IMPLEMENTING KEYMATH APPROACH FOR ASSESSMENT, LEARNING, AND TEACHING FOR AN INCLUSIVE MIDDLE SCHOOL CLASSROOM IN BRITISH COLUMBIA

by

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Abstract

Mathematical pedagogy has a large body of research as it pertains to both typically achieving students and learners with special needs, particularly math disabilities. The label of mathematical disability is dependent on several types of assessments that measure various aspects of cognition related to mathematical skill. Rural school districts, such as the one where this project started, have limited resources to assess and instruct learners with mathematical disabilities. This project made use of the KeyMath-3 diagnostic assessment in the construction of classroom math units using the KeyMath-3 diagnostic assessment that guides diagnosis of math disability. This diagnostic assessment was used as a focus for the language and types of questions used in various mathematical units. The British Columbia Ministry of Education math curriculum, KeyMath-3 diagnostic assessment, and IXL.com math program were all analyzed to find common language and goals as the focal points for the lessons. Probability, Pythagorean Theorem, Algebraic Expression, and Surface Area units were constructed at the Grade 8 middle school level using this approach to make learning accessible to learners with math disabilities while simultaneously allowing stronger math learners to fully express their levels of mathematical understanding. The combined use of diagnostic assessment, curricular goals, and support programs analyzed in this project allows for the construction of math units that could improve the understanding of all math learners, especially those students with math disabilities.

Keywords: KeyMath, assessment, mathematics, diagnosis, math disability

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The idea for this project came from my belief that research should inform teaching practice and that this leads to the best possible experience for all learners. I have created units with other teachers based around current trends in research such as universal design and backwards design and this has resulted in exceptional units that have students engaged across multiple difficulty levels, especially those with individualized education plans. This project would not have occurred without those educators who supported me in the construction of these units and those students who found that they could be good at math with the right mindset.

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Chapter 1: Introduction

The instruction of mathematics constitutes a cornerstone in the educational systems of an industrial and global society, and the importance of adult numeracy is a concern for many educators and governmental bodies (Coben, 2003; Gal, 2000; Johnston, 2002; National Numeracy Review Report Panel, 2008; Parsons & Bynner, 2005). The ability of citizens to participate in everyday finances also requires a reasonable level of mathematical competence (Northcote & McIntosh, 1999). The use of mathematics permeates modern society and the correct application of mathematical algorithms occurs in the lives of citizens on a daily basis (Northcote & McIntosh, 1999). Correct, efficient, and fluent use of mathematical concepts is a stated target of mathematical curricula (BC Ministry of Education, 2016). This importance has led to debates on the proper and effective instruction of mathematics at the junior and high school levels (Hutchinson, 2017; Winzer & Mazurek, 2011).

Scholars of mathematics education have reiterated the competence and ability of the instructor, which has significant effects on the success of students (Brownell, Sindellar, Kiely & Danielson 2010; Bouck & Flanagan, 2009). In addition, diagnoses of students' relevant academic abilities can also lead to different methods of teaching mathematical concepts and other contents of the curriculum (Ketterlin-Gellar, Chard & Fien, 2008; Seo & Bryant, 2012; Zheng, Flynn, & Swanson 2013). Modern technological applications in the classroom have greatly impacted the ability to teach mathematical concepts and have the potential to make even abstract concepts more understandable for all learners (Loong, 2014; Tezer & Kanbul, 2009; Watson 2016). These factors are in turn influenced by the philosophical beliefs and values of the teachers' involved (Winzer,

2007) while the "correct" philosophical teaching approach has undergone shifts over the history of mathematics education (Haggarty and Pepin, 2002; Katz, Jankvist, Fried & Rowlands, 2014).

Significance of the Project

Current trends in mathematics education have been readily adopted by teachers in many schools, especially at the elementary and high school levels so as to ensure success of learners (BC Ministry of Education, 2016). The idea of having all students access the material and express their understanding in unique ways has been seen as the way to bring all students into a single learning environment in an inclusive and successful way (Dieker & Rodriguez, 2013; Sayeski & Paulsen, 2010). Further significance of the developed lesson units could provide both support for the continued use of the KeyMath-3 assessment and place the assessment in the role of informing teaching practice in a more direct and innovative way, thereby providing variety for the teachers and learners.

In addition, through the developed units, students' learning needs can be predicted by the teachers and can be compensated with appropriate strategies, resources and technologies (BC Ministry of Education, 2016). The developed lesson units using the KeyMath approach could alleviate time constraints on teachers and allow for more universal participation for learners challenged with mathematic concepts across the middle school classroom, the target of the project. These learners that would typically be isolated from what the rest of the class is working on will be able to participate in a relevant and constructive manner (Daugherty, Campana, Kontos, Flores, & Shaw, 1996; Pliner & Johnson, 2004).

The middle high school math units constructed and examined in this project have been used for several years but will now be presented in the context of current assessment practices for learners with special needs.

In sum, due to the emphasis on Math learning success, acquisition of math and the need for applying math concepts in practical real-life situations for students as referenced in the new BC Curriculum (BC Ministry of Education, 2016), this project can hopefully be of significance to students' successful learning.

Background of the Project

This project is based on the researcher's teaching experiences at a middle school in northern British Columbia. It is the only middle school in this rural BC school district. The school has approximately 450 students with 150 students in Grades 6, 7, and 8 respectively. The Grade 8 class has five divisions that are taught by three math teachers. The demographics of the student population are predominantly First Nations, with the remaining students being from a variety of other backgrounds (Kelley & Legge, 2006). The school has a population of students who are predominantly from a lower socioeconomic status and there is a high proportion of students with Individualized Education Plans (IEPs) (Kelley & Legge, 2006). There are many students on waiting lists to be assessed for possible IEP designations.

This project has grown out of teachers reporting increased student success and participation when using the methods of Universal Design for Learning (UDL) and a computer practice program, IXL, in the teaching of math. The math teachers of the school found success in increasing student engagement across skill levels and have seen an increase in student initiative in math projects. These projects have allowed students to display their individual abilities to a much greater extent than traditional math projects that limited student options for expression. With the focus of this project on the inclusion of the KeyMath-3 assessment diagnostic tool, it is hoped that more student learning problems and challenges will be addressed during unit construction which was the purpose of this project.

Personal Location

For eight years I have been a math and science teacher in a rural British Columbia school district. I have taught adults and middle school students. In both environments, I have encountered individuals with barriers to their education. Some students had diagnosed disabilities while others likely had undiagnosed disabilities. Either scenario would have led to difficulties in the classroom and in the math classroom in particular.

Through multiple professional development workshops and district initiatives I was exposed to the idea of Universal Design for Learning (UDL) and the idea of removing educational barriers for students at the level of unit construction. The school district and teachers started initiatives to develop units that used the UDL approach to create several units. Success was found when these units were implemented and the units have been improved over the years.

Both classroom and special education teachers in the school have experience using the unit construction in Math subjects as a means of addressing the challenges of learners with special needs. There are still some questions as to whether the approach is truly effective. Some teachers have been hesitant to begin the process of recreating units from a student perspective. With this project, I hoped to provide innovative examples of approaches to unit construction using the KeyMath diagnostic assessments, which will have a significant impact on Special Education teachers in the production of Math curriculum units in middle school classrooms.

Purpose of the Project

The primary purpose of this project was to develop units using the KeyMath diagnostic as an approach for teachers to improve the learning environment of learners in Math classrooms, especially for special education learners at the middle school level.

The secondary purpose of the project is to circumvent this problem not through repeated practice of multiple methods but through altering the entirety of the middle school BC math curriculum unit to focus on fewer, but more universally applicable, mathematical algorithms.

Additionally, the project aims at improving teaching practices for classroom math teachers of middle school through the developed units from the current British Columbia Mathematics Curriculum that corresponds with the KeyMath-3 diagnostic assessment.

Other aims of the project are to create lessons that include exceptional learners and their potential math disabilities (MD) as recommended by many Canadian scholars of exceptional learners (Hallahan, Kauffman & Pullen, 2015; Hutchinson, 2017; Winzer, 2007).

Researcher Context

For this project, the research context was based on documentary and content analysis of relevant literature related to math assessment and pedagogies for inclusive classrooms. I used the BC middle school math curriculum (BC Ministry of Education, 2016) at the Grade 8 level to develop my units using the KeyMath-3 diagnostic assessment approach.

As the creator of the units, I synthesized a set of lessons that would target potential deficits in math ability and understanding. By using a diagnostic tool and current research to guide unit construction and directly influence my approach to lessons, I created student-centered lessons that promote accessibility to the material by determining what criteria students will be assessed on in a diagnostic assessment such as the KeyMath-3.

As earlier stated, in my current middle school, I created math teaching units using the UDL approach and continue to be an active proponent of the *IXL* computer program being used. The other teachers and I have combined our expertise to create lessons, notes, and assignments that make use of modern math pedagogy such as the Universal Design approach. By using my experience in a special education program, I hope to further refine and enhance my ability to create units that make math accessible to all learners. A major focus of this project is bringing together innovative modern math teaching and assessment tools in both general math and special education pedagogy to implement the best teaching practice possible for the math concepts being taught.

Project Overview

The following project will provide examples of the thought process behind lessons and units created using diagnostic assessments with special needs learners in mind. The units that are developed will greatly improve both student and teacher understanding of mathematical concepts by using the KeyMath approach. From the student perspective, lessons have been made more accessible and assignments have been made open-ended and have allowed for more personal expression. Learners that typically face a large number of barriers can now feel some learning satisfaction, motivation and success in the math classroom with the KeyMath approach. Student projects, such as the Pythagorean Art assignment or the student created game of chance give students a basic set of criteria that can be expanded on according to student interest. The build up to these projects includes assignments focused around concrete understanding of the fundamental concepts while allowing for all learners to express their understanding to the best of their ability. Students are taught how to manipulate formulas but are also given heuristics to complete questions quickly. An example of this is the multiple uses of the Pythagorean Theorem to find both a leg and a hypotenuse. Students are all taught how to manipulate the formula but are also given key words and clues to watch for that allow them to know which variation of the formula n to use. Approaching the unit from the KeyMath perspective has highlighted common questions that students are assessed on and this has focused the lessons towards ensuring that all students are given the tools to complete a given question.

From the perspective of teachers, the units could replace rote practice lessons with more involved lessons focusing on understanding and discussion of the concept being taught. Use of computer assistive technology is what frees the teacher to engage with students more about understanding rather than simply as a source for correct answers. Using the KeyMATH diagnostic to highlight the basic skills that are assessed in the diagnostic will focus the teachers' efforts and allow the teacher to work on areas of need and be more efficient with the limited class time available.

The next chapter of this project provides a review of literature as it relates to general math pedagogy, special education math instruction, diagnosis of math disabilities, and the use of technology in assisting mathematics instruction.

Chapter Summary

This project has come out of a perceived need to improve the math learning success of students at a British Columbia middle school that services a large number of low socioeconomic status and at-risk students. I have found that many students struggle with not simply the mathematical operations that are taught in math class, but with the language of the questions as well. Knowing that language can be a stumbling block for students, I make efforts in class to ensure that the language of the tests and quizzes parallels the language used in class to limit the confusion for students. I examined the KeyMath-3 diagnostic test to check for the language that is used in the assessment in the hopes of further focusing the language that I use in my math classes so that students who are assessed using this diagnostic will be able to more clearly express their understanding of mathematics. Math fluency has the potential to improve the lives of students as mathematical operations are a common part of daily life (Northcote & McIntosh, 1999). It is with this idea in mind that I am creating units that can be accessed by all learners so that they not only learn the material, but are able to express their learning clearly. Using diagnostic test as the starting point for unit construction gives me the language that will focus the unit towards areas commonly assessed by the KeyMath-3 assessment. By using this KeyMath approach, both teachers and learners could benefit. Students may benefit from consistent use of language that parallels the diagnostic that tests their math ability. Teachers may benefit from the KeyMath method by having their efforts focused towards areas of need and by improving the quality of student instruction in the classroom.

Chapter 2: Literature Review

This literature review goes through the current understanding of math pedagogy in both the general classroom and in special education services. The use of modern technology to assist in these learning environments is a topic of interest in both the general and special education classrooms. Common math disabilities and their solutions are also examined. The diagnosis of these math disabilities and the assessments that are used to determine if a learner has a math disability are then discussed with emphasis placed on the KeyMath assessment that is the focus of this project.

Current Understanding of Math Pedagogy

There is a broad consensus on mathematical knowledge for teaching (MKT) and what comprises this body of knowledge: subject matter knowledge and pedagogical content knowledge (Brownell, Sindellar, Kiely, & Danielson, 2010). Additionally, Brownell, Sindellar, Kiely & Danielson (2010) proposed that these two categories can be broadly translated into “what it is” and “how to teach it” respectively with regards to mathematical concepts. The authors further stated that with the current existing body of research into mathematics education, it is important to ensure that effective practices are improved and less effective practices are removed from teacher education. Brownell, Sindellar, Kiely, & Danielson (2010) stated that research should be conducted into effective mathematics instruction for the identification of practices that may not only be ineffective but also have a net negative effect on children’s understanding. Sullivan and Field (2013) suggested that current methodologies used in pre-school special education, which have a statistically significant negative effect on learners’ math progress. The existence of possible threshold effects for certain approaches to mathematics instruction could change the amount of time allocated to certain tasks or approaches that may traditionally have made up a great deal of the instructional time in a math classroom (Doabler, Clark, Stoolmiller, Kosty, Fien, Smolkowsky, & Baker, 2017). Furthermore, Doabler et al., (2017) reported that rote practice of skills and time spent on certain tasks reaches a limit of effectiveness and that time could have been spent on other tasks that would have led to greater gains in understanding (p. 107).

Another division of mathematical knowledge that also exists includes conceptual and procedural knowledge, often seen as two distinct areas of math knowledge (Agrawal & Morin, 2014). Current focus of conceptual knowledge is the general idea of a mathematical concept in the course of teaching and learning. Agrawal and Morin (2014) stated this knowledge as the linking of new phenomenon with known phenomena in an attempt to see patterns. Procedural knowledge is the following of a series of steps to solve a problem and it is mainly the ability to correctly apply and use an algorithm to produce an answer (Agrawal & Morin, 2014). A student may have an understanding of how a concept relates to others but cannot properly follow the steps to produce a correct answer. Conversely, as is the case for many students, procedural knowledge may be given in a vacuum. The students may be able to correctly follow a procedure to produce an answer but may have little knowledge or ability to see how this procedure relates to other mathematical applications. It is when students are taught, and become proficient in, both procedural and conceptual knowledge that real growth and understanding occurs.

Powell (2015) identified some current factors of effective mathematics instruction such as general awareness of mathematical concepts and the relevant pedagogy. The common misconceptions related to math concepts are also important factors so that these misconceptions can be kept from taking root in the habits and understanding of math learners. Other current pedagogical practices in the teaching of math are impacted by awareness of legislation and standards as they pertain to the grade being taught and for all grades (BC Ministry of Education, 2016). This becomes expressly important when emphasizing skills and algorithms for those students with math disabilities and focusing on areas of the greatest relevance to future interests.

Agrawal and Morin (2016) reported that one of the current practices prevalent in the teaching of mathematical concepts is that of CRA or concrete-representational-abstract. In this teaching method, hands-on activities are introduced to give students direct experience in the concept being taught (Agrawal & Morin, 2016). Representations of the manipulatives used are then given alongside abstract notation on a board at the front of the class. In this way, students are given scaffolding of the abstract concept in the form of direct interaction (concrete) and diagram and picture examples (representations) (Agrawal & Morin, 2016). Particular success has been found in teaching algebra to learners with special needs using the CRA methods (Witzel, 2005).

There is a lack of math knowledge background given by teacher educators to new teachers which affects the varied nature of mathematics instructional competence which correlates with poor or weak student academic performance (Stotsky 2009). Current emphasis on ways of how to teach math in teacher education programs is given attention in the numeracy and technology courses of teacher education programs to solve this problems (Kitchenham, 2011). Teacher confidence and competence is seen as a key factor in student academic achievement (Ekstam, Korhonen, Linnanmaki & Aunio, 2017; Kitchenham, 2011).

Current research supports and has influenced the shift of math education away from rote practice skills in isolated situations to a more student centered and problem solving set of skills (Boyd & Bagerhuff, 2009; Sayeski & Paulsen, 2010; Shin & Bryant, 2015). Furthermore, Boyd and Bagerhuff (2009) explained the use of relevant problems as having the ability to maintain student interest while allowing for the practice of applying math skills to real world situations that students are likely to encounter. This method has also been shown to improve student performance and application of learned processes and allows them to "see the point" of math and gives the students impetus to put in the necessary effort to internalize the procedures learned (Boyd & Bagerhuff, 2009). Teachers adopting this approach must relate the mathematical curriculum to real life situations and situations that hold personal relevance and importance for the student as part of the classroom practice (Hallahan, Kauffman, & Pullen, 2015; Sayeski & Paulsen, 2010). Current studies reported that by giving students mathematical problem solving situations related to the real world, the students will not question the importance or use of the material they are being taught and will connect the math to their daily lives (Sayeski & Paulsen, 2010).

The process of providing personal motivation is doubly important for special needs students as they are already at risk of low motivation due to the increased difficulty with mathematics and other academic areas (Willcutt, Petrill, Wu, Boada, DeFries, Olson & Pennington, 2013).

A classical approach to mathematics education could have very real application in modern special needs education (Katz, Jankvist, Fried & Rowlands, 2013). According to the author's report, classical mathematical proofs, especially for those concepts taught in early to middle grades, was involved, hands-on, concrete and often showed mathematical patterns in everyday life. These practical uses lend themselves to the kind of mathematical literacy that is being sought for students. An added advantage to this approach is the relevance mathematics would have on other disciplines, i.e. social studies curricula and the math of a culture, which will allow for the creation of cross-curricular units. An example of this would be the mathematics involved in the creation of monuments constructed by ancient peoples (pyramids and temples). While not necessarily a comprehensive solution to the problem of student interest, this broadens the application and relevance of mathematical processes to culturally significant or at least culturally recognized aspects of the world (Katz, Jankvist, Fried, & Rowlands, 2013). Such interdisciplinary techniques in mathematics at the secondary school level is not highly encouraged, especially by math teachers with less teaching experience and background knowledge (Hallahan, Kauffman, & Pullen, 2015; Winzer, 2007).

Current math pedagogical approaches by teachers in high schools center on the use of textbooks in line with most Canadian provincial and territorial math curriculum policies (BC Ministry of Education, 2016; Hutchinson, 2017). Textbooks are often viewed as the sentinels that can determine what is elite or appropriate information to pass on to pupils (Haggarty & Pepin, 2002). Such important documents as math textbooks should be held to a high standard but textbooks can vary greatly in their approaches to mathematical content. How students are taught to do math will determine the type of math textbooks used, from simple math books with illustrations and pictorials for junior elementary grades to a more social story textbook for middle school students. The selection of appropriate texts will be especially important for those learners challenged by and diagnosed with dyscalculia (Hutchinson, 2017).

Recent developments in math pedagogy involve moving away from sole reliance on text-based resources, due to the fact that evidence of multiple sources of representation is advantageous to the development of mathematical understanding (Loong, 2014). The increased availability of online resources and the prevalent access students have to information from multiple sources requires that students be better prepared to filter information on their own. A single text that must be taken as fact is counterproductive to the development of this skill (Tezer & Kanbul, 2009).

The use of visual representations (VR) has been shown to be effective in teaching mathematical concepts in today's math classroom at all levels. A visual representation is generally regarded as a physical representation of a student's mental processing during problem solving (van Garderen, Sindellar, Kiely, & Danielson, 2016). These are used by both special education and mainstream classroom teachers.

Emphasis towards lower-level questions as opposed to higher-level questions is a common feature of math classrooms, particularly for new teachers. This teacher approach may seem logical given the deficits of special needs learners but is counterproductive to the argument that real-world multi-step problems improve student overall understanding (Griffin, Jitendra, & League, 2009). The focus of mathematics classrooms is shifting towards fewer in number but more complicated real-world problems. The emphasis of special needs math teachers towards basic questions is widening the ability gap between special needs student and their peers without math disabilities (Bjorklund, 2012; Schulte & Stevens, 2015).

Fraction operations are challenging for teacher delivery methods as well as student learning engagement (Shin & Bryant, 2015). This area of mathematics, and the proper way to teach it, has been the focus of current math research (Agrawal & Morin, 2016; Shin & Bryant, 2015). Effective strategies often included a combination of concrete and visual representations; explicit and systematic instruction; and the use of real world problems (Agrawal & Morin, 2016; Shin & Bryant, 2015).

Use of Computer Assistive Instruction

Other strategies currently used by math teacher include the use of computer assisted learning and the general increase in creating problems that are directly relevant to students (Bouck & Flanagan, 2009). Currently, the study of probability is being used as a means to relate fractions to real world mathematics and forms the basis for one of the units developed in this project.

Research has highlighted some key factors in successful implementation of technology in mathematics education and among these is the ability to maintain student attention. This has been shown to be a key factor in the early years of mathematics instruction (Tezer & Kanbul, 2009). The internalization of the key concepts of math learned in early grades has the benefit of improving the ability of math learners to participate in the more complicated problems of advanced mathematics. Without these basics at a high enough level of competency, if not mastery, math learners cannot fully participate or conceptualize the real-world problems that are increasingly the focus of mathematics classrooms (Tezer & Kanbul, 2009).

Powell (2015) also noted that a key factor of mathematics instruction is assessing and responding to assessment and that current technology can greatly assist in this particular endeavor. Increasing the amount of self-correcting practice and immediate feedback that occurs with computer instruction can therefore be a valuable asset to teaching mathematics. In addition to the advantages of using computers in math instruction, all learners (including learners with dyscalculia) benefit from immediate and constructive feedback. Unfortunately, constraints on achieving this level of feedback exist in the classroom (Hutchinson, 2017). Using computer programs has the potential to alleviate misconceptions that can develop. The ability of students to independently practice outside the classroom is also opened up with computer based math instruction (Hutchinson, 2017), as well as allowing the development of additional practice time. Additional practice time is seen as a key factor in the approaches used to improve the skills of learners with special needs (Hutchinson, 2017; Winzer, 2007).

In terms of manipulating a computer representation or manipulating a physical object, it was found by Loong (2014) that a combination of both methods yielded better results than either option on its own. A variety in the presentation of mathematical concepts, so long as it involves manipulation and participation, improves student understanding of mathematics. The selection of these virtual manipulatives needs to have three (3) key factors considered, which include mathematical, cognitive, and pedagogical fidelity must be considered for the manipulatives (Loong, 2014). Mathematical fidelity is the degree to which the virtual environment represents the mathematical properties of the objects being manipulated and this can be seen as the mathematical validity of the representation being shown (Loong, 2014).

Cognitive fidelity is the degree to which the manipulatives represent student understanding and thinking (cognition) of the math concept; this is a measure of how much the manipulative parallels the student's own understanding of the concept (Loong, 2014).

The third type of fidelity, pedagogical, is the degree to which the manipulative represents the teachers own methods and therefore the understanding that has been conveyed in the classroom (Loong, 2014). While these factors are being applied to virtual manipulatives, the same principles to apply to physical manipulatives as well and are important factors to consider when using any manipulative or instructional example.

Any computer program or game that is used will need to have great care given during its programming to create something that uses proper math etiology and pedagogy (Bakker, van den Heuvel-Panhuizen, & Robitzsch, 2016). An important fact to consider is the use of games as a math pedagogical approach and that games can prove to be statistically significant in the improvement of student understanding, but caution must be taken to properly create such a program from its inception to its clinical practice (Bakker, van den Heuvel-Panhuizen, & Robitzsch, 2016). Indeed, teachers should take the time to familiarize themselves with any program that is used and be able to base their decision on pedagogy that they use in the classroom.

Just as important as the choice and use of technology in the teaching of math is the teacher training in such technology (Bouck & Flanagan, 2009). The use of technology as part of education should be infused throughout teacher education services as a means of broadening its application in teaching practice (Bouck & Flanagan 2009; Kitchenham, 2011).

Shin and Bryant (2017) reported that computer programs with corrective feedback and sequential difficulty have been shown to improve student performance. The authors reported that this factor would be a deciding factor in the selection of math programs for classroom use. To this end, the IXL math program was chosen to support students in this project.

Math Pedagogy in Special Education

The mathematical instruction of students in special education is a growing concern due to different philosophical and pedagogical approaches to mathematics instruction, the support given to mathematics instruction, and the reinforcement of the mathematics instruction (Bjorklund, 2012; Dieker & Rodriguez, 2013; Ekstam, Korhonen, Linnanmaki, & Aunio, 2017; Griffin, Jitendra & League, 2009; Sayeski & Paulsen, 2010).

Students in special education are already at a deficit when compared to other non-special-education students in elementary, middle, and high school grades. More traditional math pedagogies are not inclusive of these special needs, especially for those students who have IEPs (Hutchinson, 2017; Winzer, 2012). For most teachers, the focus of math pedagogies has been the constructivist approach which continues to current times (Boyd & Bagerhuff, 2009). The traditional process for instructing higher-need exceptional students has traditionally been approached from a behaviourist perspective (Boyd & Bagerhuff, 2009). These two approaches can be seen as contradictory in several respects.

Chong and Siegel (2008) conducted a longitudinal study of a group of 214 children across the second and fifth grades. The study investigated two types of computational deficits: procedural deficits and fact fluency deficits. Both of these deficits are strongly associated with math learning disability. The constructivist approach of many math teachers involves establishing an algorithm or formula that requires certain pieces of information (Chong & Siegel, 2008). This approach focuses on the procedural skills of mathematics and it was found that the gaps in procedural knowledge of the students lessened between the second and fifth grades. The author stated that if these pieces of information are correctly discerned and entered into the algorithm, the correct answer is given. Initially, only the relevant pieces of information are given to allow for practice. As students become more proficient, more information is given that requires additional algorithms to create the relevant pieces of information (Chong & Siegel, 2008). These added steps often build on the math knowledge gained in previous lessons or school years. Additionally, questions can begin to contain extraneous pieces of information that need to be filtered out of the question to obtain the correct solution. These multistep approaches and extraneous pieces of information can cause problems for special needs learners who struggle with short-term and working memory deficits (Chong & Siegel, 2008; Garrett, Mazzocco & Baker, 2006). The fact fluency deficits were found to be more stable when compared to the progress made in the procedural deficits for the same children (Chong & Siegel, 2008).

A discrepancy exists between the more constructivist approach in the main-stream classroom and the support that is given in the special needs classroom. Math support provided by teachers to learners with special needs in classrooms is focused on practiced behaviour that can be internalized through enough practice (Swanson, 2015; Zheng, Flynn, & Swanson, 2013). For special education students in lower grades, most pedagogical practices of teachers often involve the use of manipulatives in teaching basic concepts and algorithms with information given in a very consistent manner, with little or no deviation throughout the unit (Griffin, Jitendra, & League, 2009). While this method is in line with older methods of teaching mathematics it has had very little success with providing special needs students with the ability to close the gaps in their education and allow those students to join with their peers in advanced level mathematics courses or pursue more mathematics oriented careers (Swanson & Jerman, 2006).

Approaching mathematics instruction from both perspectives and amalgamating the methods of both constructivist and behaviourist approaches could have the potential to benefit special needs learners in the classroom (Boyd & Bagerhuff, 2009). This may also have the benefit of leveling the playing field and circumventing the need to retrofit lessons based on IEP goals for special education learners in all grades. Although pullout instruction may be beneficial for some special education learners in math lessons, especially as added practice, it can also limit further interaction with peers, which may be a limitation to learning socialization and collaboration skills (Zheng, Flynn, & Swanson 2013).

Teacher use of more complex problems, and the learning of skills necessary to sort out relevant information which can be used to solve a desired solution, is a typical approach to teaching many math topics. This is seen as the most “real world” approach to mathematics instruction (Shin & Bryant, 2015). The mathematical demands of the workforce require that students have these skills and this method is becoming much more common place in the math classroom, especially at the high school level, where community living transitioning strategies are taught to special needs students (Hallahan, Kauffman, & Pullen, 2015). This differs from the traditional teacher approach of providing a set of basic learning problems for students to practice in isolation of the situations where algorithms and formulas might apply. The literature supports the idea of authentic practice and real world application in math class (Shin & Bryant, 2015).

Math that is authentic has meaning for the student, regardless of practical use (Shin & Bryant, 2015). The drawing of proportional shapes on graph paper may have a purely aesthetic appeal for students, but it provides the representation of math with an artistic meaning, thereby linking the discipline to other disciplines such as the arts, one of the goals of the New BC Curriculum (BC Ministry of Education, 2016). This has relevance to this project because this approach was used in the creation of several assignments and activities in the units that were created. Artistic expression of mathematical concepts was used to improve student interest and investment in the mathematical concepts of the lessons.

Real world math has a clear and obvious practical purpose for the student, especially Low Incident Exceptionality (LIE) special needs students at the middle and high school ages, who are prepared for community transition living. Using Math in real life situations, such as the purchasing of items at the best price, is a practical learning outcome for the students (Hallahan, Kauffman, & Pullen, 2015; Hutchinson, 2017). Math content such as multiplying the sides of shapes to determine area has practical purposes for construction and other trades, where some special needs adolescents at the middle and high school level may find themselves (Hallahan, Kauffman, & Pullen, 2015). The above are both authentic and real world practice for students, which helps to maintain student interest in math topics (Bjorklund, 2012; Boyd & Bagerhuff, 2009).

Intervention at the neuropsychological level that target specific cognitive deficits rather than tackling mathematical deficits directly has been shown to improve mathematical performance (Faramarzi & Sadri, 2014). The researchers examined a group of 30 second grade girls with dyscalculia. Using a pretest, posttest and control group, the researchers used the Wechsler Intelligence Scale for Children (WISC-III-R) and the KeyMath assessment. The study found a statistically significant different between the experimental and control groups at the $p \leq 0.001$ level. This idea that neuropsychological interventions can assist students with dyscalculia opens the door to several types of interventions that can be undertaken in the special education classroom. Math exercises could be done at home with the support of family members and would not be academically demanding for parents or guardians of students. Faramarzi & Sadri (2014) found the following four domains to influence math interventions for learners, especially the special education students.

Reinforcing active memory. Poor active to non-active memory are some of the exceptionalities experienced by many students experiencing Math difficulties as well as those diagnosed with dyscalculia in middle to high schools across Canada (Hutchinson, 2017; Winzer, 2013). Various studies recommend learning activities that support this cognitive aspects which include recognition memory of hidden objects, while other teaching-learning strategies include the improvement of visual and auditory memory. Active use of memory in the forms of matching games using flashcards can strengthen this area of cognition and improve the ability of students to remember math facts quickly and automatically (Faramarzi & Sadri, 2014). By improving automaticity for memory recall, the working memory limitations of MD students can be eased and allow for the advancement of mathematical understanding (Faramarzi & Sadri, 2014).

Reinforcing attention. These activities include looking at a picture where an object is obscured by other objects; books and games that involve looking for objects in a cluttered image help to improve a student's ability to attend to certain features amongst seeming chaos (Faramarzi & Sadri, 2014). For auditory deficits, a similar activity can be undertaken using recordings where a particular message needs to be interpreted amidst background noise (Faramarzi & Sadri, 2014). The applications of these teaching and learning strategies are beneficial to special needs learners, especially those challenged by attention deficit as ADHD, Indigo and Spinal Bifida students (Hutchinson, 2017; Lipson, 2004; Rose & Holmbeck, 2007; Stegeman & Aucion, 2018).

Training executive functions. Most special needs learners are challenged by limited executive functions such as organizing and planning (Faramarzi & Sadri, 2014). These functions can be reinforced in math pedagogy by having students at the lower grades group blocks according to size, colour, thickness, or any other relevant criteria, and by constructing structures according to a reference model. This will facilitate self-training of some of these executive functions (Faramarzi & Sadri, 2014). Such math teaching-learning strategies will not only benefit special needs learners executive function deficits, but engage them in hands on learning, which enhances the psychomotor level domains of Blooms taxonomy of learning (Anderson & Krathwol, 2001).

Developing and reinforcing visuospatial perception. Faramarzi and Sadri (2014) reported several ways that visuospatial perception can be developed, especially in students with special needs. These include balancing and hand-eye coordination exercises that will improve this perception. Identifying objects without the use of the eyes (with the exception of visually impaired special needs learners) can serve to improve visuospatial perception (Faramarzi & Sadri, 2014). Graph paper is commonly used in the math classroom and the practice of copying models using graph paper can increase perception while making students familiar with the use of graph paper to maintain organization in drawing and writing (Faramarzi & Sadri, 2014). While these math teaching strategies are good and effective, it excludes some special needs learners as mentioned above, hence alternative inclusive strategies must be sought by teachers.

Reinforcing skills related to speech and language in math. Games that improve phonological awareness are abundant and are often played by parents and children. In this theory of intervention, these phonological games not only serve their directly intended purposes of improving language, but may also improve the student's math ability indirectly (Faramarzi & Sadri, 2014). Simply improving knowledge of the meaning of mathematical words and concepts by reading about them can serve as a means of improving the familiarity with the language of mathematics (Faramarzi & Sadri, 2014). As students improve their fluency in the language of mathematics, they will improve their ability to converse in and manipulate this language to achieve a solution (Faramarzi & Sadri, 2014). Furthermore, Breaux et al. (2017) reported that naming automaticity of objects was found by to be a strong predictor of math computation ability. The author's report supports the link between language and math ability.

Building all of these previous exercises and practices into the curriculum of pre-elementary, elementary, and to some extent middle and high school programs creates and environment that promotes student success in both reading and mathematics without drastically increasing academic demands (Faramarzi & Sadri, 2014). I found the use of these neurological interventions that were seemingly unrelated to math useful as a means of approach mathematical concepts with students who considered themselves to be innately poor at math. Practicing seemingly unrelated skills allows for students to feel success and progress in my classroom and the research by Faramarzi and Sadri (2008) strongly influenced the development of several lessons and activities in the units created in this project.

When cooperative teaching is used, emphasis should be placed on the “why” of particular pedagogy. This is important so that a compromise can be met between the methods of the teachers that makes full utilization of both teacher’s areas of expertise (Speijer, Gray, Peirce & Doherty, 2016). Both the classroom and special needs teacher need to consider the reasoning behind the type of instruction being given so that achievable and relevant goals can be made. When the “why” is addressed, then consideration can be given to the various ways to achieve the final goal (Dieker & Rodriguez, 2013). How students demonstrate their learning will vary based on the essential goals outlined by the teachers. When both parties explain to each other why certain math methods are used, there can be an appreciation of the perspectives that will be important in reconciling any apparent contradiction in instructional methodology. If a student is receiving contradictory math teaching methods of instruction this will lead to increased confusion for the student, especially for learners with special needs (Ekstam, Korhonen, Linnanmaki & Aunio, 2017). This is detrimental to any student but is exacerbated by a special needs student’s inherent difficulty in discerning key and salient information (Sayeski & Paulsen, 2010).

Special education teachers and classroom teachers need to recognize that special education need not be a permanent designation. In many cases students are in special education for only part of their educational career, either as a result of successful intervention, time limits, or late diagnosis (Schulte & Steven, 2015). The authors conducted a statewide longitudinal study using the database available to examine students diagnosed with a disability and students with a diagnosis across the third and seventh grades. Gaps between the groups widened over time as students were diagnosed over their time in school and were then placed in the diagnosed group. The use and implementation of successful strategies for students can have a very real impact on their educational trajectories. Any educational plans made for students should be made not only for the specific class or school year but put in the context of the student's entire mathematical education (Schulte & Steven, 2015). I found that this study highlighted a need to be aware of the diagnostic tests that students are subject to so that I as an instructor can be better prepared to spot discrepancies in student achievement and tailor my lesson to better prepare students for diagnostic assessment.

There is a need to alter the approach of special educators towards special education. The ability gaps between learners with and without special needs widen as the students proceed through the school system. Some research suggests that traditional methods of mathematics instruction or special education instruction in general have net-zero or even detrimental effects on special needs students' ability in mathematics (Speijer, Gray, Peirce, & Doherty, 2016). This study was professional learning project for Grade 9 and 10 Applied Mathematics classroom with the intent of improving communication between the Regular Classroom Teacher and the Special Education Resource Teacher. Improving communication between these main-stream classroom and special education teachers was seen as a possible way improve the understanding of students and make the most use of teachers relevant fields of expertise. For the purposes of this project, this study highlighted the need for myself as a Regular Classroom Teacher to bring the knowledge I have gained in my studies in a Special Education Program and attempt to bridge the gaps between the two positions.

The current trend towards co-teaching is not without both merit and concern. The one lead/one teach method is the most common in the secondary years (Dieker & Rodriguez, 2013). In this situation one teacher has the specific content knowledge and the other assists either during instruction or immediately after during student practice (Dieker & Rodriguez, 2013). A drawback of this method is that one teacher is often relegated to a subordinate role that can be due to ignorance or a lack of comfort with the material being taught. This can be a problem when considering that teacher comfort and confidence with material is correlated with student success (Ekstam, Korhonen, Linnanmaki, & Aunio, 2017). In these situations special needs teachers often fall into the role of soft-skill work such as homework practice and organizational skills (Rosa and Holmbeck, 2007). While these skills are necessary, they are not immediately relevant to the content at hand and can distract from the immediacy of what is being taught. A work-around for this problem can be the improved implementation of co-operation during the planning phase rather than the instructional phase. This avoids having the special needs educator being relegated to soft-skill work (Dieker & Rodriguez, 2013).

It is important for teachers to predict and recognize specific problems that student with disabilities may have and which assessments and interventions are available for those disabilities (Boyd & Bagerhuff, 2009). Technology can play a significant role in these interventions (Brownell, Sindelar, Kiely, & Danielson, 2010).. The ability and willingness to scale back on concepts to the level of the student has shown success in improving student understanding of math concepts in special education settings (Björklund, 2012).

The method and level of communication for students is an important factor in special education (Hallahan, Kauffman, & Pullen, 2015; Winzer, 2010). There are many speech and communication deficits that students may have and this may have a greater impact than just the language barrier to communicating mathematical concepts. Use of egocentric speech and talking through problems to themselves is one way that students can learn mathematical concepts, especially at the lower elementary and middle school grades (Stegemann & Aucion, 2018). In this way, a student slowly internalizes external ideas as the speech becomes internal and automatic (Vygostky, 1978). Some students may communicate through gestures and these gestures can serve as a type of egocentric speech (Hutchinson, 2017; Stegemann & Aucion, 2018). These gestures may be smaller and more subtle than other types of gesture for communication and it is important to encourage the use of such gestures as a means of working through math concepts towards improved understanding. Many of the more visuospatial concepts of mathematics, such as fractions, may need such external movements to allow for understanding to occur (Zurina & Williams, 2011).

Based on a study using meta-analyses by Ketterlin-Geller, Chard & Fien (2008), six (6) instructional strategies emerge as potentially beneficial for students with disabilities: visual and graphic depictions; systematic and explicit instruction; student think-alouds; peer-assisted learning; formative assessment data provided to teachers; formative assessment data provided directly to students (p. 35). These instructional strategies were used in the creation of two intervention groups for a study involving 52 fifth-graders in the Pacific Northwest. Low-achieving students were divided into three groups: small group instruction aimed at highlighting student's misconceptions, extended time and support for concepts taught, and a control group.

The strongest support for the above six areas of intervention can be found for visual and graphic depictions, systematics and explicit instruction, and formative assessment data that is quickly relayed to both teachers and students (Ketterlin-Gellar, Chard & Fien, 2008). Peer assisted instruction as a math pedagogy failed to have large effect sizes during a literature analysis by Zheng, Flynn, and Johnson (2013). However, whether peer-assisted instruction has an impact on learners with special needs was not clarified by the authors.

There has been evidence provided in a paper by Elliot, Kurz, Tindel and Yel (2016) that indicates special education students are receiving equal opportunity-to-learn time in mathematics when compared to their peers. This study used a group of 78 teachers and had them keep instructional logs daily with regards to a group of 162 students with learning disabilities and 165 students without disabilities. The instructional logs were examined to determine opportunities that students had to learn. There is evidence that not all of the math instruction given to special needs learners is positive or beneficial (Sullivan & Field, 2013).

The beneficial assistance that students with math disabilities receive can be limited. When compared to out-of-class tutoring that is received, students with a reading disability report far more tutoring assistance than their peers with a math disability (Willcutt, Petrill, Wu, Boada, DeFries, Olson & Pennington, 2013). This highlights the need to address the problems of students with math disability in-class and improve the quality of in-class math instruction.

The net negative effect seen in mathematics achievement may be the result of the more social aspects of education focused on in special education. Because there is an emphasis on social skills development, math skills are not at the forefront and so suffer from less time and practice. This could result in the net negative effects seen in special education at the preschool level, which will have a growing effect at the elementary, middle, and high school levels (Sullivan & Field, 2013). This study made examined groups of preschool students who received special education services and compared the achievement of these students to children who did not receive special education services at the pre-school age. The authors reported that part of the net negative effect on math achievement can be seen as a result of the behaviourist approach of special needs teachers. The students in question did see a decrease in social behavior problems that are the focus of the behavior intervention. Sullivan and Field (2013) stated that the emphasis on the behavior problems left less time for intensive and direct mathematics instruction, which could have impacted the gains in mathematical understanding. A notable finding was that there was a statistically significant increase in learning *related* behaviours that would not be directly reflected in academic achievement (Sullivan & Field, 2013). Again, this discrepancy in achievement could be reflective of an underlying behaviourist approach to special education (Morgan, Frisko, Farkas & Hibel, 2010). While this approach is not invalid and does have some positive effects on student learning behaviour, the approach needs to be adjusted if math performance is to be improved. I used this concept of potentially doing harm with my instructional strategies to carefully examine my approaches in the construction of my math units. I took into account the difficulties and misconceptions that learners with special needs may have in their understanding of mathematics. It was important for me to create lessons that did not hinder a student's future progress or ability to achieve in mathematics.

Emphasizing the most universally applicable algorithms and teaching as few of these algorithms as possible has the potential to improve student performance (Sayeski & Paulson, 2010). Such algorithms include cross-multiply and divide for the solving of many types of proportions. The breaking down of questions into clearly distinguished steps can ease the demands on working memory for students with a math disability (Swanson, 2015; Swanson & Jerman, 2006). The generalizability of students with a math disability may be limited and applying learned processes in novel ways may also be limited. Explicit instruction of the steps required may assist these students in overcoming mental deficits in problem solving ability (Seo & Bryant, 2012). Teachers should emphasize the instruction of the most applicable algorithms to improve student understanding and allow them to apply what they know to the widest range of situations. While mastery of algorithms is not the intention of the math classroom, there has been success when lowering the amount of information necessary for the special needs students to remember. Allowing for mastery of fewer skills may lead to more math competency and literacy (Sayeski & Paulsen, 2010; Shin & Bryant, 2015). If done at an early enough level, there is the potential to lessen the ability gap and improve confidence to the point where more students with math disabilities can participate in higher level math courses without the need for severe time and resource investment on the part of schools, families and students.

Criteria for Diagnosing Math Disabilities

It is estimated that between 5 and 10 percent of all students have a math disability (Xin and Tzur, 2016, p. 196). Other research estimates the number of students with MD at between 3 and 8 percent or possibly between 6 and 7 percent (Swanson & Jerman, 2006, p 250). The diagnosis of mathematics disability has a long history in both its diagnosis and treatment. It is important to recognize the growing schism between the treatment or pedagogy associated with mathematics disability education and the means by which we determine if an individual has such a need for treatment (Xin & Tzur, 2016). Traditionally, a math disability, or MD, diagnosis was the result of a discrepancy between math score assessment and overall intelligence scores, specifically the results of IQ tests (Garret, Mazzocco, & Baker 2006, p. 79). This has been found to not have strong validity in two literature analyses (Hoskyn and Swanson, 2000; Stuebing, Fletcher, LeDoux, Lyon, Shaywitz & Shaywitz, 2002). These faults are the result of the validity of IQ tests as they relate to clinical teaching practice; the inability to detect discrepancies between an underlying cognitive factor related specifically to math as opposed to an overall cognitive discrepancy; and the assessment itself not being built to validly test specific domains of cognition as they relate to various aspects of mathematics ability.

Validity of IQ tests in the measure of math disability. The overall use of testing student IQ has had questionable use in teaching practice (Dekker, Ziermans, & Swaab, 2016). A general intelligence test does not directly inform teaching practice and the use of the score to the classroom teacher as a guiding tool is limited as a result of this (Dekker, Ziermans, & Swaab, 2016). Dividing mathematical intelligence into several factors allows for examination of the interaction of these aspects and this is useful from a research perspective. The most effective predictor of math ability is the combination of these factors into a general intelligence score (Markon, Chmielewski, & Miller, 2011; Parkin & Beaujean, 2012). In addition, Parker and Beaujean (2012) suggested that this general intelligence score can then be used to guide decisions for instruction or assessment, especially when developing IEPs for students with math disabilities (Stegemann & Aucion, 2018).

Underlying cognitive factors or an overall cognitive discrepancy. Two studies have examined the differences in ability between students with an MD (math disability) diagnosis, RD (reading disability) diagnosis, MD + RD combined, and students without any diagnosis (Foster, Sevcik, Romski & Morris, 2015; Willcutt, Petrill, Wu, Boada, DeFries, Olson & Pennington, 2013; Zheng, Flynn, & Swanson, 2013). The concerns are that there may be a relationship between cognitive domains affected for math and reading diagnoses (at least in certain cases) and that these domains are not being tested for in current methods. Attempts to improve the diagnosis and address the underlying cognitive domain could improve teaching practice and allow for more gains in mathematical understanding. Comorbidity with other disabilities also makes accurate diagnosis difficult (Branum-Martin, Fletcher & Stuebing, 2013). The authors have suggested that there is a continuum of severity and this calls into question the need to separate certain learning disorders into categories. This was supported by Chong and Siegel (2008) who also noted that math disability be seen as more of a continuum that an orthographic category. This opinion resulted from findings that memory-retrieval deficits were relatively stable over time while procedural deficits improved for learners with a math disability over the same period of time. Research methodologies that allow for multiple means of solving problems may allow for differential procedures that allow learners with a math disability to compensate for cognitive deficits through the application of particular math algorithms (Chong & Siegel, 2008). Though there is a high level of relationship between reading disability and math disability, not all students show both disabilities. Students diagnosed with only a reading disability report lower academic success that those with only a math disability (Willcutt, Petrill, Wu, Boada, DeFries, Olson & Pennington, 2013).

An interesting connection between reading and math disabilities was reported by Foster, Sevcik, Romski and Morris (2015) where a significant relationship between phonological awareness and naming speed with mathematical achievement was found. While naming speed has been seen as subsidiary to phonological awareness in determining math ability, both had varying degrees of significance in different areas of mathematics. The naming speed of colours was consistently related to success on addition and subtraction tasks. Math fact retrieval was seen to trigger the same area of the brain as language processing and this could strongly influence the ability of students in certain areas of mathematics such as Time, Money, and Geometry units that have a strong verbal language component. Even the ability to detect rhyme and alliteration at age 3-5 years was related to math achievement at age 6 years (Foster, Sevcik, Romski, & Morris, 2015).

Valid detection of mathematical relevant cognitive domains. There needs to be consideration of the clinical applications of a diagnostic test and the focus on improving student achievement. Basing the diagnosis on a variety of factors has been suggested by Branum–Martin, Fletcher, and Stuebing (2013). The authors compared this diagnosis of math disability to the diagnosis of those factors that determine obesity, which is diagnosed around multiple factors that determine if a given individual is obese (height, weight, and other biometrics). Scores may be similar between non-diagnosed and diagnosed individuals but it is the *set* of scores which warrant diagnosis and therefore treatment. For mathematics, these domains may include factors such as fact retrieval, procedural knowledge, verbal understanding, etc. Students with a math disability may score similarly on a given task to students without a math disability but the suite of scores taken together would indicate a need for intervention (Branum-Martin, Fletcher, & Stuebing, 2013).

In addition to assessments not necessarily detecting the proper cognitive domains, there are a variety of assessment tools that are used to determine if a learner qualifies as having a math disability. These tests are often used in research projects and achievement criteria are set to determine groupings of students for analysis. The specific criteria for determining a math learning disability can vary between and even within studies (Chong & Siegel, 2008). The authors reported that test cut-off criteria for determining whether students do or do not have a disability can vary from 10^{th} percentile to 11^{th}-25^{th} percentile and that this shift can significantly affect results (Chong & Siegel, 2008, p. 309).

While not denying the existence of math disabilities, there is concern over both the range of characteristics tested for in diagnosing math disability (Branum-Martin, Fletcher, & Stuebing, 2013) and the relatively fluid achievement range and cut-off points that determine whether learner is considered to have a math disability (Chong & Siegel, 2008).

Affected domains of cognition and other characteristics of math disability. Math disabilities are seen as being persistent deficits in mathematical achievement separate from other academic areas. Any math deficit would be separate from overall low intelligence (Garrett, Mazzocco, & Baker, 2006). The most common deficits of individuals with an MD diagnosis are visual-spatial deficits and the inability to switch between different mathematical operations. These are only two of the most common deficits noted in for students with MD but have severe implications for mathematical ability.

Visual-spatial awareness has severe implications for the more concrete aspects of fraction understanding and geometry. A common method of mathematics education pedagogy is that of concrete/representational/abstract with the implication that concrete is the area most easily understood and forms the basis and bedrock of further understanding. The hampering of concrete understanding by lack of visual/spatial understanding causes a disconnect with the commonly accepted and empirically supported method of teaching most mathematical concepts. This issue leads to the need to fundamentally change the way that a concept has been taught successfully in the past. Basic numerical processing shares resources in the brain with areas that are commonly associated with spatial tasks (Stanescu-Cosson, Pinel, van De Moortele, Le Bihan, Cohen & Dehaene, 2000). Lack of spatial awareness would highlight a cognitive deficit that is directly linked to cognitive resources needed for efficient mathematical ability.

The inability or delayed ability to switch between operations is a significant detriment to the math ability of students with a math disability (Garret, Mazzocco, & Baker 2006). This diminishes the student's ability to solve the complex real-world mathematical problems that are increasingly becoming the focus of math classrooms at all levels of instruction. Current math pedagogy increasingly focuses on the idea that real-world situations and the relevant math will have the greatest impact and connection to students and increase student interest and therefore student mathematical ability. A difficulty arises from this approach based on the reality that "real-world" problems have multiple steps with numbers that are not always clear and may require rounding with the need to understand the need for multiple operations (Garret, Mazzocco, & Baker 2006). An inherent difficulty or inability to switch between multiple operations will greatly hinder the ability of individuals with a math disability to independently complete such work, even if the individual were given minimal assistance (Garret, Mazzocco, & Baker 2006). A subset of this problem is an inability to filter out relevant or salient information from a situation. Again this hampers an individual's ability to complete a complex real-world problem as there is often an abundance of information in the real world and the filtering of such information is an important skill in daily mathematical functioning.

Mathematical awareness can be split into the two distinct groups of symbolic and non-symbolic numerical information (Furman & Rubinsten, 2012). Symbolic structures include such things as Arabic numerals where the visual information is not directly linked to the magnitude of the concept or object. Non-symbolic numerical representation is recognizing the amount of objects in a group or differences in groups of objects and seeing that one group is larger than another (Furman & Rubinsten, 2012). Non-symbolic representation can further be broken into two (2) different ranges; the subitizing range includes small numbers (1-4) where the recognition is automatic and quick and the counting range which includes larger numbers and recognition is accomplished serially and slowly (Furman & Rubinsten, 2012). Students with MD seem to have difficulty in automatically associating symbolic and non-symbolic understanding (Furman &

Rubinsten, 2012). This is to say that a symbol representing even a small subitizing amount is not automatically processed as quickly as seeing the corresponding number of objects.

Neurological dyscalculia is the term used when a defect or deficiency may have developed at the neurological level (Faramarzi & Sadri, 2014). Support for this type of dyscalculia can be seen in the presence of abnormal brainwaves in students with learning disabilities. There are several subtypes of this dyscalculia. There may be a deficit in verbal semantic memory and connecting meaning to words. This makes it difficult for the student to quickly or accurately recall math facts or conceptual meanings. A second subtype of neurological dyscalculia is using developmentally immature methods of problem solving and making frequent errors in basic calculation. Examples of developmentally immature methods would be finger counting or simple repeated addition to solve larger multiplication problems. The third subtype of dyscalculia is a visuospatial deficit that leads to improper column placement and place value errors. This type of error can lead to problems in mathematical calculations especially for longer problems where organization is key to correctly solving a problem.

A specific area of difficulty for students with MD is the telling of time (Burny, Valcke, & Desoete, 2012). These problems can be the result of multiple number scales and also deviation from the base-10 numbering system (Burny, Valcke, & Desoete, 2012). Telling time requires a great deal of verbal knowledge and so is strongly linked to a student's phonological processing. Burny, Valcke, & Desoete (2012) studied 725 students from Grades 1-6 and found that students diagnosed with math disabilities consistently had difficulty with the tasks of telling time. These tasks included stating analog reading times, stating digital reading times, transforming between digital and analog times, and writing times represented by written or spoken phrases (quarter past 12) (Burny, Valcke, & Desoete, 2012, p. 356). The authors reported that being alert to time telling ability can serve as a basic warning sign of math disability at elementary and middle school ages (Burny, Valcke, & Desoete, 2012).

An individual's available working memory can determine what problems or level of complexity they are best able to work at. Focusing on working memory from a lesson planning perspective can give students an ability to approach subjects that may otherwise be overwhelming and frustrating for them. Use of tools that supplement working memory may also be an important intervention for those students that have deficits in this area without the need for major interventions that differentiate them from their peers. A notable finding of some of the research was that students without a math disability showed no sizeable difference in effect size between methods of instruction (Swanson 2015). This may prove important as it shows the possibility that some teaching methodologies may be harmful or overly difficult for MD students while other methods greatly assist them but neither method disadvantages those students without a math disability.

According to the studies of Stanescu-Cosson, Pinel, van De Moortele, Le Bihan, Cohen and Dehaene (2000) specific areas of the brain that are active during rote mathematical calculation are also active during verbal communication. Approximation and exact calculation of large numbers activated different areas of the brain, which would typically encode numbers in a non-verbal format. This strongly implies that there are two systems at work in the mind. One system exists for fast approximation processes and another for slower and more exact calculation processes. Certain subtypes of dyscalculia can be explained by lesions affecting one of these networks disproportionately (Stanescu-Cosson, Pinel, van De Moortele, Le Bihan, Cohen & Dehaene, 2000).

The mathematical research of Hale, Fiorello, Dumont, Willis, Rackley and Elliott (2008) highlights the need to know a learner in all aspects. One of these aspects is the domain of time. Teachers need to monitor a student's progress across not only the current school year but across several years and to plan accordingly. The inherent persistence of MD, which is also a key factor of its diagnosis (Chong and Siegel, 2008), means that gains will be small but significant and any gains should be examined and expanded upon to make the best use of the child's inherent strengths. Patterns of performance are key to ideographic interpretation of interventions that will work for that child. These are ignored if examining general scores or assessments in isolation (Hale, Fiorello, Dumont, Willis, Rackley & Elliott, 2008).

A number of studies as reported in Zheng, Flynn and Swanson (2013) have reported an overall pattern in effective math instruction for MD students. The authors conducted a literature review of eight group and seven single-subject studies. The participants of the study were organized into learners with a math disability, learners with both a reading disability and a math disability, and a control group. While there is consensus on limited research in the area of math disability diagnosis and treatment, it is important to collate and make sense of what information currently exists. In general, it was reported that high effect sizes included the following components: stating instructional objectives and/or directing students to focus on particular information (advanced organizers); fading prompts and/or providing necessary assistance (control difficulty); explaining underlying concepts and/or providing repetition within text (elaboration); distributing review and practice (explicit practice); engaging students in dialogue and asking questions (questioning); sequencing short activities (sequencing); skill modeling; reminding students to use instructed strategies or procedures (strategy cues); breaking down the targeted skill into small units (task reduction) (Zheng, Flynn, & Swanson, 2013, p. 108). I found that this study highlighted some of my own practices in my classroom. Seeing how each of the key factors was reflected in my own teaching also highlighted those factors that I tend to not focus on. I have increased my focus on all of these areas as a result of this paper and have taken them into account in the construction of my teaching units.

The listed factors above have been explained and reported by Zheng, Flynn, and Swanson (2013) as making their way into math classrooms that make use of evidence based pedagogy. These factors can be found to support several of the methods used in this project to improve mathematics learning and understanding for middle school aged learners with special education diagnoses.

Tests for Determining Math Disabilities

The assessments used to determine a math disability can be seen as belonging to one of two groups: diagnostic tests or screening tests. Diagnostic tests are thorough construct that determine a student's mathematical ability across a wide range of math topics. These tests determine that a student has a math disability. Screening tests are simpler and briefer tests that determine the need for further assessment of an individual. These tests do not determine that an individual has a math disability.

Diagnostics tests, because of their educational implications, must be properly constructed and administered. A properly constructed diagnostic assessment will directly test the differing underlying cognitive domains that can be connected to math achievement. A test should examine a student's level in these domains and create a profile of what deficits there may be in the student's cognition so as to guide intervention. As discussed earlier, a diagnosis of an individual should also take into account the treatment for that individual.

Screening tests are used at the beginning of the assessment process to guide the decision making process (Erford & Biddison, 2006). The authors reported that decisions for further assessment using diagnostic tests or more careful classroom monitoring by teachers, educational assistants, or special education teachers can be made using screening tests. Early interventions can have significant impacts on future learning and these screening tests can allow for interventions before serious problems can manifest (Erford & Klein, 2007). It was reported by Erford and Klein (2007) that intensive and effective elementary education can lead to improved mathematical performance later in school.

A distinguishing factor between screening and diagnostic tests is the level of achievement that the test detects. Screening tests are generally briefer and are designed to discriminate between low-achieving and typically-achieving students (Erford & Biddison, 2006). High achieving students will score similarly to average achieving students on these tests and will not usually be distinguished. To separate out the high-achieving students, it was reported by Erford and Biddison (2006) that more difficult items are necessary than those typically used in screening tests. Diagnostic tests, because they are much more thorough and specifically examine cognitive domains (Erford & Klein, 2007; Parker & Beaujean, 2012), can distinguish between all 3 groupings. This ability to determine more levels of skill comes at the price of being longer tests that require a greater investment of time to administer and score.

A second difference between diagnostic and screening tests as reported by Erford and Biddison (2006) is the type of error that is of concern. False positives, identifying students as needing intervention that do not actually have a math disability, are less of a concern for screening tests. These students would simply be identified as possibly needing further assistance and additional observations would likely conclude that only moderate classroom intervention would be needed instead of a full diagnostic or special education assistance (Erford & Biddison, 2006). False positives for diagnostic tests result in a great deal of professional effort to assist a learner that may have only needed moderate intervention.

Both screening and diagnostic tests are concerned with false negative errors where students needing assistance are misidentified as not needing assistance (Erford & Biddison, 2006; Erford & Klein, 2007). It was reported by the authors that false negatives are important at the screening level and diagnostic level as it eliminates a student from being a concern for special education services. This can be especially detrimental in educational systems that already have limited resources and cannot continually reassess or screen all students regularly (Erford & Biddison, 2006).

Within assessment tests there is the possibility of improving student performance through the interspersing of easier test items that reinforce understanding (Robinson & Skinner, 2002). The authors studied thirty Grade 7 Students in a rural southern school district. This study made use of subtests from the KeyMath-R diagnostic assessment. The subtests were altered to allow for interspersed easier test items. Students were shown to answer items correctly on tests that had interspersed easier items when they had answered those same items incorrectly on tests without interspersed easier items. It is important to note that this method can have differing effects for level of task demand. Verbal responses, lack of time constraints, and mental computation can be significantly affected by the interspersing of easier items in the assessment. Test items that fall in the challenging range (those questions that are not too easy or too hard) were most affected by the interspersing method. An important educational implication of this research is that typically low performing students may have a means of showing their true understanding by having easier items mixed into the test that are related to those challenging questions. Summative, screening, and diagnostic assessments may thus have their validity increased by the use of this method of interspersing easier test items. I have applied the information gained from this study in the construction of my own tests and also in how I approach assisting students. When students ask for assistance on a given math task, I emphasize the basics of the question by breaking the problem down into steps. This is a common method of assisting math learners but is difficult to do during test administration to keep the testing fair to other students. By placing easier test items before the larger and more complex questions, I hope to improve student performance on the more complex problems that immediately follow these simpler and related test items.

KeyMath-3 diagnostic assessment. The KeyMath-3 assessment is a series of 13 subtests split into the 3 general categories of Basic Concepts, Operations, and Applications. Basic concepts is made up of the Numeration, Rational Numbers, and

Geometry subtests. Operations consists of the Addition, Subtraction, Multiplication, Division, and Mental Computation subtests. Applications consists of the Measurement, Time and Money, Estimation, Interpreting Data, and Problem Solving subtests. Combined, these subtests form a robust measurement of a student's overall math ability (Williams Fall, Eaves, Darch, & Woods-Groves, 2007). As an overall measurement of math ability, the KeyMath-3 assessment also aligns with the new BC curriculum goals with its ability to examine the mathematical literacy that allows students to become competent citizens (BC Ministry of Education, 2016). The new BC curriculum has increased emphasis on the development of financial and problem solving ability that the KeyMath-3 explicitly examines. The KeyMath-3 subtest sections are very closely aligned with the British Columbian educational outcomes for Grade 8 (BC Ministry of Education, 2016). This makes this grade ideal for using the diagnostic as a planning tool for units for the purposes of this project.

The KeyMath-3 assessment has two developed forms that have been normed for Canadian students, which makes it possible to give the assessment twice over a relatively short amount of time and to examine differences between the two assessments. The KeyMath approach is useful in any proposed project that will see students receiving a pre- and post-assessment of math ability after two math units. The different forms of the assessment will limit the practice effects that could occur if the same version of the test were used in such a short period of time.

The KeyMath-3 assessment shares many characteristics with the Wechsler Intelligence Scale for Children (WISC), such as being averaged around a score of 100 with standard deviations of 15 (Parker & Beaujean, 2012; Williams, Fall, Eaves, Darch,

& Woods-Groves, 2007). The KeyMath test is also administered in a similar manner with the establishment of basals and ceilings similar to those found in the WISC (Parker & Beaujean, 2012; Williams, Fall, Eaves, Darch, & Woods-Groves, 2007).

The IXL math program. This is an online program that offers math curriculum from grades K-12 that is separated by grade, topic and unit (IXL.com). The program is currently in use in several schools in the school district where the units of this project were developed. A student is given an account and their own login and password which is controlled directly by the classroom teacher. The student accounts are attached to a teacher administrator account. This allows the teacher to monitor progress and offer assistance if students are not making progress in a given area. Teachers may also monitor the progress of students over the course of their time with the program and examine areas of prior success or difficulty. Schulte and Stevens (2015) reported that the ability to examine a student's progress through education is an important factor in making appropriate educational and instructional decision with regards to mathematics education. The student selects a topic and grade level and is then given a series of questions. The questions can be answered as multiple choice or written answer depending on the grade level selected. Points are awarded as the student gives a correct response. An incorrect answer results in a loss of points. The goal is to get to 100 points and achieve "mastery" of the topic. Questions are randomized so there can be no memorization of the correct answer in a particular order. The ability to memorize answers after trial and error is a downside of many online computer assisted instruction programs. The validity of the achievement score in such programs is questionable if the score is used as a measure of math ability (Bakker, van den Heuvel-Panhuizen, & Robitzsch, 2016). Not only are the questions randomized but they become increasingly difficult. Questions also reward fewer points as the student increases their score. This allows a teacher to gauge a student's relative ability with confidence in a given math topic area based on where the student finishes at the end of a practice period.

The hundreds of assessments that make of the IXL program are focused around single concepts that target specific mathematical algorithms or procedures. Zheng, Flynn, and Swanson (2013) reported that repetitive, guided, and relatively short activities that make use of task reduction are among the practices that produce the greatest effect sizes in mathematics classrooms.

A key part of the potential benefits of IXL.com is the immediate feedback upon answering a question. Immediate and instructive feedback is important in effective formative assessments for mathematics instruction (Ketterlin-Gellar, Chard & Fien, 2008). The student knows immediately if their response to a question is correct and does not develop misconceptions that may result in many questions being answered incorrectly. An incorrectly answered question results in an explanation appearing onscreen to highlight the error and then explain the full process again. While useful for all students, this feature has the benefit of being available to parents so they can work through the solution with their students. The ability to assist parents at home opens the possibility for more assistance and practice out of the classroom, which can be a problem for mathematics instruction and practice (Brownell, Sindelar, Kiely, & Danielson, 2010; LeFevre, Polyzoi, Skwarchuk, Fast & Sowinsky, 2010). Increased practice can help some learners and a major barrier to this can be that the only adult available with the fluent knowledge to complete the math may be the math teacher. By having the explanation highlighted and available on screen, more parents will be able to assist their children and this removes this barrier of lack of access to an expert or proficient individual (LeFevre, Polyzoi, Skwarchuk, Fast & Sowinsky, 2010).

The IXL program can provide immediate feedback and support for students, which is an important factor in the selection of a classroom math program (Shin & Bryant, 2017). The broad scope of the IXL program ensures that it covers those topics that are assessed in the KeyMath-3 diagnostic and can assist in the reinforcement of concepts and improve the quality of instruction in the classroom (Ketterlin-Gellar, Chard, and Fien, 2008). I have found that the language used in the KeyMath-3 and IXL.com can differ for similar concepts and that this can occasionally cause confusion for learners with math difficulties. While providing more context and vocabulary for high-achieving learner, the difficulties caused for other learners should not be overlooked.

Chapter Summary

The pedagogy of mathematics is a varied topic. This literature review narrated a general picture of the effective methods of mathematics instruction for average achieving students and special needs students.

The importance of making mathematical lessons relevant to students' lives is a main concern in many teaching methodologies. This importance is reflected in the curricula created by provinces and is evident in the learning outcomes of the curriculum. Teaching methodologies commonly make use of manipulatives to illustrate concepts and improve student understanding. Modern computers are increasingly used to make math concepts more accessible for learners. The IXL math program is an example of a computer program that addresses pedagogical concerns of immediate and constructive feedback and is currently in use by school districts to support mathematics instruction.

Mathematics in special education has been the result of constructivist and behaviorist approaches and this discrepancy has led to differing pedagogical approaches to mathematics instruction. Instruction that is applicable to student's lives and that addresses the specific cognitive deficits of the learner has proven to be effective in improving mathematical understanding.

The diagnosis of math disabilities determines interventions for students and these interventions guide teaching practice. Valid assessment of math disabilities ensures efficient use of resources and that teaching methods will address the correct cognitive domain that is affected by the disability. The KeyMath-3 diagnostic assessment is used in schools to determine mathematical disability and the assessment is broken into several categories that correspond to different domain of mathematical understanding.

Current trends in math pedagogies were explained with the suggestion that planning units around the special needs students first and taking their barriers into account at the time of planning may allow for the creation of units that make math accessible for all students regardless of designation. These units could receive substantial support from modern technological advances and make improved math achievement a realistic goal for all learners. With improved efficiency in unit construction, teachers will have increased time for direct student interaction and this will improve the quality of mathematics education.

Chapter 3: Research Methods

As a teacher in the school where this project is based, I have worked with learning services teachers as part of a referral process for students who are having difficulty in math classes. These teachers often use the KeyMath-3 assessment to gauge a student's ability in mathematics. I understand the need to test students based on language that is familiar to them and after examining the test that was administered to students I realized that the language of the test differed from my own in-class usage or those of my colleagues who also teach math. It was my concern that this confused students and therefore did not validly test their understanding of the concept.

This project comes from this realization that students may be scoring lower than expected on assessment tests that can be used to determine if a math disability is a likely diagnosis for the student. The success that has been found using universal design for learning approaches will be combined with the KeyMath-3 assessment to produce units where students are able to access the material of the lesson and express their learning in an authentic manner that truly reflects their understanding of the underlying math concept.

The project is based on qualitative research orientation that lends itself to an exploration of teacher practices (Creswell, 2015) especially in the teaching and assessment of mathematics inclusive classrooms reaching learners with special needs and those challenged by math disabilities (Hallahan, Kauffman, & Pullen, 2015). This research project is a non-subject type, hence the method used is primarily the documentary method of analysis (DMA), described as "the technique used to categorize, investigate, interpret, and identify the limitation of physical sources, most commonly written documents whether at public or private domains" (Payne & Payne, 2004, p. 23). Historically, educational researchers have and still use DMA in understanding and reporting government policies and documents on school practices (Creswell, 2015). The choice of DMA for this project includes the application of the method in educational research based on its flexibility and availability (Creswell, 2015; Scott, 2005). The method has and is still used by researchers to investigate and report studies that will improve the learning success of special needs students with math disabilities (Hutchinson, 2017).

Specific government documents such as the BC Ministry of Education middle high school curriculum policy document, and the BC Ministry of Education Special Education Unit, were analyzed as part of the literature and the development of the select math teaching units using the KeyMath approach in chapter four.

There is an explanation of types of primary documents as "eye-witness accounts produced by people who experienced the particular event or the behaviour we want to study… they are documents that are produced by individuals and groups in the course of their everyday practices and are geared exclusively for their own immediate practical needs…" (Scott, 2005, classified p. 10). This explanation applies to me as a practicing teacher seeking to understand my teaching practice and student success in Math subject teaching practice for over eight years. As part of my professional practice, I access, reflect, and evaluate student records such as report cards. My position allows access to school records such as report cards to analyze the math performance of students across my middle school classroom. Access to student documents and records is what allowed me to see patterns in the problems faced by students and shaped my development of using the KeyMath approach. This was done in an attempt to improve my teaching practice, the learning flexibility for students and improving the overall learning success of the students, which are goals of the new BC curriculum (BC Ministry of Education, 2016).

Further consideration of using DMA approach for this project was based on the methods' measures of validity as *authenticity* (evidence as genuine and from an impeccable source), *credibility* (free from errors and distortions) and *meaning* (clear and comprehensible evidence) (Scott, 1990). As noted by educational researchers, Creswell (2015) confirms the validity of DMA process through a triangulation process whereby researchers corroborate with other sources of data to ensure validity of a study.

Additionally, this project also used the qualitative content analysis method (CAM) that examines documents, especially texts that are related to the phenomenon under investigation (Mayring, 2000). Content analysis is defined by Krippendorff (1969) as "the use of replicable and valid methods for making specific inferences from text to other states or properties of its source" (p. 103). To this effect, I engaged in the content analysis of various texts (books and journals) related to the scope of the project on not only KeyMath policies and implementation, but on Math pedagogies in elementary and middle schools, diagnosis of math disabilities (MDs), technology and the teaching and assessment of math disabilities amongst others.

Content Selection of KeyMath-3 Assessment

Content analysis of the KeyMath-3 (2008) along with the BC Math Curriculum for Grades 8-10 (BC Ministry of Education, 2016) serves as the content being analyzed and interpreted in the development of teaching units in chapter 4 of this project.

In directed content analysis, there is an existing body of knowledge that can be extended to validate a theory or theoretical framework (Hsieh & Shannon, 2005). Patterns in the types of questions asked by the KeyMath-3 assessment were examined to determine the types of questions and language that should be used in math units that teach math concepts. By knowing the types of questions and language used by the assessment, students will be better situated to display their learning if they are being assessed using this tool. This forms the basis of the research questions of this project. The units and lessons are constructed using language that parallels the KeyMath-3 assessment in the hopes of improving student understanding of the questions that determine or otherwise guide their diagnosis of having a math disability. The IXL program was also examined to an extent so that the language of the lessons also appropriately lined up with the language of the practice program.

The KeyMath-3 assessment is currently used to assess mathematical ability at the middle school where this project was developed. The KeyMath-3 assessment looks at a range of math skills in its subtests and many of these correspond with the current BC curriculum for Science 8 (BC Ministry of Education, 2016; Connolly, 2008). Because of the close alignment with currently taught units and its use as a diagnostic assessment of math disability at the middle school, the KeyMath-3 assessment was chosen to guide the construction of units directed towards increased mathematical learning and understanding for learners diagnosed with math disability.

The KeyMath-3 assessment may guide the diagnosis of the students as having a math disability and familiarity with its language and approaches to mathematical understanding would be important in ensuring the validity of the assessment.

Examination of the KeyMath-3 assessment showed the types of questions asked of learners during administrations of the test and this would allow for in class math practice to parallel the types of questions that would ultimately determine if a learner receives a diagnosis of possessing a math disability. Using the wording and types of examples used in the assessment could potentially remove any confusion or misunderstandings that may arise during the administration of the test. Students will have encountered similar language in the classroom and therefore be better prepared to display their understanding of the math concept, rather than their understanding of the test language.

Construction of Units Around KeyMath-3 Assessment

The teaching units created for this project are built from the very beginning with the special needs learner in mind. Not only is the learner considered from the viewpoint of difficulties that they may have with math, but the assessment used to diagnose them, the KeyMath-3, will be used as a guide for the domains of math that are commonly assessed to determine math disability. This approach specifically targets deficits found in the assessment and directly teaches to the problems that students may encounter. By directly addressing typical problems encountered on the test, it is hoped that students will directly improve those skills most relevant to their diagnosis and work towards removing their MD designation.

The stated goal of removing the MD designation has several implications for learners. The first is that the designation is not inherent to the person that has the diagnosis and that through practice and study an individual’s understanding can improve and be compensated for. The second implication is that diagnoses are not permanent and that this will give students a reason to work hard at improving skills that deemed low so as to improve as learners. A student need not accept the diagnosis of low math ability and give up on mathematics based learning or career goals. By working to remove this designation, student can prove their ability to overcome obstacles in spite of cognitive deficits and move on to more academic pursuits that would otherwise have been shut off to them if they had merely accepted their diagnosis of low math ability.

The KeyMath-3 assessment is divided into multiple sections that test a range of skills. These skills are taught to student across multiple grades and are also tested at increasing levels of difficulty. By using this diagnostic tool as a guide for unit construction, teachers can create math problems with multiple levels of difficulty that students can attempt to answer according to their own ability. This means allowing students to choose how much of a question is answered. This will allow lower achieving students to answer to the best of their ability (basic concepts) and will give higher achieving students the choice to attempt to answer in greater detail and with more precision. This has the effect of not changing the question but having students choose the approach to the questions that reflect their own understanding. By opening up the criteria for answering a question, teachers do not have to provide separate work for low and high achieving learners and can just set a universal set of criteria and place the impetus on students to choose how they will answer, giving students greater control of their learning and the expression of their understanding.

Use of Computer Assisted Technology in KeyMath

The units are constructed based on the idea that rote practice and basic skills related to each lesson would be reinforced using a computer program that provides automatic and immediate feedback to improve student understanding (Ketterlin-Gellar, Chard, & Fien 2008; Shin & Bryant, 2017). The IXL program will be providing much of the feedback necessary to improve basic skills and the in-class lesson will focus on the concepts that improve understanding rather that basic skill mastery.

When providing students with technologies that automatically calculate outcomes, such as fraction calculators or other programs, it is emphasized that these tools do not replace, or make up for, a lack of understanding of the concept being taught. The technology makes some aspect of the problem easier that would otherwise cause a great deal of frustration, even though the skill that the technology compensates for may be unrelated to the math concept that is of primary concern in the lesson. This frustration might lead to student giving up on a math concept that they actually understand because of some cognitive deficit in an unrelated area of mathematics. By using tools that supplement weakness during the application of math knowledge, students can see that they do possess some knowledge of mathematics and that their deficits can be overcome with perseverance and assistance.

All of the units will have supplemental support from an online math program, *IXL.com*, that supports the lessons with basic drill practice as well as providing virtual manipulatives. This will be especially helpful for the more visuospatial tasks as a means of alleviating the need to have fine motor skills necessary in graphing or the drawing of diagrams. The immediate feedback and corrective nature of the IXL program will free up class time for the teacher to focus on the thought processes of the math concepts, rather than basic practice and marking. This will have the effect of allowing for more valuable and constructive class time while making the practice more meaningful, similar to those effects reported by Seo and Bryant (2012). The students will know immediately if they are incorrectly completing problems and these misconceptions can be dealt with before they become embedded in the student's understanding.

Chapter Summary

The use of documentary method of analysis (DMA) is a technique used to examine government documents and programs, which was a central focus of this project. The qualitative content analysis method (CAM) was also used examine texts that were related to the phenomenon under investigation, namely the diagnosis and related math pedagogy related to special needs learners with a math disability diagnosis. The KeyMath-3 diagnostic assessment and the BC Ministry of Education middle high school curriculum document were the main documents examined using this method. It was the examination of these documents that guided unit and lesson construction with special needs learners with a math disability diagnosis being the main target of the lessons produced.

The KeyMath approach to unit construction makes use of the diagnostic test that will determine if a student has a math disability and uses this assessment to guide construction of units. The questions and topics covered in the assessment are examined in order to make sure that lessons accurately reflect the test items. In this way, students will become familiar with the concepts and language of the questions that will determine their diagnosis. Students will be given the opportunity to improve their understanding of the material that determines their diagnosis. This improves the validity of the diagnostic test by ensuring that student understanding of the math concept is what is being tested, not the language or approach of the test administrator. Student understanding will be reinforced with a program that provides immediate corrective feedback to ensure the teachers can address problems or misunderstandings in a timely fashion. Students with IEPs will have equal access to the materials and will be part of the class as a whole rather than receiving different work or standing out from amongst their peers. By taking all of these factors into consideration during unit construction, mathematical understanding may be increased for all learners in the classroom.

Chapter 4: Teaching Units

The Math Concepts

The units for the project are based around the Pythagorean Theorem, Probability, Algebraic Expressions, and Surface Area portions of the KeyMath-3 and Grade 8 British Columbia Curriculum Goals (BC Ministry of Education, 2016; Connolly, 2008). These four units have a great deal of real world application and include multistep problems that are traditionally difficult for learners with special needs. All four of these units also have classical roots which means the concepts will have multiple means of expression for students to learn and approach the concept (Katz, Jankvist, Fried, & Rowlands, 2014). This is also a benefit for teachers who may adjust their approach based on the form they find most useful or with which they are most comfortable. Use of a variety of research based methods will be used to improve students understanding of not only the specific concept of these units but will be focused around general math literacy and ability that is assessed in the KeyMath-3 diagnostic.

The construction of the units takes into account the varying levels of student ability and the assessments and activities can be accessed in a variety of ways. The concern of students standing out because they are given separate work will be dealt with from the outset by having all learners producing the same assignment. Opening up the criteria for assignments will ensure that learners of all levels can express understanding of the concepts. Those students with IEPs will also have immediate access to all BC curricular topics available on the IXL program and can be guided towards math that is at their current level of understanding. Students will be encouraged to try the given assignments given to the class as a whole because the initial questions of an IXL section are meant to test basic understanding of a concept. If a student is unable to complete the basic initial questions, the instructor will already be receiving valuable information about the level of the student and will quickly be able to assign work at the student's level to ensure quality use of class time.

The Probability Unit

The concept of probability can be simplified to a series of choices and which choice is likely to occur. The options and their likelihood are often expressed as a ratio, most commonly written as a fraction. There is a subtle distinction between ratio and fraction but the two are very similar. Ratios based off of spinners or dice can be used to more clearly illustrate what is meant by a fraction. There are a limited number of outcomes available that can be directly observed by the learner. This level of concrete reference makes understanding ratios a more achievable goal for many learners, especially those diagnosed with a math disability.

Determining the numbers in a probability fraction is a case of simply counting the total number of likely results and counting the number of desired outcomes. The two numbers are then placed one on top of the other with the total number of results always going on the bottom. This simple and consistent algorithm is what makes the probability unit an appealing start for teaching fractions as a whole and limits the number of algorithms that a learner needs to remember. Students will gain an understanding of what numbers in a fraction mean in the context of mathematics and that there is consistency and logic to the number being placed in that position.

Probability makes for a very real-world introduction to the idea of ratios and fractions. Fractions is a traditionally difficulty unit for learners but lends itself well towards the use of manipulatives to illustrate concepts. The school where this project was based had multiple sets of manipulatives for the instruction of fractions. The abstract ideas of fraction math (the bottom number representing a whole) can be more concretely represented in probability, which allows for increased understanding for the learners. Probability also has the advantage of being an experimental topic where students can undertake trials and create their own data. This increases scientific literacy and allows for the active creation of a student's own data that can then be analyzed. These are valuable skills in a learner and their development will be key to increasing scientific as well as mathematical literacy.

Probability and statistical analyses is explicitly examined across a range of difficulty levels in the KeyMath-3 diagnostic. The situations and problem proposed in the diagnostic follow the common understanding of probability taught to students of middle school grades. As skill level can vary greatly at an individual level, establishing a baseline of understanding will help to make sure that all students are accessing the material. The emphasis of this unit as an introduction to the structure of fractions means that the unit draws direct parallels between fraction manipulation as it relates to the probability of given events. The fraction problems encountered in the diagnostic assessment are often encountered in the probability unit this familiarity should improve performance and understanding of the particular types of questions that are typically asked when discussing operations with fractions. The explicit connections made between this unit and fractions allows for the use of consistent language and increases the time that students can spend with the language. This increased time may help learners with special needs solidify their understanding of the concepts.

The follow unit was created using the curriculum goals of Grade 8 Math in the BC Ministry of Education New Curriculum. The following table contains the key ideas of each lesson along with math difficulties that can be encountered in topic area.

Table 1

Probability Unit Summary

Lesson	Key Idea(s)	Math Difficulties Approached by the Lesson
Lesson 1: How many choices do you have?	The denominator of a fraction corresponds to the total number of probabilities	Students can commonly have difficulty in seeing the meaning behind parts of a ratio. This lesson gives context from a particular perspective for the denominator of a fractions as a means of solidifying this concepts in the minds of learners
Lesson 2: How many of each option do you have?	The total number of possibilities does not vary for the same given situation (spinner or data table) Assistive technology that does not remove the necessary understanding to complete a problem should be encouraged	Students can commonly have difficulty in seeing the meaning behind parts of a ratio. This lesson gives further context for the parts of a fraction by showing the numerator as the outcome of the probability and the denominator as the total number of options.
Lesson 3: How many ways can I do this? How many ways can I get there?	By multiplying the number of options given, you can get a number for how many combinations are possible	Basic multiplication is a skill that is often taught by rote memorization without context. This lesson looks to provide context for a basic math skill that can be applied to real world context. The use of tree diagrams helps to illustrate this concept visually
Lesson 4: If I do this # of times, how many times should I get x-results?	A whole number multiplying a probability is the number of times you repeat the event Cross multiply and	This lesson shows the idea of expected values from a probability. This is the application of the expected probability over repeated trials to determine an expected value. Again, this gives students an application for basic math skills in a

	divide can be applied to this situation	particular, real-world context.
Lesson 5: Experiment creating your own data using dice	The experimental and theoretical values should get closer together as you increase the number of trials	MD students (and typically achieving students) can have difficulty with the abstract idea of approaching a theoretical value with enough trials. This lesson has students directly creating their own data to prove this concept.
Lesson 6: The odds found in a deck of playing cards	Probabilities related to colour, suit, and value are examined with a diagram of a deck of cards	The visual of the deck of cards illustrates the different ways that a deck can be organized and provides a visual for the fractions/probabilities to help MD learners clearly see the values of the probability
Lesson 7: Which option should I choose?	There are scenarios that exist with a logical pro/con list that includes mathematics and this can help students choose the best option. Logic can be used to reduce the number of negative or incorrect options	Applying logical algorithms to decision making processes can help students to directly see the importance of math in everyday life. This lesson helps to solidify this concept for MD students who may see less of a need for probability in their everyday lives.
Lesson 8: Games of chance and the math behind them	Many games have different levels of rewards based on the rarity of certain events occurring	This lesson is meant to give more real-world applications of probability. With enough real-world references for probability, students will have a clearer picture of the application of this area of mathematics.
Project: Game of Chance	Students can create scenarios that have certain probabilities make games of chance. These games are pending the approval of the teacher to ensure probabilities are not overly complicated	In this project, students take some probability scenario that has been previously discussed and make a game out of that scenario. This is meant to give students a unique means of expressing understanding of probability and make their learning more personal.

Lesson 1: How many choices do you have? This is an exercise in knowing the number of different outcomes available. The specific math involved is simple counting but the concept can be seen as complex. This lesson has the additional goal of teaching students that basic math can be used to describe everyday situations. A single number answer is produced by counting the options available. This idea of knowing the total number of options forms the basis of the rest of the unit. In the concept of fractions, the number being produced in this lesson becomes the denominator of the probability fraction.

Lesson 2: How many of each option do you have? This lesson requires students to know the total number of options, as discussed in the previous lesson, but also to know how many of each intended option there is. Students are putting option into subgroups from the whole. This is an important concept in fractions and marks the beginning of having a numerator over a denominator.

The activities of this lesson have student looking at spinners or data tables to determine a total number of outcomes. It is stated explicitly to student that this number is consistent for all probabilities that apply to that situation, disregarding fractions put into lowest terms. Placing fractions into lowest terms should not be considered a priority for this lesson as it is not the main goal and can lead to confusion, particularly for MD student who have trouble with multiple additional steps. Students should be instructed on the meaning of each of the terms in the probability. Writing the meaning of each term relative to the question being asked will help students to see the meaning behind the numbers. As an example, a four-section spinner has two even number options. If the student is asked to give the probability of getting an even number, the correct answer is

2/4 with a simplified version of ½. If students are encouraged to write the meaning of each number, they would then write that the 2 represents the number of even values while the 4 represents the total options. The reduced form of ½, while still correct, has no direct meaning to the spinner but stronger math students could be encouraged to take this extra step. The amount of automatic math knowledge necessary to easily put fractions into lowest terms is a significant limiting factor for many math students, especially students with a math disability.

Assistive technology in this situation can include several modern apps that automatically place fractions into lowest terms. As the main goal of this lesson is to determine the number of options that meet certain criteria over the total number of available options, assistive technology that only places fraction in lowest terms does not oversimplify the task being asked of the students. A student must have understood the concept to be able to create the initial, unreduced fraction.

Lesson 3: How many ways can I do this? How many ways can I get there?

This is a straightforward multiplication when the math is examined but explains the idea of combinations of options. This has very applicable real world application as many choices are afforded consumers in the modern economy and students will have likely encountered scenarios where this math has applied in the past.

The key idea of this lesson is that there are a set number of combinations possible from a limited number of selections. Students should be explicitly told that while the math is straightforward multiplication, the concept that the math is describing has real world implications. The use of branching tree diagrams will give a visual for students to see the choices available at splitting of the branch. The total number of options at the final tier of the diagram illustrate the total number of options for a given set of options.

The assessment for this lesson includes questions with varying number of options, such as how many sandwiches can be made from 3 cheeses, 4 kinds of bread, and 2 types of meat. The multiplication gives the answer but this concept can also be shown more concretely through the use of tree diagram that show the branching nature of the options. Tree diagrams also help to illustrate that the order will not affect the answer of the question as the final result of 24 will be achieved no matter the order that the diagram or multiplication is completed.

Lesson 4: If I do this # of times, how many times should I get x-results? This lesson is based around a probability of an event occurring over many trials. It is a fraction multiplied by a whole number but can be shown using concrete examples. If a 6-sided die is rolled a total of 24 times, how many times can we expect to get the number 4? This works out to 24 (the number of trials) multiplying 1/6 (the probability of getting a 4) which gives the value of 4. We can expect to get the number 4 a total of four times. Students should be explicitly told the whole numbers multiplying the probability values represent number of trials. Use of this vocabulary is important in tying the concepts of this lesson with the upcoming lab and the idea of repeating events to gain more data.

When discussing the examples, emphasis on the ability of cross-multiply and divide to give an answer will give all students a powerful and nearly universally applicable tool to determine an accurate result. The use of cross-multiply and divide and the ability to set up ratios to form proportions where this method applies can be an extremely powerful tool for mathematics. This tool comes up in many mathematical situations and meets the needs of many MD students to have fewer but more applicable algorithms. For this lesson specifically, the students should be taught that the bottom numbers of each ratio represent total amounts (for both probability and number of trials) and that the only top number known is for the probability of the given event. As there is only 1 missing value from the fractions, this gives students the pattern required for cross-multiply and divide to be used.

Lesson 5: Experiment creating your own data using dice. Giving expected (theoretical values) versus experimental (actual values) after a series of trials using multisided die. Students complete several sets of trials and patterns in the experimental value are compared with theoretical values and the two should become closer as the number of trials increases. Students will actively observe the difference between the two values and how this difference of what “should” happen and what “does” happen can be reconciled by increasing the number of trials for an event.

Students are creating their own data and examining the discrepancies between what they theoretically know should happen and what is actually happening as they increase the number of trials.

Lesson 6: The odds found in a deck of playing cards. This lesson makes use of the probabilities found in a deck of playing cards. Students are taught about the probability of drawing the various cards based around colour, suit, number, and combinations of these results. A handout is given to students that has the cards of the deck organized in rows according to suit in ascending order of card value. This gives a visual reference for the various probability questions that can be asked (What are the odds of drawing a 7?) that will aid students in answering the question but also aid teachers and educational assistant in explaining questions to students. Having the cards in a diagram rather than using an actual deck of cards helps to keep the information clear without becoming overwhelming or requiring the organization of an entire deck of cards each time a card problem needs to be solved.

Lesson 7: Which option should I choose? Scenarios of choice are given to the students the outcome that has the most likely beneficial outcome is explained. This lesson focuses around the ideas of logic and using pros and cons from a mathematical standpoint to make choices based around numbers rather than preferences. Game theory makes use of this type of math to determine the best possible choice in a given situation. While the scenarios described may seem isolated to students, making analogies to choices they make in their own lives will help to connect this math and allow students to make mathematically informed decisions in the future.

Activities for students mainly involve the use of logic to reduce the number of choices and improve the likelihood of a correct choice. Many of the example involve multiple choice problems where there are some obviously incorrect choices. The change in the probabilities due to the removal of incorrect choices is mathematical proof of the advantage that using logic on multiple-choice problems can have. Many tests in the future, including KeyMath assessments, will have multiple-choice questions to which this process can be applied.

Lesson 8: Games of chance and the math behind them. The main focus of this lesson is the explanation of games of chance and how their probabilities are determined. Games such as roulette and poker are examined with emphasis being placed on why certain events are less common and therefore have higher payouts. A key take-away from this lesson is the idea that a less likely event should have more value associated with it. Roulette is an example of one such game. Many students encounter these games and this will give some logic and reasoning behind these games. False beliefs about gambling should also be discussed in this lesson to dispel any misconceptions of gambling, such as the gambler's fallacy. This concept is related to the idea of independent events found in the curriculum.

Project: Game of chance. Students will create and describe a game of chance. Several examples are given and the project leads to a carnival day where all of the games are played in class and students move about taking turns playing the games. The game needs to have its probability stated. Many modern games, both board games and computer games, make use of probability. Students are encouraged to research these games and make connections to their own hobbies and interests. The knowledge that their hobbies make use of the basic mathematical principles they have been studying will give the students a greater appreciation of both mathematics and the role it plays in the activities they enjoy.

The Pythagorean Theorem Unit

This unit is an excellent example of the concrete/representational/abstract approach due to its classical discovery and use. The math unit makes extensive use of a visual tool of the students' own creation, involving them in the process of learning the Pythagorean Theorem. The theorem can be shown using manipulative and videos that clearly illustrate the concept. This unit has traditional been seen as quite difficult but recent success has been found using the tools built in this unit with these assignments as assessment. The theorem's classical origins have allowed centuries of examples to be developed along with application to many real world situations including the trades and more academic applications; this allows for appeal to many typically achieving individuals from a range of backgrounds.

Assignments do not deviate from a set idea or require students to alter the algorithm of the day in any significant fashion. All students will have equal access to the lessons and will be taught as a single group.

Higher achieving students are encouraged to find more accurate answers or to reinforce concepts on IXL. An example of this is the number of decimal places in a square root. An acceptable answer for the square root of 150 would be "between 12 and 13" because the square roots of 144 and 169 are 12 and 13 respectively. This can be found using the visual tool that the students create. A more accurate answer would indicate that the answer is closer to 12 than it is to 13. A decimal answer would also be acceptable but is not considered the "best" answer due to the fact that a calculator can be used and may not reflect understanding. The explanation of the answer is what makes it better, not the accuracy of the number.

The importance of providing proof is reinforced by the marking scheme used for the questions. This is also an opportunity to teach the importance of showing organized thinking through proper mathematical reasoning in a way that can be easily followed by a reader. Marks are given for the inclusion of the formula, the work of substitution, squaring, and taking the square roots, as well as a correct final answer. In the case of longer word problems, both a diagram depicting the question and a sentence answer are also required. Students become aware of the many aspects of the questions but also see the basics that are repetitive throughout the application of the theorem. The basics of the theorem's formula and substitution as well as the labeling of a right angle triangle are present in all questions and give students the potential to never receive "0" on any question by always providing at least these basic pieces of knowledge. Receiving partial marks has the dual effect of increasing student confidence and allowing the teacher to be aware of at least a basic understanding of the concept.

The unit makes extensive use of the visual tool that is created in the first several days. This constantly ties the questions using the theorem to the understanding of what it means to be a square number and how the roots of these numbers operate. This will create a conceptual reference point for students and prevent the more complex problems from becoming too abstract for students to understand.

While not explicitly examined in the KeyMath-3 diagnostic, all of the requisite mathematical operations and understanding necessary for the Pythagorean theorem are separately examined across various subtests. Student are asked to solve the sum of two square numbers (Ex. $3^2 + 5^2$) and there are multiple questions regarding students understanding of angles, particularly of 90° angles. Awareness of type of triangles is also tested for. Ability to take a square root is also tested. Use of formulae in a variety of applications is also examined. Students will be given clues to watch for in the question that allow them to understand if they are looking for a leg or a hypotenuse. These clues are explicitly connected to variations of the formula given to them in the notes. Students are shown how to manipulate the initial formula but the use of clues and pre-manipulated formulas takes some of the burden off of struggling learners and allows them to more actively participate in the topics of the day. All of these separate pieces of mathematical knowledge are utilized during any given theorem question and fluency with these terms is major goal of this unit.

The follow unit was created using the curriculum goals of Grade 8 Math in the BC Ministry of Education New Curriculum. The table gives the keys ideas for each lesson as well as some math difficulties that can be encountered in the subject area.

Table 2

Pythagorean Theorem Unit Summary

Lesson	Key Idea(s)	Math Difficulties Approached by the Lesson
Lesson 1: Square Numbers	The reason we refer to numbers as "square" is because they literally create perfect square shapes	Students with MD can have difficulty in understanding the reasoning for certain terms. This lesson gives explicit instruction for the meaning of square numbers, which forms the basis for the unit.
Lesson 2: Create the Square Number Rainbow Sheet	Square numbers have a pattern that can be shown visually for easy reference	The tool created in this lesson is a strong visual tool that simplifies and illustrates the relationship between squares and square roots while helping students to memorize the pairings of root and square.
Lesson 3: Use of the Rainbow Sheet	The creation of a tool can help to reinforce concepts and facts	This lesson further uses the tool to allow students to become proficient and internalize the number pairs of the tool.
Lesson 4: Estimating Square Roots	Square roots do not have to have exact decimals and have a reasonable range based on their values in relation to perfect squares	This lesson tries to make it easier for MD student to represent an estimate by giving a range for the values of a root, rather than giving exact number or just simply using a calculator to give a number they don't truly understand. This gives an accurate gauge of the students understanding of roots and the ability to use the tool they constructed in previous lessons.
Lesson 5: Introduction of Types of Triangles	There are various types of triangles with consistent characteristics	Visuospatial awareness can be a problem for MD students and this lesson seeks to highlight the differences between seemingly similar objects (triangles) and how they can be grouped together into

		subgroups based on certain characteristics.
Lesson 6: Pythagorean Theorem Introduction (Concrete and Representational)	The theorem has many concrete proofs that show the relationship between the legs and hypotenuse of a right-angle triangle	General math instruction can be improved by using concrete examples. This makes these ideas more accessible for MD learners and helps to improve their understanding and involvement in the class. The Pythagorean has many such concrete examples available in online videos or through the use of math manipulatives.
Lessons 7-9: Use of scaffolding and single ideas (manipulations of the algorithm to solve various kinds of Pythagorean problems)	When given the legs of a right-angle triangle, you add the squares and then take a root to determine the length of the hypotenuse	This lesson does not deviate from a set pattern of being given the legs of a triangle to determine the hypotenuse. It can be difficult for MD students to constantly change algorithms and this lesson was meant to focus and improve understanding of a single application of the theorem.
	When given the hypotenuse and a leg, subtract the square of the leg from the hypotenuse square to determine missing leg	Again, this lesson focused around a single concept to leave less room for confusion of MD student who have difficult with multiple types of questions. Giving students a list of key terms or hints to watch out for will help them learn to understand when and where they can apply the various forms of the theorem
	During word problems, determine if you have a hypotenuse or not to determine if the question is an addition or subtraction problem	The main focus of this lesson was to give student the ability to determine the presence of an hypotenuse either through drawing a diagram or recognizing language in a question. This ties in to the previous lessons and allows students to identify the algorithm necessary to solve the problem.
Project: Pythagorean Art assignment	Right-angle triangles can be placed together in a multitude of ways to form images with mathematical meaning	This assignment allows for a great deal of student independence while creating achievable goals for learners who are having difficulty. MD students can show their understanding at whatever level they feel most comfortable. This assignment also allows students to create their own simple problems and then solve them.

Lesson 1: Square numbers. Students are given a table and create a list of numbers that are multiplied by themselves (1x1, 2x2, 3x3, etc.). Each expression is then

drawn as a set of rows where 1 is 1 row of 1, 2 is 2 rows of 2s, etc. This has the effect of creating a square and shows the reason for calling these numbers “square”. The amount that can be used to create the square is referred to as a square number and the number of rows is the square root. Students are given a visual representation of the definition with this method and this will help to internalize the concept.

Lesson 2: Creating the square number rainbow sheet. This is a visual representation developed as a tool to use in solving square number and square root problems. Students use graph paper to create a graph that shows the relationship between increasing numbers (1, 2, 3…) on the horizontal axis and their respective squares (1, 4, 9…) on the vertical axis. The graph ends up create a square that consists of the exact number that corresponds to the root. There is a square of 9 blocks at 3 and a square of 25 blocks at 5. Students are able to see the increasing value and proportions of square as the numbers increase. This tool also allows for easily finding the root of a number or the square of a number. The lines are coloured and students need only start at the corresponding root or square and then follow the line back to the corresponding axis. This method also works for non-perfect squares as an estimation tool. Students may not determine the exact root but will be confident that it is between two square roots and will likely be able to say which of the roots the actual solution is closer to. A sample of this tool can be found in Appendix-B.

Lesson 3: Use of the rainbow sheet. In this lesson the sheet created during the previous class is utilized to determine squares of numbers and their respective roots. Use of the tool may highlight any errors used in the construction of the tool. Lines may not be lined up properly and so may give incorrect answers. This lesson serves as a calibration of the tool that the students have developed. Students are made aware of this fact and are therefore taught the skill of testing to determine accuracy of a tool or concept.

Lesson 4: Estimating square roots. Use of the developed tool is further refined in this lesson. Students are given notation to show non-perfect roots and squares. Showing that a root is between two numbers allows students to not simply recite a memorized answer or do basic calculator work. This type of answer requires use of the tool and a thought process that shows an understanding of what the true answer might be. This lesson also shows students that the square root estimates have a narrower range than square estimates and the idea of amplifying error through multiplication.

The non-decimal method of displaying square roots can be given by the following example. If a student is attempting to determine the square root of 45, they would locate the relative position of 45 on the rainbow squares sheet. As 45 falls in between 36 and 49 on the table, the student takes the square root of 36 and 49 and determines that the root of 45 must fall between 6 and 7, the roots of 36 and 49. As a means of displaying this result the student could write 6-----7, indicating a range that the root must fall in. As a further display of understanding, the student could then indicate that the root likely falls closer to 7 because 45 is closer to 49. That can be indicated with 6----x-7. The student has shown several levels of understanding and has not resorted to using a calculator or relying on decimals. For students with a lack of symbolic understanding of the relative value of decimals, this method provides them the means to show their level of understanding. This method also gives other learner another means of expressing their own understanding of an estimated square root.

Lesson 5: Introduction of types of triangles. The major groupings of triangles are discussed with images and key characteristics displayed. This is done to show that not all triangles will work with the theorem and how to identify a right angle triangle compared to other types of triangles.

The specific types of triangles mentioned in the lesson include, scalene, obtuse, right-angle, isosceles, oblique and equilateral. The triangles are visually displayed and students are shown the relationship of angles and the importance of the right-angle triangle in the context on this unit. Identifying a right-angle triangle and the ability to infer that a right-angle triangle is being discussed are key components of this lesson. An example of such a situation could be a boat that leaves a dock and goes south for 8 km and then east for 6 km. It is implied by this question that the turn would have been a right-angle even if it is not explicitly stated. Inferring this sort of information will be important in setting up diagrams and knowing that the theorem can be applied. An advantage of IXL reinforcement at this stage is the access to assignments that can quickly reinforce these terms with immediate feedback to students. The program will allow for more practice that a worksheet would allow and will give rapid feedback to ensure that all learners have a strong foundation for the terms that will be used in the unit.

The visuospatial deficits of some MD learners will keep them from being able to recognize or recreate some of these triangles. Focus should be given to identifying right-angle triangles over the ability to recognize the other types of triangles. The right-angle triangle forms the basis of this math unit and this should be made explicit to all learners.

Lesson 6: Concrete and representational forms of the Pythagorean Theorem. The theorem itself can be illustrated using a manipulatives to clearly and concretely illustrate the concept of the $a^2 + b^2 = c^2$. There are many videos that illustrate the concept in a fluid manner. Students are given several example questions with numbers small enough to be solved using both manipulatives and the square numbers tool. Those triangles that do not have a perfect square hypotenuse are estimated using the tool.

During this lesson, various videos are shown displaying scenarios that display how the theorem functions. A key aspect of this lesson is the multiple means of providing evidence that the theorem functions. It is not enough for students to simply accept that a theorem works in the modern curriculum. Emphasis is being placed on the ability of students to prove that an answer is what they say it is by providing accurate evidence. The visual aids that have been developed for the theorem provide a body of evidence that gives increasing support for the theorem. In this way students are exposed to the idea of math as not a set construct but an ever expanding body of knowledge that can be examined in new ways, even after thousands of years.

Lessons 7-9: Variations of the Pythagorean Theorem. These lessons show the manipulation of the Pythagorean Theorem and then make consistent use of that single change to the theorem.

The first scenario is the basic $a^2 + b^2 = c^2$ where both a and b are given in the equation. The squares are added and the root determined to obtain c. This method is then consistently used for the remainder of the lesson to avoid confusion. Questions are kept within the range of the rainbow sheet initially to keep material simple for students. After the initial assignment, moving to larger numbers that require calculator work with require training so that student use the square root functions properly. It is possible to maintain the line method with calculator work to help reinforce the idea of the decimal being between two square roots. Students can give their answers in the form of the line method rather than simply transcribing the number from a calculator. This requires that students take the time to actively think about the decimal and determine which number it is closest too. This exercise can help reinforce the ideas of estimation for all learners but will be particularly useful for MD learners who may not have fully developed the concept.

The second lesson's scenario adjusts the formula to $c^2 – a^2 = b^2$ and students are given the hypotenuse of the triangle and then asked to determine the missing value. How this formula is arrived is shown explicitly through the manipulation of the original $a^2 + b^2 = c^2$ as student are shown how equation can be manipulated to determine the values for other variables. The formula $c^2 – a^2 = b^2$ is then consistently used for the remainder of the lesson to avoid confusion. This prevents students who have difficulty in manipulating formulas from having a barrier to their progress that is not reflective of their ability to use the formula provided and complete the assigned work.

The third lesson and any further practice sessions that are deemed necessary examine the use of the theorem in word problems that reflect real world scenarios and applications. It is during this lesson that patterns and sets of numbers that meet the criteria of the theorem are highlighted and elaborated upon. These number sets have been known to mathematicians for thousands of years and the student now have enough theoretical foundation to prove that the numbers sets meet the theorem's criteria.

These lessons can also make use of direct measurements made by students from around their classroom and school environment. The students can make two measurements of right angle triangles that occur in these environments and make predictions of what the third number should be. The students can then make the final measurement and compare predicted and measured values. Reasons for any discrepancies can then be discussed or hypothesized. In this way, all students see the real-world application of the theorem and also apply the scientific method of prediction, measurement, and critical reflection.

Project: Pythagorean art assignment. The project can be completed in a variety of ways and is very student driven. Completing the project has the student essentially creating their own practice problems by creating a series of right angle triangles on graph paper. The process of creating overlapping right angle triangles has the effect of creating an abstract picture of triangles with the associated math being shown on a separate page. Students may also choose to create an image using right-angle triangles as the building blocks of the image. This imposes a limitation that sparks creative and novel use of triangles that students may have never otherwise encountered. The idea behind the project is to have a student driven art project that links the concepts of mathematics and artistic ability. Examples of this project can be found in Appendix D.

Algebraic Expressions Unit

The key concept of the algebraic expression unit is the representation of numbers in multiple ways. A number can be represented symbolically by variables or numbers may be represented through expanded expression with operations that give that number as a simplified result. A number being represented by a variable is given by a=3. When the student sees an "a" in the expression, then that "a" represents the value "3". A number being expressed as a series of operations is 3 + 4 – 5. When solved, this operation represents the value of 2. The number 2 can also be represented by 10 / 5, which gives a result of 2. Numbers have both a simplified and expanded expression and this unit focuses on the manipulation of mathematical concepts to either simplify or expand the representation of a given value.

Symbolic representations are a difficulty for many students with MD and this unit will be a test of this cognitive domain. The emphasis is not only simplifying expressions but on recognizing that a mathematical expressions is an expanded representation of simpler term. Assistance that is given to learners with a specified problem in the area of symbolic representation should emphasize the direct correspondence between the given variable value (m = 5) and the expression (3m + 2). It is important to clearly show these students that the "m" is really a "5" in this question. The added complexity of introducing variables for substitution that must be solved by order of operation will make these expressions more difficult than normal but explicit instruction will help to alleviate these problems.

The KeyMath-3 assessment makes extensive use of questions that have their origins in the concept of algebraic expressions. Students are often asked to simplify operations and substitutions during the assessment and the concept finds its way into several of the test categories. This highlights the fundamental nature of algebraic expression in understanding more complex mathematics. The ability to compute and simplify for algebraic expressions is responsible for a significant portion of a student's score on the KeyMath-3 assessment.

The follow unit was created using the curriculum goals of Grade 8 Math in the BC Ministry of Education New Curriculum. The following table describes the key ideas of each lesson and the potential math difficulties that can be encountered when teaching the lesson.

Table 3

Algebraic Expressions Unit Summary

Lesson	Key Idea(s)	Math Difficulties Approached by the Lesson
Lesson 1: Basic Expressions and Simplification	Mathematical expression are complicated representations of single number	This lesson begins to show students a new way of approaching mathematics and how expressions represent the number that is their “answer”. This will be a very difficult task for MD learners who have difficulty with symbolic representation.
Lesson 2: Order of Operations	The order in which we solve problems can effect the outcome	The difficulty that many student will have with this lesson is seeing how an expression can be solved multiple way but that there is a correct way. Continued practice and use of manipulative examples where order matters will help MD students.
Lesson 3: Substitution into Expressions Using Variables	Variables in a question simply stand for a given value and are then solved as normal	This lesson will be difficult for those learners who have trouble with symbolic representation. Explicit instruction on the variable becoming a number will hopefully lessen the difficulty of this topic.
Lesson 4: Simplifying Algebraic Expressions with Variables	Parts of an equation can be grouped together to create a simplified expression that may not be a single number	There is a specific approach that can be used to treat math as a language just like any other. This concept is particularly useful for algebraic terms and will help MD learners with their understanding of like terms using algebra tiles or other manipulatives.
Lesson 5: The Distributive Property	A term multiplying a group of terms in a bracket, applies to all terms inside the brackets equally	This concept should be taught carefully to MD learner and it should be made explicitly clear that the multiplying term is how many sets you have of what is in the brackets. MD learners will have trouble with the abstract and multistep nature of the concept.
Lesson 6: The Steps of Simplifying Expressions	By using the logical sequence of simple to complicated, it is possible to put the steps of already solved expressions in order	Applying logic to the steps of the problem such as knowing which step has more terms will help students to see that they can place a solved equation in order without the need to directly simplify each step. The pattern recognition may be difficult for MD learners.
Project: The Math	A single term is	Some student with MD may have trouble

Map	represented in a multitude of ways with project criteria guiding how the term is expressed to ensure that all methods from the unit are represented	with the amount of freedom that this project gives and will need clearer and more direct instruction to produce a final product. Having an EA or teachers help to create examples within the students assignment and then having the learner create a similar expression may alleviate this problem

Lesson 1: Basic expressions and simplification. Student will initially be shown a binder that has been organized into 4 sections using dividers. Each section has a number of pieces of paper that may be blank or may be worksheets relevant to the topic of the section (Math, Science, Socials, etc.). Students are asked what the object is. Students will answer binder and some may answer book. The binder will then be opened and the sections taken out in their entirety. The binder is now described as 4 sections with each having a label. Now, rather than calling the binder a binder can we not then name if for its four sections (A Math-Science-Socials-English)? While more complicated, it is still factually correct and we are still naming the same object. Now each of the sections has its papers taken out with lined paper going into a pile and handout going into another. Can the initial binder now be called lined paper and worksheet from this section, lined paper and worksheets from that section, lined paper and worksheets from that section and lined paper and worksheets from that section and finally lined paper and worksheets from that section. This is a yet more complication way of describing the same initial binder. Finally the individual papers are counted in a final and overly complicated fashion. On the board or projector, this pattern can be shown as an expanding pyramid with either the binder on top expanding towards the bottom (the reverse of a typical problems) or just as easily in the reverse (a typical pyramid shape).

In this example, we have highlighted both the overall pattern of how mathematical expressions are simplified and the idea of representing objects by their constituents in an increasingly complicated way. This is meant to show students that both simple and complicated description are accurate and still represent the same value. The same can be done for numerical values. This will be demonstrated concretely for students through the use of math manipulatives.

In this lesson students are shown basic expressions ranging from simple (3 + 7) to the more complex (3+5-7+12). These expressions will have varying operations but will not yet be complicated to the point of needing order of operations knowledge, which is the subject of subsequent lessons. The expression will then be simplified by solving the various operations to reduce the amount of numbers. Students are encouraged to come up with 3 term expressions that have a real world application. An example of this might be the term 4 – 3 + 12 which might represent a student having 4 pencils, giving 3 to her friends and then taking 12 from Mr. Stovel's extra pencil tin. Students will be given manipulatives for each expression and then be asked to demonstrate other ways that the same amount could be represented with those same manipulatives. The expression for each representation will be written down to further enforce the idea of written expressions representing real world values. Such values are the focus of several curricular competencies such as representations, justification, modelling, and estimation. These terms can be directly referenced in the subsequent activities of the unit to give clear examples of student competency in these areas.

Lesson 2: Order of operations. In this lesson student will be using concrete manipulatives to show that operations done in various orders can result in different answers. A key aspect of this lesson is how do we write or ask a question that tells someone to do the steps in the order we intended or in the order that reflects what actually happens when those numbers and operations are applied to real world scenarios.

The demonstration for this lesson involves an initial amount of math counters. For this example we can say there are three counters. We can then say we start with a group of 3, subtract two of them and then have five copies of the group (subtracting 2 and multiplying by 5). We then do the reverse order by subtracting 5 copies of 2, which is simply subtracting 10. This results in two different answers that are written on the board. One of the answers cannot be illustrated with math counters and provides a brief introduction to the idea of negative integers. The students are then asked which one is correct. We have to then explain the order of what happened showing the ways of writing the operations. When writing mathematical expression, we need to examine the interpretations of the expression. For 3 x 5 + 2, is this three times five and then we add two resulting in 17, or is this three times the five and two added together (7), which gives 7 copies of 3 resulting in 21. Communicating which operation should be done first is the basis of understanding the order of operations and that there is a correct way to interpret a given set of operations.

Students will be placing their math counters into stacks of a given number, asked to split that stack that represents the initial stack, and then repeat this several times. This will allow the student to show the original stack in multiple ways. As a wrap up activity for the students, they can be given personal whiteboard and placed into groups. A number between 10 and 20 is placed on the front board and students must come up with multiple means of expressing that number. As an added piece of competition within the group, students receive a point for each novel expression they have that no other member of their group managed to create.

Lesson 3: Substitution into expressions using variables. The key idea of this lesson is that a missing value of an expression is given and then "plugged-in", or substituted, into an expression that allows the expression to be simplified.

From a concrete standpoint this is shown using math counters with the inclusion of a container. The counters are placed into stacks and the container is left empty. The only action that can be taken is adding counters to or removing counters from the container. By adding or removing counters to the container, we can then change the total number of counters in there are in the container. This provides the student with the essence of what a variable is. A variable is the portion of an expression that can be changed or varied to give a different result.

Students will be given counters and containers to represent expressions. They will set up the stacks of counters and then put the container next to the stacks. Students will then write the corresponding mathematical operation of the counters and containers in front of them with a variable being represented by the container. This will represent a series of addition substitutions. An example of this is 5 + c. A stack of 5 counters is placed next to an empty container.

In the case of subtraction, a separate type of counter can be used (this would help to introduce the idea of negative values). An example of this might be 5 – c. Because this "c" represents a subtraction, it might be a different colour (red if the initial container was blue) than the container used for the previous example of addition. For subtraction, the students could also physically remove counter. This second option may be less abstract and appealing to those students who have difficulty with understanding that adding negative value object counts as subtraction.

In the case of multiplication questions, the number of counters in the container might represent the number times a particular stack is being copied. In this way, the number of counters in the container are almost like a set of instructions, telling the learner how many sets of the stack they need to make. It could also represent the number of rows or groups of variables that are being created. An example of this might be a stack of 5 with 3 counters in the container. This would be giving the student instructions that there need to be 3 stacks of the initial 5 counters.

For division, the number of counters in the container could represent the counters that will make up each new stack. An example of this might be a stack of 10 with 5 counters placed in a container. This would mean that the 10 needs be split into stacks of 5. Students would count out the stacks and determine that they now have 2 even groups with no counters left over. The number of stacks that can be made represents the simplified expression. Extra care should be taken to make sure number work out well and that situations that do not solve evenly (12/5) unless students have an understanding of remainders and these situations should be avoided for simplicity. Even if students are not comfortable with remainders, this method gives them an idea on what remainders signify and could lead to further discussion about what could be done (breaking the counters into pieces). This may lead to increased understanding of decimals and fractions.

This lesson focuses on giving students the understanding that a variable is the part of an equation that can change and that the amount of change is determined by the amount of the variable. This concrete basis for the operations gives a reference point for all student and helps to make the concept less abstract for those students with difficult with symbolic representation. See Appendix C for examples of each scenario.

Lesson 4: Simplifying algebraic expressions with variables. In this lesson, the main idea for students is that certain terms can be combined to make the expression simpler. The math counters provided should have some sort of overall organizing characteristic, such as size, shape, or colour. The amounts should initially be grouped inefficiently so that students can practice organizing, or simplifying, the groups. This will allow students to learn that variables can be grouped together as well as numbers (constants). An example of this would be the expression $3a + 2 – 2a + 5 – 6$. Those terms with variables ($3a$ and $– 2a$) can be combined and might be represented by blue and red squares. Blue representing positive integers with the "a" variable and red squares representing negative integers with the "a" variable. The constants (+2, +5 and -6) can be combined and would be represented with red and blue circles with red being negative and blue being positive. A review of integers, especially the use of integer tiles, would be necessary for this unit to succeed. A review could be done as part of the unit or this algebra unit to follow a unit on integers. Given that integers are now mainly found at the Grade 7 level, a standalone review of integers may be necessary to ensure student success and learning. The final simplified expression will be $a + 1$ and would be represented by a single blue square and a single blue circle.

When asking students to set up the initial conditions (groupings), it will be beneficial to have pictures or drawings of the initial setup on the board for students to copy and then simplify. If numbers are used on the board, part of the exercise is made redundant by not having students determine on their own what number are being represented and therefore not showing understanding of what the visual representations are meant to express. The use of pictures rather than numbers also makes the lesson more accessible to those students with MD who may have difficulty with symbolic representations.

Once students have shown competency in setting up given scenarios, students may be given the opportunity to set up their counters and containers and have other students determine the initial expression. This will test students' ability to understand the mathematical expression of a given situation and will provide opportunity for students to challenge one another.

Lesson 5: The distributive property. This lesson focuses on the concept of multiplication affecting the terms inside of a set of brackets. The key concept to gain from this lesson is that the outside term influences both inside terms equally. This can be proven to student using the concrete manipulatives of the earlier lessons. Using the idea that one type of counter indicates the number of copies being made (multiplying coefficient outside of the brackets) and two separate groups of counters can be used to represent those values that are being copied (the terms inside of the brackets). Students should be shown that the outside term or the number of copies does not change the final result if the inside terms are combined before applying the multiplier. An example of this is $2(3 + 4)$. The 2 can be applied to the inside terms first to produce $6 + 8$ which gives 14. The inside terms could be combined first to gives $2(7)$, which also gives 14. This idea illustrates that there is often more than a single way to solve a problem and determine a correct solution. There are more efficient ways of solving problems, such as order of operations, and this fact should be directly stated to students.

Lesson 6: The steps of simplifying expressions. In this lesson, several long expressions have been simplified over several steps to give a single correct answer. The entire simplification process is then cut into strips and jumbled up and combined with other expressions and their steps. Students have to then correctly assemble the strips into a correct order. Rather than directly testing mathematical calculation knowledge, this activity allows students to use problem-solving skills and reason to determine what order the steps should occur in. The overall pattern of increasing complexity from single term to complex term is what will allow the students to place the steps into the correct order. Some mathematical knowledge will be necessary to determine which numbers correspond to previous or subsequent steps but there is never an over reliance on computing the equation from beginning to end.

Project: The math map. This project is a variation of an earlier activity in the unit where students had to create multiple ways of expressing a single number that was placed on the board. In the project, students will be creating a mind-map on a piece of 11 x 14 paper bubbles emanating from a central number. This initial central number will range from 10 to 20 depending on student ability but a minimum number of expressions must be met, such as needing 2 additions, subtractions, multiplications, additions, distributions, etc. Some of the expressions must also be visual representations, like those used earlier in the unit. Students must also make use of variables in at least some of their expressions and give the substituted values for those variables that will give the simplified term in the central bubble.

This project is an extensive open-ended question that has students displaying their understanding according to their ability. Learners with math disabilities will need guidance in creating their expression and may need to be able to express themselves orally or with manipulatives to show that they can produce expression that are representative of their chosen number.

Surface Area Unit

This unit makes extensive use of concrete models that will give students a reference point for the majority of the work in the unit. The unit starts with the use of nets to create 3-dimensional shapes and these shapes will serve as the basis of concrete examples for the subsequent lessons. The vocabulary lesson is an overview of many of the terms that will be encountered in the unit with both formal and informal definitions to give students a clear understanding of what is meant by each term. The area and surface area lesson make use of the models built at the beginning of the unit to break down the nets and shapes that constitute the 3-D shapes and will help students to understand what formulas will need to be applied to calculate total surface area. The project that sums up the unit is actually a selection of 4 projects that students may select from based on their own interest or ability.

The readily apparent practical nature of area calculations and the use of hands-on models will help to develop and maintain student interest. Student interest and the recognition of the importance of a topic are factors that can improve student performance on mathematical tasks. Learners with math disabilities can have particular troubles in attending to tasks in math and this unit serves to remove this obstacle through direct application of the math to student’s lives and the use of concrete models to improve students understanding. The unit’s models and projects also help to appeal to more kinesthetic learner by allowing students to construct models that show their understanding.

The follow unit was created using the curriculum goals of Grade 8 Math in the BC Ministry of Education New Curriculum. The following table describes the key ideas of each lesson and the potential math difficulties that can be encountered when teaching the lesson.

Table 4

Surface Area Unit Summary

Lesson	Key Idea(s)	Math Difficulties Approached by the Lesson
Lesson 1: Folding of 2-dimensinoal Nets Into 3-dimensional Shapes	3D shapes can be represented as 2D nets that are comprised of multiple 2D shapes	MD learners that require concrete examples of concepts will be able to see the relationship between 2D net and 3D object. There are no calculation so MD learners with dyscalculia will not be affected by this lesson
Lesson 2: Geometry vocabulary	There are a wide variety of terms that exist to describe shapes in the surface area unit	MD learner can also have reading disabilities (RD). These are addressed by the use of informal definitions using language developed by the class to more clearly define certain terms. Cloze versions of the definitions are available. Use of labeled images helps to reinforce concepts.
Lesson 3: Anatomy of a 3-dimensional Objects	Various shapes have differing numbers of edges, faces and vertices, There are patterns in these numbers amongst certain types of shapes	MD learners can struggle with pattern recognitions. The patterns are clearly explained and written out using mathematical formulas. This also helps with those MD students who have memory difficulties and removes the number of individual facts that need to be retained.
Lesson 4: Creating Pictures Using Area Formulas of 2-dimensional Shapes	There are formulas for the areas of shapes that apply to total surface area	MD learners are repeating the formulas to improve memorization. There are no calculations so dyscalculia students are not at a disadvantage.

Lesson 5: Surface Area of Rectangular Prisms	3D shapes are comprised of multiple 2D shapes with area formulas. Marking scheme emphasizes displaying how to solve a problem, not solving the problem	This is a multistep process that will strain the working memory of learners. Choosing to focus on one aspect, such as nets or formulas, will limit demands on student working memory. Receiving marks for multiple pieces of information ensure that students receive some credit for displaying some of the knowledge learned in class.
Lesson 6: Surface Area of Triangular Prisms and Pyramids	3D shapes are given comprised of multiple 2D shapes with area formulas. Marking scheme emphasizes displaying how to solve a problem, not solving the problem	This is a multistep process that will strain the working memory of learners. Choosing to focus on one aspect, such as nets or formulas, will limit demands on student working memory. Receiving marks for multiple pieces of information ensure that students receive some credit for displaying some of the knowledge learned in class.
Lesson 7: Surface Area of Cylinders	3D shapes are comprised of multiple 2D shapes with area formulas. Marking scheme emphasizes displaying how to solve a problem, not solving the problem	This is a multistep process that will strain the working memory of learners. Choosing to focus on one aspect, such as nets or formulas, will limit demands on student working memory. Receiving marks for multiple pieces of information ensure that students receive some credit for displaying some of the knowledge learned in class.
Project: 3-floor House Plan, 1-floor Apartment Model, 3-dimensional Stellated Platonic Solid, Novel Cereal Box	Students are given multiple means of expressing themselves and their understanding of surface area mathematics.	MD learners can have difficulty with calculations and spatial reasoning. The last 2 projects are meant to address this by having student create basic or repetitive objects that fit together easily with the assistance of adults. The varied number of projects going on ensure that no students stands out as needing assistance.

Lesson 1: Folding of 2-dimensinoal nets into 3-dimensional shapes. The unit begins with students selecting 3 nets from a selection of options. The nets fold into a variety of 3-dimensional shapes that will be examined at various points throughout the unit. The shapes include objects such as rectangular prisms and cylinders as well as more

complicated objects like dodecahedrons and octahedrons. Students are asked to colour and decorate each of their models and then fold them into their 3-dimensional shape. It is explicitly stated that the models will be used by themselves and other students as examples, demonstrations, and tools to solve problems.

This lesson gives students the opportunity to be creative from the outset and also allow them to create their own tools that will be used later in the unit. This connection to later lessons gives the seemingly creative and fun lesson a sense of purpose that drives students to complete their models.

Lesson 2: Geometry vocabulary. The geometry vocabulary sheet is a 3-column handout that is given to students to go over the relevant vocabulary for the unit. There is a cloze-sentence version for those students who may have writing difficulties so that they can still participate and follow along with the notes. The goal of this lesson to give a list of key vocabulary terms that will be used throughout the unit (including reference to the models made in the previous lesson) and to give formal and informal language definitions of the terms along with an image illustrating the concept.

The first column of the handout contains the term already typed out. The second column is labeled "Definitions" and is where both the formal and informal definitions will be written. The formal definition will contain jargon terms and language that not all students will be familiar with. The informal definition that will be written in this space is a simplified version of the language to give students a clearer picture of what the formal language is saying. An example of this would be the definition of the term vertex on the vocabulary sheet. The formal definition is "the point on the object where several edges meet" and is itself a relatively simple definition. The informal definition would be written based on student feedback and conversation about what this formal term means. An example of the informal definition might be something like “the pointy and pokey part of a pyramid”, which is an accurate description of vertex, even though it is not necessarily a complete definition that uses jargon. In this way, students are participating in the construction of the language that describes the objects and concepts that will be studied in the unit.

The final column is for an image that describes the concept in as simple a way as possible. This image may include labels or sketches of the objects that were made in the first lesson as a means of describing the vocabulary term in question.

Lesson 3: Anatomy of a 3-dimensional objects. In this lesson students will be using the objects construction in the first lesson and describing them according to the vocabulary defined in the second lesson. Students will be given a sheet that lists 3D objects made in the first lesson, along with several that were not created in the first lesson (such as spheres or octagonal prisms). Students are asked to state how many faces, edges, and vertices each of the objects possesses. Use of the objects that were made in the first lesson, in addition to models that may be brought in from a math resource room, will give students a physical object to interact with so they might better count the faces, edges, and vertices.

Upon completing the activity in this lesson, the class then looks for patterns in the faces, edges, and vertices for shapes that are related. Formulas are then produced to help students who may not be able to remember the exact number of ease measurement but can be applied to arrive at a correct answer. An example of this is the pattern seen in prisms. The number of sides will vary between rectangular, triangular, and octagonal prisms but the total number of sides will always be 2 (the number of bases) + the number of sides of the base. This pattern will be developed from the information on the activity sheet so that students are given the opportunity to create formulas that describe patterns from a list of data. Other such shape patterns occur for edges and vertices and can be applied to the different types of pyramids.

The IXL program provides activities that are similar to this for additional practice to reinforce the ideas learned in class.

Lesson 4: Creating pictures using area formulas of 2-dimensional shapes. This lesson is a review of the area formulas for triangles, squares, rectangles, and circles. For circles, the formula for circumference is included as circumference is needed to solve for the surface area of cylinders.

The activity of this lesson is writing each of the formulas multiple times in order to create the shape of the object to which it applies. This will reinforce the relationship between the formula and the shape to which it corresponds. The triangle formula (1/2 b x h) is repeatedly written to fill the shape of a triangle, the square formula (s^2) is written to fill the shape of a square, etc. For circles, the circumference formula is used to write along the circumference of the circle. Students are encouraged to use colours and to arrange multiple shapes to create an image that is made of the various formulas. Most learners will able to complete this assignment with modification being made for those learners who have difficulty writing having the shapes already drawn for them and repeating the formulas a minimal number of times.

The numerical practice of the formulas will make use of IXL as it gives immediate feedback and can be completed at the student's own pace.

Lesson 5: Surface area of rectangular prisms. This lesson focuses on rectangular and square prisms and the shapes that comprise them. Students are shown what the nets are for the prisms (including cubes) and the number of each shape that needs to have its area determined. For examples a square prism has 2 squares and 4 rectangles that are all the same and a cube has 6 sides that are all the same. The associated formulas for each shape are shown and students are given a clear process to go through when they write out their calculations.

An introductory activity for this lesson is giving students several different configurations of the nets of the shapes. Students are placed in small (2-3 members) groups with several different nets for a given shape. Some of the nets will fold to create rectangular prisms and cubes while others will not work. At the end of the activity, those configurations that create the complete prism are placed together while those that did not create prisms are placed together. For learners with math disabilities, emphasis should be placed on those configurations that do work so as to limit confusion and different configurations that are memorized.

Marks are assigned so that students will needs to draw out the nets, label the sides with the correct lengths, give formulas for each different shape, substitute values into the formulas, show work for completing the formulas, give the areas of each shape, and then show a final step where they add all of the individual areas together to obtain a total surface area. The fact that these questions are very long is explicitly stated but students are told that many of the marks come from simply drawing the shape, labeling sides, and giving formulas. It is possible to achieve sufficient marks to pass a question by simply drawing nets, giving formulas, and substituting. The calculations simply add to the mark.

This is done to alleviate the stress of needing to obtain a final answer and instead focusing on illustrating understanding of what is required to get a final answer. This removes the barrier that some students with math disability have with calculating correctly. Students are also reminded that, while the questions are long, each individual step is a basic formula question that just happens to apply to a more complicated 3D shape.

Clearly delineating the distribution of the marks for each question will help students to see the different steps and will serve as a modified program for those learners that warrant modification. Teachers will be able to set specific goals related to parts of the question to determine student math competencies in determining surface area.

Lesson 6: Surface area of triangular prisms and pyramids. This lesson is similar to the previous lesson but applies to triangular prisms and pyramids. The nets for each are drawn out and the marking schemes are similar. These shapes may give students more difficulty as these prisms are not encountered as often and the calculations involve fractions.

Lesson 7: Surface area of cylinders. Cylinders are one of the most complicated shapes to obtain surface area for because they make use of multiple formulas and the numbers cannot always be made round because of the nature of pi. This situation is partially accommodated for with the marking scheme used for previous shapes that highlights the need to show how an answer can be obtained, not necessarily just the ability to produce an answer.

Project: Applications of surface area. The project for this unit is actually a selection of 4 different projects that students may choose from based on interest. The 3-

floor house plan is a drawing of the floor plans for 3 floors of a home that students design. Each floor must have a minimum of 4 rooms. The cost of painting and flooring the rooms must also be determined using a price list provided to the students. The 1-floor apartment model is similar to the house plan but has the students actually construct the model but still calculate costs to paint and floor the apartment. The 3-dimensional stellated Platonic solid requires students to pick 2 of the following: octahedron, dodecahedron, or dodecahedron. Students then add the appropriate pyramids to each surface to create a stellated version of that shape. Students need state the number of faces, edges and vertices of both the original and stellated shapes as well as calculating the total surface area before and after the object has been stellated. The novel cereal box project has students creating a 3-dimensional shape that can be used as a cereal box. Students must decorate their cereal box and decide on what kind of cereal they think it should be used for. The students must also calculate the cost of producing enough cereal boxes to supply all of the grocery stores in their area.

When introducing the project, it will be helpful for students to be given examples of projects and the math involved in each. A worksheet that has scaled down versions of each project and the associated calculation will help guide the decisions of students to select a project that both interests them and is within at their ability level.

Each of these projects makes use of the knowledge that students have acquired in the unit but have significant variation in how they accomplish this. The projects also vary significantly in how a student can give a creative expression of their work. Because student project will vary greatly from one another, students that struggle in math will not stand out as receiving additional assistance as each student is doing something quite unique from the other students around them. Learners that have reached the substitute-only stage and are able to measure and count the relevant information can be given adapted version of projects. The practice of taking measurements and then placing them into formulas will be hands-on practice and is a skill that can be focused on as the main educational goal that can be reported on with regards to student achievement.

IXL Reinforcement for Units

All of the above units will have reinforcement of their relevant ideas with assigned *IXL* sections. The program will give immediate feedback and rote practice for the basic skills necessary to fully comprehend the concepts being taught. It will be emphasized that *IXL* be used as independent practice whenever possible because of its immediate feedback for students.

It is encourage that educators preview the IXL activities they wish to use before teaching a lesson so that notes may be tailored towards the language of IXL. This is partially because teacher have no control over the types of questions asked by particular IXL problem set and students may not be aware of how to answer a question that is asked in an unfamiliar manner.

Chapter Summary

These units are constructed with the special needs learner in mind but still cover the necessary material for all learners. The units make use of concrete examples to ensure that learners with math disabilities may better access the material as part of the class. Student choice was given in several instances to allow for multiple means of expressing understanding. The emphasis on students expressing how to get a correct answer, rather than just getting a correct answer, opens the door to having learners that struggle with computation still being able to give an acceptable solution on how to solve a problem. The lessons listed do not constitute the entirety of each unit and will need to be supplemented with practice classes or additional sheets and materials. These units are general guides that have taken into account the needs of learner with MD. Certain subsets of math disability will require different considerations depending on the math lesson. Certain lessons will be differentially affected by particular deficits and these have been pointed out to educators in each of the lesson summaries found in the appendix. By using these general guidelines, teachers can create lessons that take into account the needs of learners with MD while still maintaining the integrity of the lesson and its concept.

Chapter 5: Reflection and Conclusion

Reflection

The intention of this project was to summarize and make use of the current research on mathematical pedagogy with respect to special education and to use a diagnostic tool, KeyMath-3, to guide construction of instructional math units. By building units around possible problems that students with math disability might encounter, the unit and its concepts can be made more accessible for all learners.

In my research, I encountered a wealth of information about the various ways that students can exhibit mathematical learning difficulties and that students will not necessarily exhibit all of these deficits. It is therefore important to provide varied instruction that does not stress a single cognitive domain, as this will make mathematics an overwhelmingly difficult task for those learners who have difficulty in that domain. The varied approach to mathematical instruction gives students multiple viewpoints of the same topic. This gives average and high achieving students a broader understanding of the material while giving low achieving students or students with math disabilities an access point for the material.

A major take-away from this research project was the separation of mathematical operations from the scenarios that they describe in common math pedagogical practice. By this I mean that I have begun to notice that the basic operations learned in the elementary years are new and distinct from one another but after the middle school years there is relatively little that is new in terms of operations. Students learn counting, addition, subtraction, division and multiplication and it is these operations that form the basis of all later mathematics. In many situations, the previous skills are only applied in different scenarios and to describe different real-world situations. This has led me to place a great deal of importance on the explicit instruction of the operations that are necessary for a given concept and to describe the real-world scenario to which the math is meant to apply. An example of this would be in the surface area unit that has formulas involving multiplication and addition that are applied to multiple shapes and then combined to give a total surface area. The overall amount of math can be overwhelming for students but breaking down each step and showing the basic operations (multiplication, addition, etc.) can help to alleviate anxiety in students. Giving students these instructions will hopefully alleviate the confusion and anxiety that can accompany learning a “new” type of math. This viewpoint also makes the mastery of, or at least functional ability to complete, these basic operations of paramount importance to success in math.

The ability to automatically mark a student’s work should not be underestimated. The time it takes to mark an assignment can greatly impact the time spent discussing errors or correcting student misunderstandings. By reducing time spent marking, not only do students get immediate feedback as to whether they are correct, but teachers are freed to directly interact with students for more time or to spend time on other activities that can directly address the misunderstandings of students.

The end projects of each unit often involved some form of artistic expression of the math concept being taught for that unit. Mathematics can become a one-dimensional subject for students with its only expression being numbers and correct answers. By using projects that encourage students to artistically represent math concepts, it was my hope that these learners no longer hold on to the stigma that math class needs to be only for those with number driven dispositions. Artistic expression has its place mathematics. By expressing learning in this manner, I also hoped that students would be driven to create more math expressions in order to create more art. This intrinsic drive could be a more powerful motivator than simply completing more math problems that I, as the teacher, had created. The intrinsic drive to create art projects around mathematical concepts would also increase the relevancy of the mathematics for the student, which has been shown to improve student performance in mathematics (Boyd & Bagerhuff, 2009; Sayeski & Paulsen, 2010; Shin & Bryant, 2015).

Any methods taught should be taught to the class as a whole and any goals should be made achievable by the class as a whole. This is not to say that students who have higher or lower levels should all produce the same end project but that all students should be able to access the same information and produce the same baseline product (Dieker & Rodriguez, 2013; Sayeski & Paulsen, 2010). Equitable access to the material does not mean there will be equitable production of material. The differentiating factor for the learners should be what they produce, not what they are taught.

Students need to understand why they are learning a given topic and this is an important question to consider not only from the perspective of students but also from the perspective of the teacher. Teachers need to understand the *why* of the education being administered and how that education will be useful to the student in the future. Considering this question at the time of unit construction will ensure that there is also an end-goal and purpose to each lesson with an overarching goal for the unit beyond simple mastery of basic skills (Zheng, Flynn, & Swanson, 2013). Having the students know how they will use the math later in real-life situations will provide more incentive to practice the skills now or in the present learning situation. Lack of interest can lead to lack of motivation which further leads to a lack of ability, as the student never practices the basic skills.

Using the KeyMath-3 to guide the construction of the units and assignments has allowed me to word questions and build the understanding of students that will directly influence any future assessment they may be given. When these students are assessed using a KeyMath diagnostic in the future, the questions will be reflective of material they have encountered in the classroom using a variety of means that were designed to improve their understanding of that specific concept. The learners will have encountered concrete examples of the concepts that improve understanding (Agrawal & Morin, 2016; Witzel, 2005) and will have been allowed to express understanding in a variety of ways at various stages of solving the problem. When the KeyMath-3 assessment is administered in the future to these students, it will be a more valid assessment because of the thorough way in which these learners have been exposed to the material that are being tested on.

The goal of this project was successful in that it has allowed me to use the knowledge and practices I have gained through my research to create entire math units that will improve instruction to all learners. Research has informed my teaching practice and I now have a greater understanding of not only math pedagogy but student difficulties with mathematics and the assessment of student math ability. All of these factors allow me to teach mathematics to all students effectively and work with other professionals to advance student understanding of mathematics.

Conclusion

By using the KeyMath approach to unit construction, the language of the math lessons was tailored towards the assessment that would lead to diagnosis of math disability. This allowed the units to be tailored towards deficiencies in mathematical understanding and ability while simultaneously considering the need for stronger math learners to be able to express their level of mathematical understanding. The units constructed tied together both the KeyMath-3 and the BC Ministry of Education Grade 8 Curriculum in a way that guided teaching practice towards preparing students for possible assessment using the KeyMath-3 diagnostic assessment. This practice will hopefully improve student performance on diagnostic tests and increase student confidence in their own ability to be proficient in mathematics. Students should be encouraged to see special education diagnoses as temporary and that they can improve their level of mathematical understanding to the point where their designation may be removed. Mathematical understanding is not a static personality trait and the KeyMath instructional approach is one way of improving special needs learner's mathematical understanding and performance.

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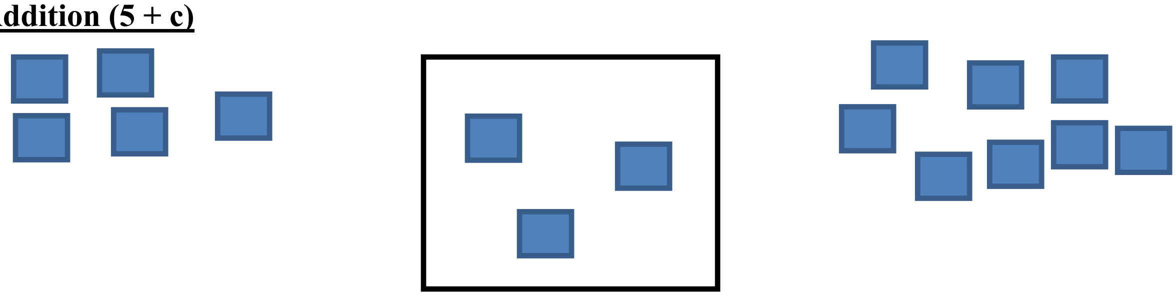
Appendix A: Sample Rainbow Squares Table

121											
100											
81											
64											
49											
36											
25											
16											
9											
4											
1											
	1	2	3	4	5	6	7	8	9	10	11

* This is just a sample of this assignment. On typical graph paper, students are able to create a table that includes up to 25 x 25. This is often sufficient for most questions that will be asked in this unit.

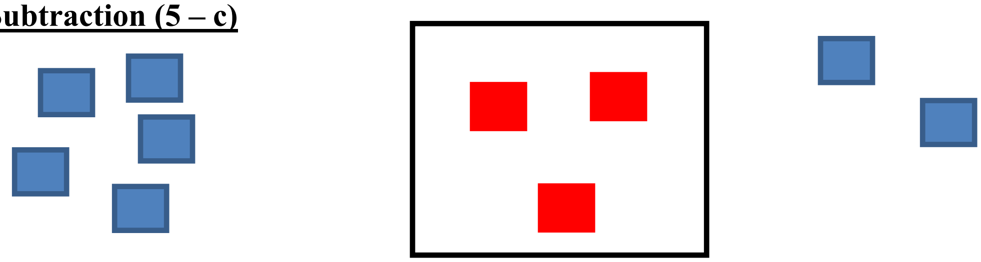
Appendix B: Samples of Algebraic Container Models

<u>Addition (5 + c)</u>



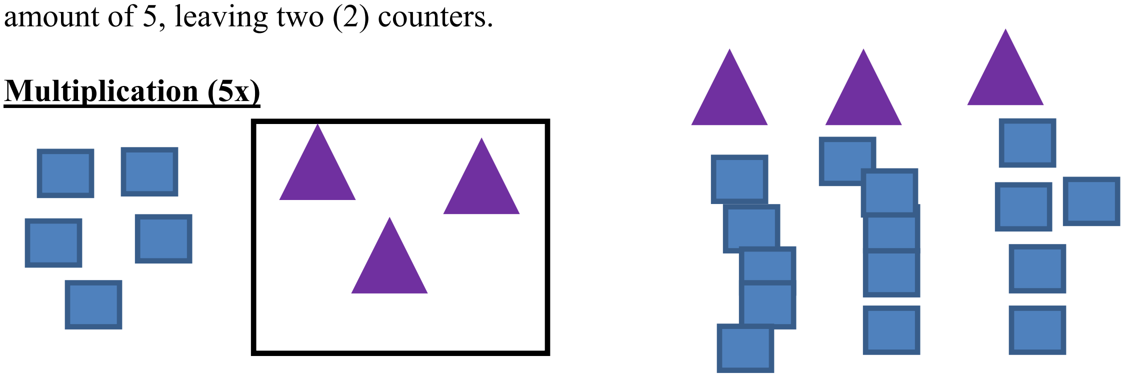
The five (5) counters outside are set in the question. The container is what can be varied and has three (3) added to it. Students count the total number of counter and see that there are eight (8).

Subtraction (5 – c)



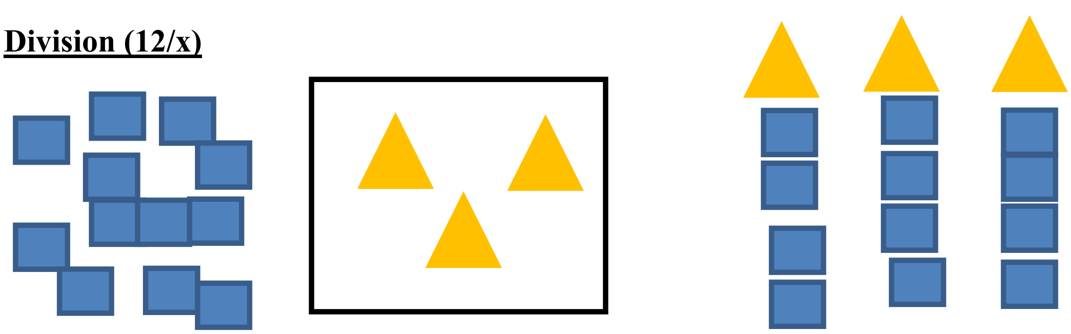
The container now has three (3) red counters added which are removed from the initial amount of 5, leaving two (2) counters.

Multiplication (5x)



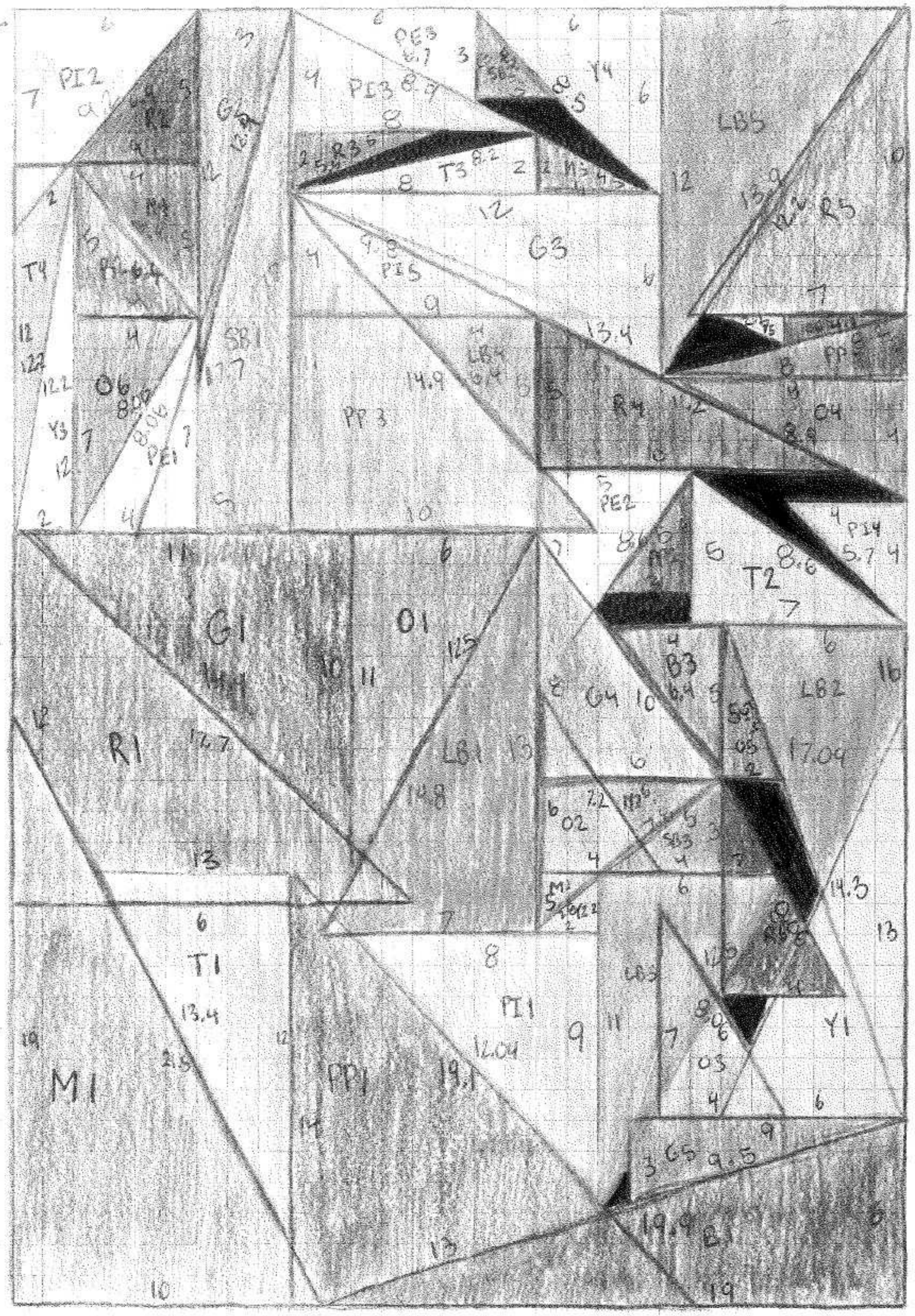
The container now has three (3) counters that vary distinctly to show the different operation and thought process. The counters of the container illustrate the number of stacks of the initial value, which now give a total of 15 of the initial counter.

Division (12/x)



The initial amount is now nine (9) and the container has 3 counters indicating division, which will be the number of even groups into which the initial set of counter is split (divided). The number of counters under each "divide" counter is the answer to the question.

Appendix C: Samples of Pythagorean Art Projects


PI2
PE3
PI3
Y4
LB5
T3
G3
R5
PI5
SB1
T4
O6
Y3
PE1
PP3
R4
O4
PE2
PI4
T2
G1
O1
R1
LB1
G4
B3
LB2
O2
SB3
T1
M1
PI1
PP1
Y1
O3
G5