value within the interval [0.780, 1.692] and NE (Ri) is within [10.09%, 21.56%]. For one-unit increase in

S&P500 market index, NFLX stock can be increased by 0.78 to 1.69. Compared to the model 1, $N\beta_1$ and NE(Ri) are moderately similar, suggesting marginally similar uncertainty in returns and volatilities.

Model 3

This is another two-factor model. $N\beta_1$ is found to be within the interval [0.822, 1.755] and NE(Ri) is shown as [8.40%,24.53%]. Among all four models, this model shows the widest range in both $N\beta_1$ and NE(Ri), representing the highest level of uncertainty. According to this model NFLX potentially yield the highest return for S&P500 index but also comes with the maximum risk under this scenario.

Model 4

Model 4 is developed to capture the effect of all three independent variables S&P500, GDP and CPI on NFLX stock. In this model, $N\beta_1$ is [1.019, 1.453] and NE(Ri) is [13.44%,18.40%] which implies the shortest range of both $N\beta_1$ and E(Ri). The narrowest and moderate ranges of $N\beta_1$ and NE(Ri) suggest lower uncertainty and a more stable expected return. Moreover, $N\beta_{1L}$ approximately equals to one and the interval range of beta indicates that the stock is steadily aligned with market volatility.

In brief, scenario 01, $N\beta_1$ provides an averaged metric to compare the overall risk-adjusted volatility of each model. The results of model 3 shows wider range and indicate higher potential returns but also come with increased risk, while the results of model 04 express the more stable returns with smaller risk. A relatively shorter range of $N\beta_1$ value in the model 04 suggests consistent performance. This lower range $N\beta_1$ implies more stability but less sensitivity to market changes compared to the other models. Models 02 and 03 have more similar results with moderate risk and volatility.

Scenario 02: NASDAQ Index with original data.

MODEL	$N\beta_{1L}$	$N\beta_{1U}$	$NE(R_{iL})$	$NE(R_{iU})$
MODEL 1	0.609	2.483	10.70%	43.52%
MODEL 2	0.590	2.568	13.49%	43.89%
MODEL 3	0.617	2.508	9.33%	47.51%
MODEL 4	1.071	1.385	22.43%	23.76%

Table 5.2.1.2 Scenario 02: NASDAQ Index with original data.

Model 1

Under the second scenario, the $N\beta_1$ in model 1 is in the interval [0.609, 2.483]. This wider range shows the change in volatility from low-risk scenarios to high-risk scenarios. $N\beta_{1L}$=0.609 indicates low risk meanwhile $N\beta_{1U}$ =2.483 indicates high risky market. It simply captures the unpredictable nature of the stock market and reveals that NFLX stocks can be moderately sensitive to the market changes or else it can be highly sensitive to the market changes. The interval of $NE(R_i)$ strongly justifies the values of the $N\beta_1$.The $NE(R_i)$ is [10.70%,43.52%] and it shows that the riskiness of NFLX stock can fluctuate within lower risk scenarios to higher risk scenarios. Overall, the model 1 has high volatility but it offers a reasonably high average return. Comparatively, $N\beta_1$ and NE(Ri) are higher than the model 1 in 1st scenario. This balanced risk and return make this model as a solid option for whom seeking a mix.

Model 2

This model has a wider $N\beta_1$ range compared to the model 1. But lower boundary of the NE(Ri) is slightly higher, and the upper boundary remain similar to the model 1. The interval of $N\beta_1$ is [0.590, 2.568] meanwhile the interval of NE(Ri) is [13.49%, 43.89%]. These differences may occur due to the effect of the 2nd factor, GDP in model 02.

Model 3

Similar to model 1, the limits of $N\beta_1$ vary in between 0.617 and 2.508. But NE(Ri) shows the lowest value 9.33% as well as the highest value 47.51% in this scenario. These values indicate that model 3 gives the widest range of the expected return NE(Ri), indicating that investor can expect the lowest return of 9.33% as well as the highest return 47.51% for this stock. By comparing the NE(Ri) of model 02 and 03, we can have an idea about the effect of the GDP and CPI factors on this NFLX stock movements.

Model 4

In this model, the interval of $N\beta_1$ is [1.071, 1.385]. Hence, model 4 recorded the highest $N\beta_{1L}$ which is very close to 1 and it indicates that the NFLX stock increase/decrease very similar to NASDAQ index. Accordingly, for 1-unit increase in NASDAQ stock, NFLX stock will increase by 1.071-units. Nevertheless, $N\beta_{1U}$ is the lowest $N\beta_{1U}$ among all. The interval of NE(Ri) is [22.43%, 23.76%] and it shows the most narrow range of NE(Ri). But 22.43% is the highest NE(Ri) in the lower limit and 23.76% is the lowest NE(Ri) in the upper limit. However, among all these models, this model shows the least risky and most stable results.

Overall, extremely higher range of neutrosophic betas and neutrosophic expected returns can be seen in this scenario. The result of each model depicts a different balance of risk and return, offering various investment strategies and risk tolerance levels. With the use of NASDAQ index models 1, 2, and 3 display a wider range of $N\beta_1$ indicating higher sensitivity to the market volatility. Additionally, the result for the expected return is relatively higher than the scenario 01. Similar to scenario 01, model 4 shows a shorter range, indicating it is less reactive to market changes and thus, potentially more stable.

Scenario 03:S&P 500 Index with MA.

MODEL	$N\beta_{1L}$	$N\beta_{1U}$	$NE(R_{iL})$	$NE(R_{iU})$
MODEL 1	0.640	1.680	7.10%	20.10%
MODEL 2	0.734	2.024	8.75%	23.32%
MODEL 3	0.778	2.011	4.60%	28.00%
MODEL 4	0.182	0.480	0.33%	4.74%

Table 5.2.1.3 Scenario 03:S&P 500 Index with MA.

Model 1

The value of $N\beta_1$ of this model is [0.640, 1.680] and the $NE(R_i)$ is [7.10%, 20.10%]. Compared to model 1 in other scenarios, this is the lowest limit of $N\beta_1$. This results implies that for 1-unit increase/decrease in S&P500, NFLX will increase/decrease by 0.640-units to 1.680-units respectively. Accordingly, we can see the lowest $NE(R_i)$ interval and consideration of MA, may be the reason.

Model 2

In model 2, intervals of $N\beta_1$ and $NE(R_i)$ are [0.734, 2.024] and [8.75%, 23.32%], respectively. Compared to scenario 01, the upper limit is higher. This upper limit indicates that for 1-unit increase/decrease in NASDAQ, the NFLX stock can increase/decrease by 2.024-units. However, this increase /decrease may vary from 0.734 units to 2.2024 units.

Model 3

The interval of $N\beta_1$ of this model is [0.778, 2.011]. The sensitivity of this model fluctuates from lower sensitivity to higher sensitivity. The interval of the $NE(R_i)$ is [4.60%, 28.00%]. Through out this analysis, the smallest lower boundary for $NE(R_i)$ is recorded under this scenario in this model. Further the model has the highest upper boundary and the widest range for $NE(R_i)$ with respect to S&P500. Thus, this model is the best for individuals who aim higher gains and can tolerate significant market fluctuations.

Model 4

This 3-factor model gives a totally different perspective. Specially, $N\beta_1$ is unusually low. The interval of $N\beta_1$ is [0.182, 0.480] which is the lowest boundaries among all models. Similar to the sensitivity, the NE(Ri) is also very low [0.33%, 4.74%]. This model, indicate the lowest sensitivity and expected return, which is suitable for very risk-aversion investors and perfect for very conservative investors who prioritize stability over growth.

After considering MA, the lower limit of $N\beta_1$ compared to scenario 01, has been decreased in each model indicating moderate sensitivity to the market changes than the 1st scenario. Moderately wider beta ranges can be observed in models 1, 2, and 3 showing some sensitivity to market fluctuations. These results give a clear picture of the unpredictable stock market behaviours to the investors. The results of model 4 shows a much smaller range with lower sensitivity to market changes while deviating from the results in other three models.

Scenario 04: NASDAQ Index with MA.

MODEL	$N\beta_{1L}$	$N\beta_{1U}$	$NE(R_{iL})$	$NE(R_{iU})$
MODEL 1	0.674	2.387	10.50%	38.28%
MODEL 2	0.710	2.781	12.12%	43.16%
MODEL 3	0.725	2.534	6.89%	47.79%
MODEL 4	0.193	0.470	1.06%	5.66%

Table 5.2.1.4 Scenario 04: NASDAQ Index with MA.

Model 1

This model shows quite similar results for the $N\beta_1$ in the original NASDAQ scenario (2nd scenario) and the $N\beta_1$ interval is [0.674, 2.387]. The value of expected return NE(Ri) is [10.50%, 38.28%] which offers a decent return in safer conditions and much riskier return in risky scenarios.

Model 2

This model gives the highest upper limit for $N\beta_1$ among all models across all scenarios. The $N\beta_1$ interval is [0.710, 2.781] and the NE(Ri) interval is [12.12%, 43.16%]. This model indicates that the sensitivity of the market can change from a low-risk situation to an extreme risky situation. This model indicates that the worst case may be 1-unit decrease in NASDAQ stock may result in decreasing the NFLX beta by 2.781 units and vise versa.

Model 3

This model shows the highest upper limit of NE(Ri) and widest range of return among all models across all scenarios, and it is 47.79%. The interval of the NE(Ri) may vary from lower

12.12% to this highest value. This model suggests significant sensitivity in safe situations and enormous sensitivity in risky scenarios. $N\beta_1$ interval is [0.725, 2.534].

Model 4

This model-results also highly deviate from all other results except the results in scenario 3, in model 4. This model gives minimum sensitivity, and lowest return in safe scenario as well as risky scenarios. $N\beta_1$ interval is [0.193, 0.470] and this is the narrowest range among all models across all scenarios.NE (Ri) interval is [1.06%, 5.66%] which is comparatively low.

Under this scenario, the application of MA does not exhibit large deviations from all other results except for model 4. These models show wider $N\beta_1$ ranges. This suggests high sensitivity to market instability. These models show slightly improved performance in certain metrics with the introduction of the MA, while Model 4 results experience a significant decline.

5.2.2. Results and Discussion based on the Models (15-years)

In this section, we conduct a model-based discussion across all four scenarios

Scenario	Scenario 01	Scenario 02	Scenario 03	Scenario 04
MODEL	S&P500 Original Data Set	NASDAQ Original Data Set	S&P500 with MA	NASDAQ with MA
MODEL 1	[0.794,1.686]	[0.609,2.483]	[0.640,1.680]	[0.673,2.387]
MODEL 2	[0.779,1.692]	[0.589,2.568]	[0.734,2.024]	[0.709,2.780]
MODEL 3	[0.822,1.754]	[0.617,2.508]	[0.777,2.010]	[0.725,2.534]
MODEL 4	[1.019,1.453]	[1.071,1.385]	[0.180,0.475]	[0.190,0.470]

Table 5.2.2.1 $N\beta_1$ Summary for 15-year period (2009-2023).

Scenario	Scenario 01	Scenario 02	Scenario 03	Scenario 04
MODEL	S&P500 Original Data Set	NASDAQ Original Data Set	S&P500 with MA	NASDAQ with MA
MODEL 1	[9.87%,21.79%]	[10.70%,43.52%]	[7.10%,20.10%]	[10.50%,38.28%]
MODEL 2	[10.09%,21.56%]	[13.49%,43.89%]	[8.75%,23.32%]	[12.12%,43.16%]
MODEL 3	[8.40%,24.53%]	[9.33%,47.51%]	[4.60%,28.00%]	[6.89%,47.79%]
MODEL 4	[13.44%,18.40%]	[22.43%,23.76%]	[0.33%,4.74%]	[1.06%,5.66%]

Table 5.2.2.2 $NE(R_i)$ Summary for 15-year period (2009-2023).

Model 1

This is the single factor NCAPM model. Neutrosophic expected return of NFLX stock are calculated using this model with respect the market fluctuations. This NCAPM model indicates varying sensitivity across scenarios with moderately wider beta ranges in some cases such as the interval [0.609,2.483] in second scenario, representing that it captures more market variations. With the original data set, and MA of the data set, S&P500 gives more similar results for beta [0.794,1.686] and [0.640,1.680] respectively. On the other hand, in these two scenarios with NASDAQ also have comparatively similar beta results.

But the results for the returns for both cases are slightly different. However, NASDAQ offers a higher variability and potentially higher returns. This may be due to more volatile nature of NASDAQ market index. The use of MA reduces the range of expected returns for both S&P500 and NASDAQ, indicating a potential trade-off between stability and the magnitude of the returns.

Models 2

This is the one of the 2-factor NAPT models. The output for NE(Ri) of NFLX is obtained with respect to market index and GDP. According to this model, $N\beta_1$ in all four scenarios slightly deviate from the model 1. This model gives the highest $N\beta_{1U}$ value among all four models recording 2.780. Apparently, the results should be different from other models because, apart from market volatility, this model 2 captures the risk associated with GDP. Evidently, the expected returns are affected using MA, often stabilizing performance for Model 2. NASDAQ dataset steadily shows higher returns compared to S&P 500.

Model 3

This is another two-factor NAPT model. The results for NE(Ri) of NFLX are determined with respect to the market index and CPI. Similar to model 2, $N\beta_1$ values in all four scenarios show minor deviations from those in model 1. This may be due to the risk associated with CPI in addition to market volatility. NE(Ri) is evidently affected using the MA technique, which often stabilizes performance of model 3. This model gives the highest $NE(R_{iL})$ as 47.79% in 4^{th} scenario. It is important to note that this model in our calculations always gave the highest $NE(R_{iU})$ in all four scenarios and hence, drive to hold the highest range of NE(Ri) among these models.

Model 4

This is a 3-factor NAPT model, which considers the effects of three factors together (market index, GDP and CPI) on NFLX stock. The results of this model consistently deviate from model 1, 2 and 3. It shows narrowest $N\beta_1$ ranges and reduces the variability and expected returns across all scenarios. But compared to other scenarios, in scenario 01 and 02 this model gives more reliable results. It implies a more stable but less sensitive model. $N\beta_1$ intervals of these scenarios are [1.019,1.453] and [1.071,1.385]. This model also gives a fair return in scenario 01 and 02. It implies a more stable but less sensitive model. But after considering the smoothing technique, the model results are totally different. $N\beta_1$ shows a huge drop compared to other models and hence substantial decrease in NE(Ri) as well.

Among the models 1,2 and 3, the results of $N\beta_1$ in model 1 shows the minimum upper boundary across all scenarios. Model 04 is more stable and less reactive, providing a distinct perspective on market risk and return. With the use of MA, all models in scenario 03 show comparatively higher ranges of $N\beta_1$ and NE(Ri) compared to scenario 01. Scenarios 3 and 4 tend to reduce the range of $N\beta_1$ values for all models. Except model 4, all other models with NASDAQ index have resulted in the highest range of return. However, in each scenario, the results of model 4 provided a narrow range, suggesting very low volatility and high stability.

Overall, these findings suggest that each of these NCAPM and NAPT models capture different market variations. Additionally, NASDAQ give more volatile results compared to S&P500.These results may help investors and analyst to have a good understanding about the relationship between risk and return.

5.2.3. Results and Discussions based on average $N\beta_1$ and average NE(Ri) on scenarios. (15-years)

With the aim of simplifying the results into a single representative value, as an illustration the average (mean) of all $N\beta_1$ and NE(Ri) are calculated. This approach reduces the complexity of dealing with interval to have a clear picture on the correlation among the models and corresponds to the classical statistics, which deals with the precise crisp data value.

$N\beta_1$	MODEL 1	MODEL 2	MODEL 3	MODEL 4
SCENARIO 1	1.240	1.236	1.289	1.236
SCENARIO 2	1.546	1.579	1.563	1.228
SCENARIO 3	1.160	1.379	1.394	0.331
SCENARIO 4	1.530	1.745	1.630	0.331

Table 5.2.3.1 Average $N\beta_1$ for each scenario for 15-years period (2009-2023).

According to the average values in scenario 01, all models show similar average $N\beta_1$, which fluctuates between 1.26 to 1. 289. These results indicate a strong volatility of the stock compared to the market index. Average $N\beta_1$ values range from 1.228 to 1.579 in scenario 02. According to the 3rd and 4th scenarios, the average volatility fluctuates from 0.331 to 1.394 and 0.331 to 1. 745 respectively. This lowest similar average $N\beta_1$ values are due to the use of model 4. Except that, all other results varying among 1.160 to 1.745 showing higher volatility.

E(Ri)	MODEL 1	MODEL 2	MODEL 3	MODEL 4
SCENARIO 1	15.83%	15.83%	16.46%	15.92%
SCENARIO 2	27.11%	28.69%	28.42%	23.10%
SCENARIO 3	13.60%	16.04%	16.30%	2.53%
SCENARIO 4	24.39%	27.64%	27.34%	3.36%

Table 5.2.3.1 Average NE(Ri) for each scenario for 15-years period (2009-2023).

According to scenario 01, average NE(Ri) ranges from 15.83% to 16.46%. In scenario 02, the average NE(Ri) is fluctuated among 23.10% to 28.69% and it indicates considerable higher average return compared to other scenarios. This clearly indicates the difference between use of S&P500 and NASDAQ market indices. The effect of MA technique can be observed in the 3rd and 4th scenarios. Except model 4, all the average NE(Ri) values under the scenario 3 vary from 13.60% to 16.30%. While these in scenario 4 are between 24.39% to 27.64%. These results slightly deviate from the results of the original data set. But it is not a huge deviation. Model 4 predict slightly lower average NE(Ri) ranges such as 2.53% and 3.36 % in Scenario

3 and 4, respectively. Above results suggest that different models may perceive different risk levels and market influences. Overall, these average returns make these models attractive for its potential returns despite moderate risk.

5.2.4 Results and Discussion based on $N\beta_1$ for 5-year periods from 2019-2023, 2017-2019 and 2009-2013

The table below illustrates the results for $N\beta_1$ in each scenario for each 5-year period from 2009 to 2023. This table clearly point outs the results of each scenario at different time periods and the analysis represents the possible variations of sensitivity in each model within the 5-year period.

Scenario	Scenario 01	Scenario 02	Scenario 03	Scenario 04
5-year period	S&P500 with Original Data	NASDAQ with Original Data	S&P500 with MA	NASDAQ with MA
2019-2023				
MODEL 1	[0.764,1.927]	[0.462,2.812]	[1.114,1.900]	[0.768,2.467]
MODEL 2	[0.728,1.924]	[0.418,3.051]	[1.421,2.769]	[0.813,3.298]
MODEL 3	[0.761,1.917]	[0.440,2.675]	[1.412,2.320]	[0.826,2.385]
MODEL 4	[1.674,1.785]	[1.605,2.119]	[0.664,1.106]	[0.406,1.089]
2014-2018				
MODEL 1	[1.051,1.942]	[0.768,3.202]	[0.828,1.253]	[0.890,2.483]
MODEL 2	[0.941,1.865]	[0.709,3.367]	[0.667,1.167]	[0.815,2.787]
MODEL 3	[0.874,1.783]	[0.752,2.945]	[0.571,1.004]	[0.817,2.329]
2009-2013				
MODEL 1	[0.701,1.164]	[0.695,1.561]	[0.037,1.631]	[0.285,2.239]
MODEL 2	[0.751,1.229]	[0.699,1.561]	[0.234,1.961]	[0.376,2.406]
MODEL 3	[0.649,1.288]	[0.639,1.866]	[-0.068,2.468]	[0.194,3.766]

Table 5.2.4 Discussion based on $N\beta_1$ for 5-year periods from 2019-2023, 2017-2019 and 2009-2013.

According to the recent 5-Year results, there is a rapid increase in $N\beta_1$ intervals. Specifically, the upper limits of most of the intervals are expanded. The upper limits of the models in scenario 01 is raised nearly to 2.00, meanwhile the upper limit of the 2nd scenario exceeds 3.0 which indicates an extremely risky situation. With the use of MA technique, the results of first 3 models indicate most volatile situation but model 4 shows incredibly stable and less risky result.

During 2014-2018, the NFLX stock seems to be highly volatile stock as it results in very high $N\beta_1$ intervals. The results may fluctuate moderately risky situation to very high level except the $N\beta_1$ in model 3 in the 3rd scenario.

During the 5-year period from 2009-2013, we note a moderate result with the original data. With the smoothing technique, the results fluctuate in between lower sensitivity situation to considerably higher sensitivity. The $N\beta_1$ for model 3 gives slightly negative output for 3rd scenario, indicating that 1-unit increase in the market index may cause decrease in the NFLX stock by 0.068-units. These results may be able to give a caution to the investors to be ready for that kind of situation as well.

5.2.5 Results and Discussion based on 5-Year Neutrosophic Expected return from 2019-2023

Based on the results of $N\beta_1$ intervals in resent 5-year period, four different models give the below $NE(R_i)$.

5 years	Scenario1		Scenario2		Scenario3		Scenario4	
MODEL	$NE(R_{iL})$	$NE(R_{iU})$	$NE(R_{iL})$	$NE(R_{iU})$	$NE(R_{iL})$	$NE(R_{iU})$	$NE(R_{iL})$	$NE(R_{iU})$
MODEL 1	10.87%	25.70%	8.64%	52.38%	11.93%	19.44%	11.10%	35.09%
MODEL 2	10.49%	23.52%	15.43%	53.62%	17.51%	27.73%	14.38%	43.17%
MODEL 3	10.79%	25.71%	9.10%	49.53%	10.85%	23.66%	8.13%	41.05%
MODEL 4	7.83%	13.97%	-1.42%	34.01%	4.80%	8.99%	-2.88%	10.32%

Table 5.2.5.1. 5-Year NE(Ri) under different Scenarios and Models from 2019-2023.

In 1st scenario, all models show relatively narrow range of expected returns NE(Ri). There is a wider range of NE(Ri) in 2nd scenario, indicating higher uncertainty or volatility compared to all other scenarios. The model 1 shows the widest range under this 2nd scenario, which fluctuates from 8.64% to 52.38%. However, the model 4 of this scenario gives negative lower bound of NE(Ri). This model predicts that the NE(Ri) can vary between -1.42% to 34.01%. Surprisingly, use of MA technique does not show much variation during this time period as scenarios 3 and 4 provide moderately similar results to scenario 1 and 2, respectively. Model 4 shows negative lower bound values in 4th scenario, indicating a potential loss, while other results show considerably positive higher return.

Slightly different results can be observed in two different time periods ,5-years and 15-years. The changes in economic policies, natural disasters, market cycles, and sector-specific movements which happen during these two periods may caused to these deviations. Specially, the effect of COVID-19 drives the stock performance of NFLX in to another level.

5.2.6. Results and Discussion based on 5-YearAverage NE(Ri) on Scenarios and Models from 2019-2023.

Average	MODEL 1	MODEL 2	MODEL 3	MODEL 4
Scenario1	18.29%	17.01%	18.25%	10.90%
Scenario2	17.17%	19.47%	17.41%	6.28%
Scenario3	30.51%	34.52%	29.31%	16.30%
Scenario4	32.15%	35.57%	30.19%	19.40%

Table 5.2.6.1 5-YearAverage NE(Ri) under different Scenarios and Models from 2019-2023.

The 5-year average returns give a simple idea about the NE(Ri) of the stock over the short period. These results offer a clear idea about the average stock return under each scenario with the use of each model. Under the scenario 01, the average NE(Ri) from the four models range from 10.90% to 18.29% suggesting the effects of varying factors and different market conditions being considered in each model. The average NE(Ri) ranges from 6.28% to 19.47% in 2^{nd} scenario which, gives more conservative result compared to scenario 1. In the scenario 3, the average NE(Ri) are higher, ranging from 16.30% to 34.52%. Using the smoothing technique, results give a more bullish market scenario. The 4^{th} Scenario is the most optimistic scenario among the four models and NE(Ri) range from 19.40% to 35.57%. Except Model 4, all other models give more or less similar returns in each scenario. Model 4 always give the minimum average return.

5.3. Results and discussion for $N\beta_2$ and $N\beta_3$

The primary focus of this study is on the $N\beta_1$ which explain the NFLX stock movement with respect to the Market Index, and how does it affect the neutrosophic expected return when using different factors and, in different time frames. In this case it is vital to accurately calculate $N\beta_2$ and $N\beta_3$. Hence $N\beta_2$ and $N\beta_3$ are also carefully calculated, and we present the results and discuss those results briefly to provide a comprehensive picture of all model in each scenario separately.

5.3.1 Neutrosophic Beta 2 ($N\beta_2$)

$N\beta_2$ values represent how much units may increase /decrease in NFLX stocks with respect to the one-unit increase/decrease in GDP value. Below, we have studied factor GDP in Model 2 and Model 04 only under all four scenarios. The $N\beta_2$ values for 15-year period (2009-2023) and most resent 5 Years (2019-2023) are given in Table 5.3.1 below.

$N\beta_2$	Scenario 01		Scenario 02		Scenario 03		Scenario 04	
	$N\beta_{2L}$	$N\beta_{2U}$	$N\beta_{2L}$	$N\beta_{2U}$	$N\beta_{2L}$	$N\beta_{2U}$	$N\beta_{2L}$	$N\beta_{2U}$
2009-2023	FOR 15YEARS							
MODEL 2	-0.480	0.371	-3.869	1.358	-0.930	0.878	-1.377	1.597
MODEL 4	-3.396	-1.168	-7.354	-0.770	-0.158	0.228	0.097	0.564
2019-2023	FOR 5 YEARS							
MODEL 2	-0.022	1.155	-3.994	1.629	-1.529	-0.164	-1.475	1.650
MODEL 4	-3.466	-1.089	-0.796	0.063	0.040	0.101	0.058	0.169

Table 5.3.1 $N\beta_2$ values for 15-year period (2009-2023) and most resent 5 Years (2019-2023).

Model 02 measures the impact of market index and GDP together on NFLX stock price. It is noted that Model 02 reflected a wider range of $N\beta_2$ values from negative to moderately positive values across different scenarios, indicating higher variability and uncertainty in the 15-year period as well as for the 5-year period.

Model 04 measures the impact of combined effects of market index, GDP and CPI, on NFLX stock price. It exhibits extreme values of $N\beta_2$, strongly negative for the 15-year period. This implies a strong inverse relationship with the market. Again, this provides evidence for the sensitivity of the model on the effect of economic fluctuations during the period such as the effect of COVID-19. In the recent 5-year period, Model 4 displays both negative as well as positive correlations with a generally weaker relationship compared to the 15-year period. The values of Nβ_2 demonstrate how different models and different scenarios capture unpredictable degrees of market correlation and uncertainty over time.

5.3.2 Neutrosophic Beta 3 (Nβ_3)

Measure Nβ_3 represents how much units may increase /decrease in NFLX stocks with respect to the one-unit increase/decrease in CPI respectively. In this study, we consider CPI in Model 3 and Model 04 under all four scenarios. Table 5.3.2 below provides Nβ_3 values for all 15 years and most resent 5 Years under all four scenarios.

Nβ_3	Scenario 01		Scenario 02		Scenario 03		Scenario 04	
	Nβ_{3L}	Nβ_{3U}	Nβ_{3L}	Nβ_{3U}	Nβ_{3L}	Nβ_{3U}	Nβ_{3L}	Nβ_{3U}
2009-2023	FOR 15YEARS							
MODEL 3	-1.073	1.139	-0.900	2.134	-2.301	2.478	-2.609	4.325
MODEL 4	-1.018	-0.861	-0.957	-0.921	-1.468	-0.845	-1.505	-1.284
2019-2023	FOR 5 YEARS							
MODEL 3	-0.026	0.062	-0.187	0.374	-1.780	0.176	-1.741	3.360
MODEL 4	-9.194	-13.999	-13.309	-2.776	0.040	0.101	-4.561	-2.840

Table 5.3.2 Nβ_3 values for 15-year period (2009-2023) and most resent 5 Years (2019-2023).

It is observed that the sensitivity of NFLX stock, in general, to the changes in CPI varies considerably across different scenarios and models. Both models 3 and 4 have resulted in more variability across scenarios and for different time periods. Model 3 tends to show more negative as well as positive sensitivities with varying ranges and Nβ_3 shows broader range of values indicating higher uncertainty. This may reveal less predictability meanwhile model 4 exhibits more consistent results, especially in the 2009-2023 timeframe, with a narrow range of values. Model 4 generally shows negative sensitivities, mainly in the more recent 5-year period. These negative $N\beta_3$ values in the two models indicate that the NFLX stock return tends to move inversely with the CPI.

According to the results, $N\beta_3$ values are more negative. NFLX is a technology stock and some of the technology stocks might have negative CPI betas, if they have high input costs that rise with inflation, and they cannot easily pass these costs on to customers.

5.4. Correlation coefficient

The consistent lower NE(Ri) values from Model 4 in Scenarios 3 and 4 need to be carefully addressed. The Model 4 predicts a narrow spread compared to other models. There may be some reasons for these results such as multicollinearity among factors, specific factor selection, data transformation techniques or any other reason. Hence, correlation coefficient of each two factors is calculated to check for the multicollinearity due to correlated factors.

Table 5.3.1 provides the correlation coefficients between S&P 500 and NASDAQ financial indices (original and with MA) and economic factors, GDP and CPI.

Factors	S&P500	NASDAQ	S&P500 with MA	NASDAQ with MA	GDP
GDP	-0.037	-0.035	0.382	0.298	1
CPI	0.105	0.045	0.243	0.104	0.270

Table 5.3.1 Correlation coefficients matrix of the Factors.

From the above table, it is noted that the market indices with original S&P500 and NASDAQ data set do not have a direct correlation with GDP and CPI individually as the values of correlations are very low. When considering the MA of the S&P500 and NASDAQ, there is a weak to moderate positive correlation. This suggests some influence of GDP on MA of

S&P500 and MA of NASDAQ. The correlation coefficient between GDP and CPI is weakly positive. This indicates that even though they are related, the relationship is not strong enough in this dataset. Further, there is no sign of presence of multicollinearity.

5.5. Model Verification

In what follows now, under all four scenarios, we describe only the Model 01 fit significance and accuracy to understand the process. In neutrosophic statistics, 2-factor or 3-factor model verifications are more complicated. Hence, we do focus on Model 01 with one factor only.

5.5.1. Hypothesis and t-statistics for each scenario of Model 01

Test statistic: t-statistic is calculated for Model 01 in each scenario to measure the significance of the regression coefficient in this simple linear model. We will present the results in each scenario separately [Degrees of freedom = 178]

Critical t-value(T) at α_N= [0.01 ,0.05] is [1.973,2.617]. Then,

$$\{T(1-\alpha_N)\} = [1.645, 2.33]$$

$$\mathrm{Max}\{T(1-\alpha_N)\} = 2.33$$

1. Validate the results for 1st scenario :S&P 500 Index with original data

Null hypothesis: NH_0: Nβ_1 ∈ [0.794,1.686]

0Alternative hypothesis: NH_a: Nβ_1 < 0.794, or NH_a: Nβ_1 > 1.686 or NH_a: Nβ_1 ∉ [0.794,1.686]

t- statistic [2.848,5.465]

Hence

$$\{[t-\text{statistic}] - [\text{ctitical t value(T)}]\} = \{[2.848, 5.466] - [1.973, 2.617]\}$$

$$\{[t-\text{statistic}] - [\text{ctitical t value(T)}]\} = [0.231, 3.493]$$

Therefore

$$\mathrm{Min}\{[\mathrm{t}-\mathrm{statistic}]-[\mathrm{ctitical\ t\ value(T)}]\}=0.231$$

According to the results, $\mathrm{Min}\{[\mathrm{t}-\mathrm{statistic}]-[\mathrm{ctitical\ t\ value(T)}]\}<\mathrm{Max}\{\mathrm{T}(1-\alpha_{\mathrm{N}})\}$.

Therefore, do not reject the null hypothesis for scenario 01. Hence, we can conclude that $N\beta_1 \in [0.794,1.686]$.

2. Validate the results for 2nd scenario: NASDAQ with original data set.

Null hypothesis: NH_0: $N\beta_1 \in [0.609,2.483]$

Alternative hypothesis: NH_a: $N\beta_1 < 0.609$, or NH_a: $N\beta_1 > 2.483$ or NH_a: $N\beta_1 \notin [0.609,2.483]$

t-statistic [3.041,8.396]

Hence

$$\{[\mathrm{t}-\mathrm{statistic}]-[\mathrm{ctitical\ t\ value(T)}]\}=\{[3.041,8.396]-[1.973,2.617]\}$$

$$\{[\mathrm{t}-\mathrm{statistic}]-[\mathrm{ctitical\ t\ value(T)}]\}=[0.424,6.423]$$

Therefore

$$\mathrm{Min}\{[\mathrm{t}-\mathrm{statistic}]-[\mathrm{ctitical\ t\ value(T)}]\}=0.231$$

According to the results, $\mathrm{Min}\{[\mathrm{t}-\mathrm{statistic}]-[\mathrm{ctitical\ t\ value(T)}]\}<\mathrm{Max}\{\mathrm{T}(1-\alpha_{\mathrm{N}})\}$.

Therefor do not reject the null hypothesis for scenario 02 and hence we can conclude that $N\beta_1 \in [0.609,2.483]$.

3. Validate the results for 3rd scenario :S&P 500 Index with MA

Null hypothesis: NH_0: $N\beta_1 \in [0.640,1.680]$

Alternative hypothesis: NH_a: $N\beta_1 < 0.640$, or NH_a: $N\beta_1 > 1.680$ or NH_a: $N\beta_1 \notin [0.640,1.680]$

t-statistic [2.138,4.824]

Hence

$$\{[\mathrm{t-statistic}]-[\mathrm{ctitical\ t\ value(T)}]\}=\{[2.138,4.824]\ -1.973,2.617]\}$$

$$\{[\mathrm{t-statistic}]-[\mathrm{ctitical\ t\ value(T)}]\}=[-0.479,2.851]$$

Therefore

$$\mathrm{Min}\{[\mathrm{t-statistic}]-[\mathrm{ctitical\ t\ value(T)}]\}=|0.479|$$

According to the results, $\mathrm{Min}\{[\mathrm{t-statistic}]-[\mathrm{ctitical\ t\ value(T)}]\}<\mathrm{Max}\{\mathrm{T}(1-\alpha_{\mathrm{N}})\}$. Therefor do not reject the null hypothesis for scenario 03 and hence we can conclude that $N\beta_1 \in [0.640,1.680]$.

4. Validate the results for 4th scenario: NASDAQ with MA

Null hypothesis: NH_0: $N\beta_1 \in [0.673,2.387]$

Alternative hypothesis: NH_a: $N\beta_1 < 0.673$, or NH_a: $N\beta_1 > 2.387$ or NH_a: $N\beta_1 \notin [0.673,2.387]$

t-statistic [3.197,7.714]

Hence

$$\{[\mathrm{t-statistic}]-[\mathrm{ctitical\ t\ value(T)}]\}=\{[3.197,7.714]\ -[1.973,2.617]\}$$

$$\{[\mathrm{t-statistic}]-[\mathrm{ctitical\ t\ value(T)}]\}=[0.580,5.742]$$

Therefore

$$\mathrm{Min}\{[\mathrm{t-statistic}]-[\mathrm{ctitical\ t\ value(T)}]\}=0.580$$

According to the results, $\mathrm{Min}\{[\mathrm{t-statistic}]-[\mathrm{ctitical\ t\ value(T)}]\}<\mathrm{Max}\{\mathrm{T}(1-\alpha_{\mathrm{N}})\}$. Therefor do not reject the null hypothesis for scenario 02 and hence we can conclude that $N\beta_1 \in [0.673,2.387]$.

Overall, the results for the application of the Neutrosophic Hypothesis test allowed positively validating the result of all four scenarios of this study with a level of significance of up to 99%.

5.6. Model Accuracy of multi-factor Models for the Expected Returns

We calculate and discuss the Neutrosophic mean absolute deviation (NMAD) measure for predictive accuracy of the fitted models for the NE(Ri) under four scenarios in Table 5.6.1.

Scenario	Model	MAD	Length of lower boundary to upper boundary
Scenario 01	Model 01	[0.326,0.380]	0.054
	Model 02	[0.328,0.377]	0.049
	Model 03	[0.299,0.394]	0.096
	Model 04	[0.294,0.409]	0.115
Scenario 02	Model 01	[0.109,0.371]	0.263
	Model 02	[0.105,0.344]	0.238
	Model 03	[0.069,0.385]	0.316
	Model 04	[0.254,0.306]	0.052
Scenario 03	Model 01	[0.262,0.367]	0.106
	Model 02	[0.229,0.351]	0.121
	Model 03	[0.182,0.392]	0.210
	Model 04	[0.415,0.435]	0.020
Scenario 04	Model 01	[0.080,0.333]	0.253
	Model 02	[0.031,0.317]	0.286
	Model 03	[0.015,0.369]	0.354
	Model 04	[0.406,0.427]	0.022

Table 5.6.1 Model Accuracy measure (NMAD) for the NE(Ri) under Four Scenarios.

Models 01 and 02 exhibit lower NMAD values and shorter lengths in scenario 1 and 2 with original data, but moderate values with MA. Model 03 shows slightly higher NMAD values and wider lengths in all four scenarios. Model 04 consistently have shorter NMAD ranges and narrow lengths. This may indicate very low volatility and high stability across all scenarios.MA (scenarios 3 and 4) tends to smooth out the data, resulting in lower NMAD values for Model 04 and indicating reduced volatility. Models 01, 02, and 03 also show reduced volatility with moving averages but still exhibit moderate variability compared to Model 04.

5.7. Further Model Applications

To test the neutrosophic model appropriateness, the $N\beta_1$ and NE(Ri) of Amazon and Apple stocks have been also calculated using the four new models.

5.7.1. Results of Amazon stock (AMZN) based on classical and neutrosophic CAPM

The beta and expected returns of AMZN stock under S&P index and NASDAQ index using the classical model are presented below in Table 5.7.1.1.

(2009-2023) Period	S&P500	NASDAQ
beta	0.990	1.088
E(Ri) of AMAZN	10.23%	15.40%

Table 5.7.1.1 Beta and E(Ri) of Amazon stock based on classical CAPM.

According to the classical model E(Ri) of AMZN stock with respect to S&P500 and NASDAQ are 10.23% and 15.40% respectively. Similar to NFLX , AMAZN also shows highest E(Ri) with respect to highly volatile NASDAQ index.

We provide the calculation for average $N\beta_1$ of AMZN stock based on neutrosophic calculations across four scenarios and four models in Table 5.7.1.2.

AVERAGE $N\beta_1$-AMZN STOCK	MODEL 1	MODEL 2	MODEL 3	MODEL 4
Scenario 1	1.049	1.039	1.068	1.541
Scenario 2	1.385	1.424	1.394	1.482
Scenario 3	0.837	0.983	0.965	0.202
Scenario 4	1.206	1.368	1.268	0.235

Table 5.7.1.2 Average $N\beta_1$ of Amazon stock based on neutrosophic calculations.

According to the results in the above table, average $N\beta_1$ of the models are moderately similar in each scenario except model 04 and the NE(Ri) fluctuates accordingly. Scenario 02 gives the highest average $N\beta_1$ in each model meanwhile scenario 03 gives the least average $N\beta_1$. However, the results of model 4 shows the highest average $N\beta_1$ in scenario 01 and 02, but the lowest average $N\beta_1$ in scenarioc03 and 04.

Based on the $N\beta_1$ intervals, NE(Ri), of AMZN stock are calculate under each scenario for all four models using the neutrosophic methods and presented as below in Table 5.7.1.3.

NE(Ri) (%) OF AMZN	MODEL 1	MODEL 2	MODEL 3	MODEL 4
Scenario 1	[7.76,19.36]	[7.61,19.32]	[3.46,25.30]	[25.92,21.15]
Scenario 2	[8.24,40.60]	[10.79,41.30]	[6.86,44.66]	[31,31.17]
Scenario 3	[5.77,14.29]	[6.53,15.79]	[3.96,21.38]	[0.29,4.86]
Scenario 4	[8.72,30.07]	[9.77,33.21]	[5.46,39.05]	[0.70,6.20]

Table 5.7.1.3 NE(Ri) of AMZN under 04 scenarios and 04 models (NCAPM and NAPT).

Use of NCAPM and NAPT models provide the values of NE(Ri) of AMZN stock. These values reflect inherently indeterminant nature of stock market. Under the different market conditions and the different calculation methods in each of four scenarios, the results imply the probable returns. Sophisticated investors can carefully use these models in trading as they are aware of the risk boundaries. This may lead them to avoid having unexpected losses. Four different statistical models offer distinct approaches to tackle the indeterminacy and risk, based on their preference.

According to the results of AMZN stocks, models 1 and 2 give pretty much similar fair return meanwhile model 3 gives most sensitive output and model 4 gives considerably narrow range compared to the other models, specially in scenario 3 and 4.

5.7.2. Results of Apple (AAPL) stock based on classical and neutrosophic CAPM

The beta and expected returns of AAPL stock under S&P index and NASDAQ index using the classical model are presented below in Table 5.7.2.1.

(2009-2023) Period	SP500	NASDAQ
Β	1.063	1.064
E(Ri)	10.92%	15.07%

Table 5.7.2.1 Beta and E(Ri) of Apple stock based on classical CAPM.

The classical model gives moderately similar beta for both indices S&P500 and NASDAQ, but the E(Ri) are 10.92% and 15.07% respectively. This difference in the expected returns may occurred due to the different nature of the two indices.

We also provide the average $N\beta_1$ of AAPL stock based on neutrosophic calculations below.

Average $N\beta_1$ of AAPL STOCK	MODEL 01	MODEL 02	MODEL 03	MODEL 04
Scenario 01	1.26	1.26	1.26	0.34
Scenario 02	1.56	1.59	1.57	0.43
Scenario 03	0.97	1.12	1.13	0.21
Scenario 04	1.21	1.35	1.28	0.20

Table 5.7.2.2 Average E(Ri) of AAPL stock based on neutrosophic calculations.

In the first scenario, the average $N\beta_1$ values from models 1, 2 and 3 are the same 1.26 except model 04 where it is 0.34. The models 1,2 and 3 in scenario 02 have the average values of $N\beta_1$ in the range from 1.56 to 1.59. However, model 04 provides less volatile values of $N\beta_1$ from 0.20 to 0.43, calculated under all scenarios. We can see a moderately similar average $N\beta_1$ in all models in each scenario except model 4.

Based on the $N\beta_1$ intervals, the neutrosophic expected returns, NE(Ri), of AAPL stock under each scenario and four models for the (2009-2023) period have been calculated using the neutrosophic models and presented below in Table 5.7.2.3.

NE(Ri) AAPL stock (%)	MODEL 01	MODEL 02	MODEL 03	MODEL 04
Scenario 01	[8.77,23.48]	[8.77,23.33]	[2.47,28.56]	[7.81,11.47]
Scenario 02	[8.40,46.56]	[11.26,46.45]	[6.68,50.54]	[10.62,15.59]
Scenario 03	[8.69,13.86]	[9.54,16.14]	[7.89,19.63]	[0.99,2.73]
Scenario 04	[12.14,26.30]	[12.98,29.49]	[9.82,35.10]	[0.50,3.39]

Table 5.7.2.3 NE(Ri) of AAPL under 04 scenarios and from 04 models (NCAPM and NAPT).

AAPL stock also gives pretty much similar results to all the models in each scenario except model 04. Generally, scenario 02 shows the largest intervals for all four models implying highest volatility. The broadest intervals of NE(Ri) are obtained from the model 3 and narrowest intervals of NE(Ri) are recorded from the model 4. These values of expected returns, NE(Ri), can be considered as risk boundaries which indicate the lower expected and highest expected returns. However, these different risk boundaries under different scenarios with distinct models help investors to conduct a more sophisticated analysis of potential risks and rewards under different economic scenarios. On the other hand, investors can choose the appropriate neutrosophic model, which satisfies their requirements and may take their investment decisions based on these risk boundaries.

Chapter 06

Conclusions, Benefits, Limitations and Future Scope

6.0. Introduction to Conclusions, Benefits, Limitations and Future Scope

This study provides the new neutrosophic models-based approach to capture the indeterminacy and uncertainty in the volatile stock market behaviour. The neutrosophic statistics enhance the traditional CAPM and APT models by incorporating the neutrosophic logic, making it more applicable to real-world scenarios where the data sets are regularly incomplete or fuzzy. However, this neutrosophic approach also has limitations, which need to be carefully addressed. The complexity of the neutrosophic model is one such limitation. Also, the complexity of financial world is increasing day by day. Hence further research into these financial models with advance technology will benefit to offer even more sophisticated models, which can better capture the market behaviour. This attempt is to address these benefits, limitations and future scope of these new models while concluding the outcomes.

6.1. Conclusions

The Neutrosophic logic-based approach provides advanced techniques to control the indeterminacy and uncertainty in the stock market. Based on these models, investors and financial analysts can improve risk assessment and their decision-making in complex financial markets.

Each model exhibits a different balance of risk and return, offering various investment strategies and risk tolerance levels to the investors. Even though the classical model gives a precise value for beta, it is far more complex. In a highly competitive market, we cannot expect a smooth flow as it is often disrupted by market manipulations, fake news, indeterminate events, and incomplete and vague data. Hence, based on these comprehensive results of four models and in each scenario considered in this study, investors and financial analysts may be able to identify their risk boundaries, and they can select their portfolios based on their financial goals. This strategic approach will help investors to mitigate the unnecessary fear or excessive risk taking up to a certain extent and they can make informed decisions based on the individual risk tolerance whether risk neural, risk lover or risk averse. These models demonstrate different levels of neutrosophic expected returns and market volatility, encapsulating the indeterminacy and range of possible outcomes in stock performance.

Primarily, the outcome of each scenario and each model represent the indeterminacy of the stock market. The results give a clear picture of the price fluctuations. The differences across the four scenarios may arise due to the use of different factors, neutrosophic calculation techniques, different market indices and different time frames.

Specially, the results obtained from NASDAQ index show more volatility compared to S&P 500. These deviations may occur due to the differences in index composition, sensitivity to economic changes, historical performance, and inherent volatility. In general, these two indices lead to different behaviour and performance patterns. Specially, NASDAQ market index is more volatile than the S&P 500 index. Because of this high sensitivity, the model results with NASDAQ may give wider prediction ranges and more significant deviations in model results, which are reflected in the model predictions.

Moreover, the results obtained during the 15-year period are somehow different from the results in the 5-year period. The differences highlight the uncertainty, different market conditions and range of possible outcomes of NFLX stock, during different timeframes. Overall, the 5-year average returns provide a quick overview of the stock's potential performance under various circumstances, helping investors understand the range of possible outcomes and associated risks during that period.

According to the results of diffrent models, the range of intervals of neutrosophic expected returns NE(Ri) fluctuates between lowest return to the highest returns indicating the adverse condition to the more optimistic outlook, respectively. Specially, the results during the most recent 5-year period show higher volatility and higher returns. These results may reveal the effects of economic turnovers during that period especially the effect of the pandemic, COVID-19. During this economic lockdown period, there was an extremely volatile situation of the NFLX stock which these neutrosophic model results also reflected. The changes in consumer behaviour during the pandemic may highly affect these results. These changes in consumer behaviours negatively impacted on most of the companies. However, the stock prices of the tech companies such as NFLX skyrocketed during this period, as most of the consumer changed their market behaviour and did operations on the online platform. The lockdowns and stay at home orders led to uplift the streaming demand and it positively affected the NFLX's subscriber growth and stock price. Similar to this situation the diverse nature of the results can be observed in different time frames.

Conversely, the changes in regulations such as data privacy, content licensing and high competition led to fluctuations in the stock price in different time frames. On the other hand, broader economic conditions, including changes in consumer spending, interest rates, and inflation, may affect stock prices. For instance, during economic downturns, the spending on entertainment can decrease. Furthermore, market sentiment; advancement in AI may also influence NFLX stock.

The results of NE(Ri)s are affected by the $N\beta_1$, $N\beta_2$ and $N\beta_3$ intervals. $N\beta_1$ is the one of the main factor which affects the stock market return, but the coefficients of GDP and CPI indexes may also affected the NFLX stock in different ways. The diverse nature of the results in different models proved it. For example, the negative coefficient of CPI sometimes positively affected the stock price. Inflation indicates squeeze profit margins, reduce consumer spending, higher interest rates, create uncertainty, and impact investor sentiment negatively. Deflation can increase real purchasing power and reduce costs, potentially boosting profit margins and consumer spending. Lower interest rates can benefit growth stocks, but persistent deflation can signal weak demand and economic issues. The results of these models are important for investors and analysts, for informed decision making, asset allocation, risk management, and portfolio diversification. The results highlight the varying impacts that GDP and CPI changes can have on NFLX stock, depending on the scenario and model used. This analysis facilitates understanding how each model performs under various scenarios, different time frames and in diverse conditions and how the level of ambiguity changes accordingly.

This study is an effort to capture the uncertainty associated with the systematic risk factors. Investors continuously evaluate these risk factors in their assessment of stock's future growth prospects and risk profile. Hence, these models will be helpful up to a certain point to capture the risk boundaries. Specifically, the effects of market index, GDP and CPI on NFLX has been tested using these models and these models can be improved by switching the factors according to the investor's requirements.

By integrating these models, investors can privilege the benefits such as enhanced strategies and better manage risks. All these benefits may guide them to make informed investment decisions. Finally, it can be concluded that this study suggests strong findings across neutrosophic components with different data treatments, techniques, and methodologies.

6.2. Benefits of proposed models

Selecting a portfolio based on beta involves balancing stocks with different beta values to align with investors risk tolerance and investment goals. A higher beta indicates higher risk and potential return, while a lower beta suggests more stability. By diversifying across sectors and periodically rebalancing the portfolio, investors can manage risk and strive for optimal returns.

Benefits for Investors:

- Financial markets are full of uncertain, imprecise, incomplete, and contradictory information. These neutrosophic models can address these issues and provide useful decision-making tool.
- **N**CAPM offer a clear formula to calculate expected return of an asset based on its systematic risk ($N\beta_1$) with respect to the market factor capturing the uncertainty of the market.
- NAPT provide different formulas to calculate the expected return of an asset considering macroeconomic factors, which affect asset returns. Also, it provides a more comprehensive risk assessment based on the systematic risk ($N\beta_1$, $N\beta_2$ and $N\beta_3$), while capturing the indeterminacy in each factor.
- All these models help investors to be aware of the risk boundaries such as the best case as well as the worst case.
- These strategic models help investors to mitigate the unnecessary fear and excessive risk taking.

- By incorporating neutrosophic statistics, these models provide a more sophisticated understanding of risks compared to traditional models.
- Investors and financial analysts can use these models to assess the investment performance against expected returns and market benchmarks.

6.3 Limitations

Classical logic can be extended to handle uncertainty, imprecision, vagueness, and inconsistency in financial stock market using Neutrosophic logic. However, this branch of philosophy, the neutrosophic logic and this analysis is a more complex phenomenon and has several limitations.

- Neutrosophic logic introduces indeterminacy, and this is complex than the classical logic. Neutrosophic models are challenging to develop, understand, and interpret.
- Neutrosophic multiple regression models are complex to interpret and solve compared to traditional regression models. Adding the number of factors can be considered to further improve the models but solving these equations are a big calculation hurdle.
- Appropriate Software need to be developed to deal with neutrosophic multiple regression models as manual calculations are harder.
- Implementing efficient algorithms for neutrosophic multiple regression analysis can be challenging, potentially leading to longer computation times and higher costs.
- Neutrosophic model validation is a challenge, especially for models with more than one factor. As Neutrosophic logic is still an emerging trend, it is difficult to define appropriate validation metrics that consider the neutrosophic components.

- In this new philosophy and growing field, it suffers from lack of standardized methodologies and techniques in financial modeling.
- Specialized knowledge in the filed of neutrosophic statistics need to implement these models.
- These models solely focus on systematic risk factors and ignore unsystematic risk which can be significant in investment decision making.
- This novel neutrosophic statistical approach which can be used to handle uncertainty and imprecision in financial stock market analysis comes with considerable limitations. These include increased complexity, lack of computational intensity, validation challenges, lack of standardization, and the need of specialized expertise. Careful consideration of these limitations is essential when dealing with this new approach in financial modeling. Comprehensive investment strategy involves not only multi-factor models but also combine with other fundamental and technical analysis along with expert portfolio management practices.

6.4 Future Scope

- There is a never-ending competition in the stock market. Capturing the indeterminacy is essential to compete confidently and take wise decisions. Hence, future study can be conducted to capture the different systematic risk factors such as liquidity risk, political risk and exchange rate risk (currency risk), which highly affect stock prices.
- Further studies can be conducted to capture the multicollinearity due to non-linear relationships or partial multicollinearity to improve the model results. Hence, multicollinearity diagnostic tools can be developed to detect the presence of multicollinearity in a neutrosophic analysis.

- As a future study, neutrosophic model verification technique can be developed. Verification of 2-factor and 3-factor models is a main limitation of this study.
- Future works should be deliberated towards utilizing a wider number of systematic risk factors by extending NAPT model to capture the effect of different factors, but solving these neutrosophic regression equations is another major limitation of this study.

"Absolutely, there is no unique neutrosophic model to a real-world problem. And thus, there are no exact neutrosophic rules to be employed in neutrosophic modelling. Each neutrosophic model is an approximation, and the approximations may be done from different points of view. A model might be considered better than others if it predicts better than others. But in most situations, a model could be better from a standpoint, and worse from another standpoint – since a real world problem normally depends on many (known and unknown) parameters. Yet, a neutrosophic modelling of reality is needed in order to fastly analyse the alternatives and to find approximate optimal solutions."

-Florin Smarandache-

7.0. References

1. Abdelfattah, B. A., Darwish, S. M., & Elkaffas, S. M. (2024, January 12). *Enhancing the prediction of stock market movement using neutrosophic-logic-based sentiment analysis*. MDPI. https://www.mdpi.com/0718-1876/19/1/7

2. Abraham, B., & Ledolter, J. (2006). *Student solutions manual for Introduction to regression modeling*. Thomson Brooks/Cole.

3. Al Aita, A., Aslam, M., & Smarandache, F. (2024). *Row-column designs: a novel approach for analyzing imprecise and uncertain observations*. Infinite Study.

4. Alqisie, A., & Alqurran, T. (2016). *Validity of Capital Assets Pricing Model (CAPM)* (empirical evidences from Amman Stock Exchange). Journal of Management Research, 8(1), 207. https://doi.org/10.5296/jmr.v8i1.8494

5. Assagaf, Aminullah. (2015). *Analysis of Relevance Concept of Measurement Capm Return and Risk of Shares*. International Journal of Business and Management. 10. 10.5539/ijbm.v10n10p194.

6. Baltussen, G., van Bekkum, S., & van der Grient, B. (2018). Unknown unknowns: Uncertainty about risk and stock returns. *Journal of Financial and Quantitative Analysis*, *53*(4), 1615–1651. https://doi.org/10.1017/s0022109018000480

7. Bea Data Application Programming Interface (API). (n.d.). https://apps.bea.gov/api/signup/

8. Bevans, R. (2023, June 22). *Multiple linear regression: A quick guide (examples)*. Scribbr. https://www.scribbr.com/statistics/multiple-linear-regression/

9. Bolos, M.-I., Bradea, I.-A., & Delcea, C. (2019, August 7). *Modeling the performance indicators of financial assets with neutrosophic fuzzy numbers*. MDPI. https://www.mdpi.com/2073-8994/11/8/1021

10. Boloș, M.-I., Bradea, I.-A., & Delcea, C. (2021, May 21). *Optimization of financial asset neutrosophic portfolios*. MDPI. https://www.mdpi.com/2227-7390/9/11/1162

11. Boloș, M.-I., Bradea, I.-A., & Delcea, C. (2023, January 23). *Modeling the covariance of financial assets using neutrosophic fuzzy numbers*. MDPI. https://www.mdpi.com/2073-8994/15/2/320

12. Brav, A., & Heaton, J. B. (2002). Market indeterminacy. *J. Corp. L.*, *28*, 517.

13. Broumi, S., Talea, M., Smarandache, F., & Bakali, A. (2016, December). *Decision-making method based on the interval valued neutrosophic graph*. In 2016 Future Technologies Conference (FTC) (pp. 44-50). IEEE.

14. Carhart, M. M. (1997). *On persistence in mutual fund performance.* The Journal of Finance, *52*(1), 57–82. https://doi.org/10.1111/j.1540-6261.1997.tb03808.x

15. Davis James L, Eugene, Fama F. and Kenneth French R. (2000), *Characteristics, Covariances and Average Returns*: 1992 to1997, Journal of Finance 55(1), 389-406.

16. Dimson, Elroy (1979*), Risk Measurement When Shares are Subject to Infrequent Trading*, Journal of Financial Economics 7(2), 197-226.

17. Dybek, M. (2024, January 27). *Netflix Inc. (NASDAQ:NFLX): Income statement*. Stock Analysis on Net. https://www.stock-analysis-on.net/NASDAQ/Company/Netflix-Inc/Financial-Statement/Income-Statement

18. FAMA, E. F., & FRENCH, K. R. (1992). *The cross-section of expected stock returns.* The Journal of Finance, *47*(2), 427–465. https://doi.org/10.1111/j.1540-6261.1992.tb04398.x

19. Fama, E. F., & French, K. R. (1993). *Common risk factors in the returns on stocks and bonds*. Journal of Financial Economics, 33(1), 3-56. doi:10.1016/0304-405X(93)90023-5

20. Fama, E. F., & French, K. R. (2004). *The Capital Asset Pricing Model: Theory and evidence.* Journal of Economic Perspectives, 18(3), 25–46. https://doi.org/10.1257/0895330042162430

21. Fama, E. F., & French, K. R. (2015). *A five-factor asset pricing model*. Journal of Financial Economics, 116(1), 1-22. doi:10.1016/j.jfineco.2014.10.010

22. Gibbons, M. R. (1982). *Multivariate tests of financial models. Journal of Financial Economics,* 10(1), 3–27. https://doi.org/10.1016/0304-405x(82)90028-9 "Relevance of Capital Asset Pricing Model – A Review. Journal on Banking Financial Services and Insurance Research, , 1, 85–97.

23. Graham Partington and Stephen Satchell. https://www.aer.gov.au/system/files/Report to the AER - *Alternative Asset Pricing Models* - 30 June 2020.pdf

24. Guan, H., He, J., Guan, S., & Zhao, A. (2020, January 20). *Neutrosophic soft sets forecasting model for multi-attribute time series*. Zenodo. https://zenodo.org/recor

25. Hawaldar, Iqbal. (2011). RELEVANCE OF CAPITAL ASSET PRICING MODEL – A REVIEW. 1. 85 - 97.

26. Hayes, A. (n.d.). *Multiple linear regression (MLR) definition, formula, and example*. Investopedia. https://www.investopedia.com/terms/m/mlr.asp

27. Hirota, S., Huber, J., Stöckl, T., & Sunder, S. (2022). Speculation, money supply and price indeterminacy in financial markets: An experimental study. *Journal of Economic Behavior & Organization*, *200*, 1275-1296.

28. *Stock market quotes & financial news*. Investing.com. (n.d.). https://www.investing.com/

29. Jha, S., Kumar, R., Son, L. H., Chatterjee, J. M., Khari, M., Yadav, N., & Smarandache, F. (2019). Neutrosophic soft set decision making for stock trending analysis. *Evolving Systems*, *10*, 621-627.

30. Jha, S., Kumar, R., Son, L. H., Chatterjee, J. M., Khari, M., Yadav, N., & Smarandache, F. (2018, July 18). *Neutrosophic soft set decision making for stock trending analysis - evolving systems*. SpringerLink. https://link.springer.com/article/10.1007/s12530-018-9247-7

31. Kenneth R. French - data library. (n.d.). https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

32. Kenton, W. (n.d.). *Capital Asset Pricing Model (CAPM): Definition, formula, and assumptions*. Investopedia. https://www.investopedia.com/terms/c/capm.asp

33. Khan Academy. (n.d.). *Mean absolute deviation (MAD) review (article)*. Khan Academy. https://www.khanacademy.org/math/statistics-probability/summarizing-quantitative-data/other-measures-of-spread/a/mean-absolute-deviation-mad-review

34. Kumar, P., Moazzamigodarzi, M., & Rahimi, M. (2024). *Neutrosophic analysis of experimental data using neutrosophic Graeco-Latin Square design*. *Axioms*, *13*(8), 559. https://doi.org/10.3390/axioms13080559

35. Laopodis, N. T. (2021, December 14). *Financial Economics and Econometrics*. Routledge & CRC Press. https://www.routledge.com/Financial-Economics-and-Econometrics/Laopodis/p/book/9781032070179

36. Laubscher, E. R. (2002, April 1). *A review of the theory of and evidence on the use of the capital asset pricing model to estimate expected share returns.* Meditari Accountancy Research. https://www.emerald.com/insight/content/doi/10.1108/10222529200200007/full/html

37. Laubscher, E. R. (2002, April 1). *A review of the theory of and evidence on the use of the capital asset pricing model to estimate expected share returns*. Meditari Accountancy Research.

https://www.emerald.com/insight/content/doi/10.1108/10222529200200007/full/html

38. *Lesson 5: Multiple linear regression: Stat 501*. PennState: Statistics Online Courses. (n.d.-a). https://online.stat.psu.edu/stat501/lesson/5

39. Merton, R. C. (1973). An intertemporal capital asset pricing model. *Econometrica: Journal of the Econometric Society*, 867-887.

40. Nagarajan, D., Broumi, S., Smarandache, F., & Kavikumar, J. (2021). Analysis of neutrosophic multiple regression. *Neutrosophic Sets and Systems*, *43*, 44-53.

41. OpenAI. (2023). *ChatGPT (GPT-4)* [Large language model]. OpenAI. https://chat.openai.com/

42. Pham, C. D., & Phuoc, L. T. (2020). Is estimating the Capital Asset Pricing Model using monthly and short-horizon data a good choice? *Heliyon*, *6*(7). https://doi.org/10.1016/j.heliyon.2020.e04339

43. Phuoc, L. T., Kim, K. S., & Su, Y. (2018). Re-examination of estimating beta coefficient as a risk measure in CAPM. *The Journal of Asian Finance, Economics and Business*, *5*(1), 11–16. https://doi.org/10.13106/jafeb.2018.vol5.no1.11

44. Ray, S., Savin, N. E., & Tiwari, A. (2009). *Testing the CAPM revisited. Journal of Empirical Finance,* 16(5), 721–733. https://doi.org/10.1016/j.jempfin.2009.07.006

45. Roll, Richard (1977), *A Critique of the Asset Pricing Theory"s Test: Part I, on Past and Potential Testability of the Theory*, Journal of Financial Economics 4(2), 129-176.

46. Roll, Richard (1981), A Possible Explanation of Small Firm Effect, Journal of Finance 36(4), 879-88.

47. Ross, S. A. (1976). The arbitrage theory of Capital Asset Pricing. *Journal of Economic Theory*, *13*(3), 341–360. https://doi.org/10.1016/0022-0531(76)90046-6

48. Satolix.io. (2024, March 4). *Market trends: Uptrend, downtrend and range*. Medium. https://medium.com/@satolix.io/market-trends-uptrend-downtrend-and-range-ed989df2bc8c

49. Sharpe, W. F. (1964b). Capital asset prices: A theory of market equilibrium under conditions of risk*. *The Journal of Finance*, *19*(3), 425–442. https://doi.org/10.1111/j.1540-6261.1964.tb02865.x

50. Sherwani, R. A. K., Shakeel, H., Saleem, M., Awan, W. B., Aslam, M., & Farooq, M. (2021). A new neutrosophic sign test: An application to COVID-19 data. *Plos one*, *16*(8), e0255671.

51. Smarandache, F. (2014). *Introduction to neutrosophic statistics*. UNM Digital Repository. https://digitalrepository.unm.edu/math_fsp/33/

52. Smarandache, F. (2015, June 15). *Neutrosophic precalculus and neutrosophic calculus: Neutrosophic Applications*. Google Books. https://books.google.com/books/about/Neutrosophic_Precalculus_and_Neutrosophi.html?id=HLIvDwAAQBAJ

53. Smarandache, F. (2022). Neutrosophic statistics is an extension of interval statistics, while plithogenic statistics is the most general form of Statistics (second version). *International Journal of Neutrosophic Science*, *19*(1), 148–165. https://doi.org/10.54216/ijns.190111

54. Sp500 (https://finance.yahoo.com/world-indices), amazon (https://finance.yahoo.com/quote/AMZN/), apple (https://finance.yahoo.com/quote/AAPL/)

55. Tanuwijaya, Billy & Selvachandran, Ganeshsree & Son, Le & Abdel-Basset, Mohamed & Huynh, Hiep & Pham, Van-Huy & Ismail, Mahmoud. (2020). *A Novel Single Valued Neutrosophic Hesitant Fuzzy Time Series Model: Applications in Indonesian and Argentinian Stock Index Forecasting.* IEEE Access. PP. 1-1. 10.1109/ACCESS.2020.2982825.

56. U.S. Bureau of Labor Statistics. (n.d.). *Registration engine*. U.S. Bureau of Labor Statistics. https://data.bls.gov/registrationEngine/

57. Villafuerte, J. L. R., Torres, L. D. T., & Jimenez, L. T. (2020). *Neutrosophic Hypothesis to validate a modification for Article 630 of the Integral Organic Criminal Code of Ecuador* (Vol. 37). Infinite Study.

58. Wahyuny, T., & Gunarsih, T. (2020). Comparative analysis of accuracy between capital asset pricing model (CAPM) and arbitrage pricing theory (APT) in predicting stock return (case study: manufacturing companies listed on the Indonesia stock exchange for the 2015-2018 period). *Journal of Applied Economics in Developing Countries*, *5*(1), 23-30.

59. Ward, M. (2000). The CAPM in an options pricing framework. *Investment Analysts Journal*, *29*(52), 35–44. https://doi.org/10.1080/10293523.2000.11082411

60. *Welcome to stat 501!: Stat 501*. PennState: Statistics Online Courses. (n.d.-c). https://online.stat.psu.edu/stat501/

61. Woodall, W. H., Driscoll, A. R., & Montgomery, D. C. (2022). A review and perspective on neutrosophic statistical process monitoring methods. *IEEE Access*, *10*, 100456-100462.

62. Yahoo! (2024, May,20). *Netflix, Inc. (NFLX) stock price, news, Quote & History*. Yahoo! Finance. https://ca.finance.yahoo.com/quote/NFLX/

63. Yahoo! (2024a, August 13). *Nasdaq composite (^IXIC) charts, Data & News*. Yahoo! Finance. https://ca.finance.yahoo.com/quote/%5EIXIC/

64. YouTube. (n.d.-a). YouTube. https://www.youtube.com/

65. Yun, S. (2021). Decision-making under Market Indeterminacy. *Journal of Finance Research*, *5*(2), 22-27.

66. Zhang, H., Wang, J., & Chen, X. (2014). Interval neutrosophic sets and their application in multicriteria decision making problems. *The Scientific World Journal*, *2014*, 1–15. https://doi.org/10.1155/2014/645953

8.0.Appendix

8.1.Coding Appendix

Python Code for Graph 4.1.1

```
import yfinance as yf

import matplotlib.pyplot as plt

# Define the ticker symbol

ticker_symbol = 'NFLX'

# Define the start and end dates

start_date = '2009-01-01'

end_date = '2023-12-31'

# Download monthly historical data

df = yf.download(ticker_symbol, start=start_date, end=end_date, interval='1mo')

# Plotting the time series

plt.figure(figsize=(10, 4))

plt.plot(df.index, df['Adj Close'], label='Adjusted Close')
```

```
plt.title('Adjusted Close Price for NFLX for the period 2009-2023')

plt.xlabel('Date Period')

plt.ylabel('Adjusted Close Price')

plt.legend()

plt.grid(True)

plt.xticks(rotation=45)

plt.tight_layout()

plt.show()
```

Python code for Graph 4.1.2

```
import yfinance as yf
import pandas as pd
import matplotlib.pyplot as plt

# Fetch the data for Netflix (NFLX) from Yahoo Finance
ticker = 'NFLX'
startDate = '2009-01-01'
endDate = '2023-12-31'
data = yf.download(ticker, start=startDate, end=endDate, interval='1mo')

# Keep only the necessary columns: High, Low, and Adj Close
df = data[['High', 'Low', 'Adj Close']]

# Plot the data
plt.figure(figsize=(12, 6))

plt.plot(df.index, df['High'], label='Monthly High', color='blue')
plt.plot(df.index, df['Low'], label='Monthly Low', color='green')
```

```
plt.plot(df.index, df['Adj Close'], label='Adjusted Close', color='red')

plt.title('NFLX Monthly High, Low, and Adjusted Close Prices for the period (2009-2023)')
plt.xlabel('Date Period')
plt.xticks(rotation=45)
plt.ylabel('Price (USD)')
plt.legend()
plt.grid(True)
plt.show()
```