TRUE QUANTUM TELEPORTATION

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B. Sc., University of Northern British Columbia, 1999

PROJECT SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

in

MATHEMATICAL, COMPUTER, AND PHYSICAL SCIENCES - PHYSICS

THE UNIVERSITY OF NORTHERN BRITISH COLUMBIA

August 2001

ABSTRACT

Quantum teleportation has been investigated experimentally, for a variety of physical systems. However, it has been suggested that most methods of teleportation do not achieve true teleportation. This is because a complete Bell-operator measurement cannot be performed without interaction between the quantum particles involved in the teleportation. Since the Bell-operator measurement is a key factor in the teleportation procedure, teleportation cannot be realized in the manner proposed in the pioneering paper on teleportation. In this project, it is verified that, without interaction between the quantum particles involved in the teleportation procedure, true teleportation cannot be achieved.

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ACKNOWLEDGEMENTS

I would like to thank Professor Mark Shegelski for all the time and effort that he put into helping me complete my project. Even when he was extremely busy, he always found the time and patience to consider any problem I might have been having. I would also like to thank him for being there to teach me my courses and be a member of my committee.

I would also like to thank Professors Lee Keener and Patrick Montgomery for taking the extra time in their busy schedules to teach me my courses.

I would also like to acknowledge my committee members Professors Erik Jenson, Lee Keener, and the external reviewer, Zbigniew Gortel.

Introduction

Science fiction movies and novels brought about the concept of teleportation, where an object would disappear from one location and instantaneously reappear at another distant location. In classical physics, all properties of an object can be known. Teleportation in the classical sense can be defined as the physical *transfer* of this system from one location to another. Alternatively, classical teleportation can be thought of as follows: a *copy* of the system can be replicated at the receiving location while the original could be destroyed at the initial location. The difference between the two is that the "transfer of the system" involves the breakdown of the system into several smaller components. These components are then sent to the final location. The "copying of the system" involves analyzing the system and sending the information about the system to the final location. Once the information is sent and the copy reconstructed at the final location, the original copy could be destroyed. This difference would be comparable to sending a letter and faxing a letter.

In quantum mechanics, it is not possible to know all properties of a system at any one time. This is a consequence of Heisenberg's uncertainty principle. At the quantum level, for example, the exact position and momentum of a particle cannot be known at the same time. This principle also applies to other pairs of physical observables. Therefore, teleportation in the classical sense cannot be achieved since a complete analysis of the object to be teleported cannot be obtained.

In 1992, Charles H. Bennett, Gilles Brassard, Claude Crépeau, Richard Jozsa, Asher Peres and William K. Wootters [1] examined the question of transporting the quantum mechanical *state* of a particle instead of transporting the particle itself. This question led to the idea of quantum teleportation, where the quantum state of a particle is teleported from one particle onto another. It can be said that the quantum state of a particle has been teleported from one particle to another if the final state of the second particle is equivalent to the initial state of the first particle. Quantum teleportation of the state consists of three main concepts. These concepts involve the Einstein-Podolsky-Rosen (EPR) effect, a Bell-operator measurement, and the classical transmission of the outcome of the Bell-operator measurement in order to perform a unitary operation. This will be further outlined after a discussion of the EPR effect.

The EPR effect

One concept needed for quantum teleportation is the idea behind the EPR paradox, originally described by Albert Einstein, Boris Podolsky and Nathan Rosen [2]. The teleportation scheme involves the use of the EPR effect, or entanglement. When two particles are entangled, the pair of particles *must* be thought of as one entity, *not* as two separate particles; i.e. the pair of particles must be described as an entangled two-particle system. Identical particles in a two-particle system are indistinguishable so it is only known that there are two particles in the system where each particle cannot be distinguished from the other. Before discussing the state of an entangled two-particle system, we will review the notion of the spin state for particles. The spin state will be used to describe the state of an entangled two-particle system.

Spin-½ particles, such as electrons or protons, can have a component of ½ or –½. These can alternatively be described as spin "up" and spin "down". A spin-½ particle can be prepared in an up or down state. In general, before the state of the particle is measured it is only known that the spin state, $|\phi>$, is a linear superposition of the up and down states, or $|\uparrow>$ and $|\downarrow>$. This superposition of states can be written as

$$|\phi> = \alpha|\uparrow> + \beta|\downarrow>,$$

where α and β are complex numbers that satisfy the equation

$$|\alpha|^2 + |\beta|^2 = 1.$$

Note that the state ket is prepared in a definite direction and is therefore a pure state and not a mixed state. When the spin state of a spin-½ particle has an equal chance of being measured in a spin up or down state, we may choose $\alpha = \sqrt{½}$ and $\beta = \sqrt{½}$. Therefore, the equation for the state $|\phi>$ before the spin state is measured is

$$|\phi> = \sqrt{½}\,(|\uparrow> + |\downarrow>).$$

Once the state of the particle is measured, $|\phi>$ is projected either onto $|\uparrow>$ or $|\downarrow>$ by the action of the measurement. However, each time the spin state $|\phi>$ is measured there is a 50% chance of being projected into the state $|\uparrow>$ or $|\downarrow>$.

An entangled two-particle system has the states of the two particles in a linked superposition of states. This linked state is identified as an entangled two-particle state.

A consequence of being in an entangled two-particle state is that a measurement of one particle's quantum state allows you to know the quantum state of the other particle. With the measurement of only one particle's state, the knowledge of the two quantum states is instantaneous. This knowledge is a consequence of the entanglement. Entanglement will next be described for spin–½ particles.

Consider two spin-½ particles prepared in an entangled two-particle state such that the total spin of the system is 0. If one particle is measured along one direction and projected into the (½) or up state, then the other particle is projected into the (-½) or down state along the same direction. Before the measurement, the two entangled particles can be separated by an arbitrary distance. Since the two particles are entangled, when the state of one particle is measured along a given direction, the state of the other particle in that same direction is instantaneously known. This occurs for any chosen distance between the two particles. Upon the measurement of one particle, that particle has a 50% chance of being in a spin up or down state. Due to the entanglement, the other particle must be in an opposite state. Therefore, if one particle is measured in a spin up state then the other particle must be in a spin down state and vice versa. This assigning of states follows from the conservation of angular momentum.

The instantaneous effect of entanglement can give rise to great confusion since it seems to defy Einstein's theory of relativity. This effect is the knowledge of the states of both particles in an entangled pair upon the measurement of one particle. One could conclude that the knowledge of the state of the distant particle is being sent from the site of the measured particle at a speed that is faster than the speed of light. However, after careful consideration, it is seen that entanglement does not violate causality. Consider two particles, say A and B, that are entangled. The state of one of the particles, let's say A, is measured by observer A. Observer B does not know that the state of particle A has been measured. This means that observer B does not have the knowledge that particle B has been projected into the opposite state of particle A. Without making a measurement, B will not know that its particle's state is in the opposite state of particle A until A sends that information classically to B. Once B knows that A has measured its state then B can check its own state and verify that it is in the opposite state of particle A. Since the information is sent via classical transmission from A to B, the entanglement does not defy the laws of relativity. Observer B does not initially know and can not know that particle B is in the opposite state of particle A until observer B makes measurements to confirm its state.

In order to understand how strange entanglement is, we can look at this in the classical picture. Let's take two items, have them interact classically, and then separate the two some distance apart. A measurement upon one item's state would tell you what the other item's state would be. However, each subsequent measurement would give you the same results. This means that if item one was found to be "up", then item two would be "down" and in each subsequent measurement, item one would always be "up" and item two would always be "down". However, in quantum mechanics, the situation is much different. Consider the spin-0, two-particle state for two spin-½ particles. A measurement of one particle's state can give either spin component up or spin component down. This process can be repeated many times for a number of identical particles prepared in the same initial state, and measuring the spin component along the same direction. Before making a measurement, it is only known that the particle being measured has a 50% chance of being "up" or "down". However, every time the one particle is found to be up, the other particle is always found to be down, and vice versa. There is no classical analogue for the phenomena.

Quantum Teleportation

When two spin-½ particles are entangled in a pure state, a measurement of one particle's spin state in one direction results in the knowledge of the other particle's state in the same direction. The same knowledge can result for three particles being entangled. Teleportation of a quantum state using a two-state system can be illustrated as follows. The two-state system to be considered here will involve the two spin states of a spin-½ particle. The basic scheme involves the use of three particles: A, B, and X. A and B are an entangled pair of indistinguishable particles that are prepared in a two-particle state $|\Psi_{AB}>$. Particle X is in the state $|\phi_X>$ that is to be teleported. Particles A and B can be both accessed by the sender, "Alice", and the receiver, "Bob". For convenience, it is said that Alice has access to A and Bob has access to B. Alice wants to send the state $|\phi_X>$ to Bob. Alice can use entanglement to send the state of X to Bob, i.e. to teleport the state $|\phi_X>$ to Bob. Alice will split the information involving $|\phi_X>$ into two parts, a classical part and a quantal part, and send these parts to Bob using two distinct methods. With the two different sets of information, Bob can reconstruct the state $|\phi_X\rangle$ at his location. The details of this process will be forthcoming shortly.

Before the teleportation begins, the system can be illustrated as follows, where a continuous line indicates an entanglement between the two particles: see Figure (1). Here, particles A and B are in a two-particle entangled state $|\Psi_{AB}\rangle$ and particle X is in an unknown state $|\phi_X\rangle$. It is very important to keep in mind that, initially, particle X is not entangled with particles A and B. The state $|\phi_X\rangle$ is prepared beforehand and is the state to be teleported.

It is important to note that the state $|\phi_X\rangle$ is unknown to both Alice and Bob. If the state $|\phi_X\rangle$ was known to Alice or Bob then it could be possible that this knowledge was used to change the state of particle B to be the same as the initial state of particle X. Thus true teleportation would not be achieved. As the knowledge of the initial state of particle X is unknown to both, then it cannot be said that Alice or Bob "accidentally" changed the state of particle B.

The state of the three particles is a pure product state between the unknown state $|\phi_X\rangle$ and the entangled pair. This product state can be expressed as $|\Psi_{XAB}\rangle = |\phi_X\rangle \otimes |\Psi_{AB}\rangle$. In this instance, no measurement upon the entangled pair can result in the knowledge of the state $|\phi_X\rangle$. This is because there is no classical information exchanged between the two particles or quantum entanglement between the two states, $|\phi_X>$ and $|\Psi_{AB}>$.

In order to teleport the initial state of $|\phi_X>$ from Alice to Bob, Alice needs to perform a measurement, referred to as a "Bell-operator measurement", on particles A and X. In a Bell-operator measurement, one starts with two particles that are initially not entangled with each other. The measurement projects the two particles into an entangled state. The entangled state is one of the following four states, referred to as the Bell-operator basis:

$$|\Psi^{(\pm)}> = \sqrt{½} \, (|\uparrow>|\downarrow> \pm |\downarrow>|\uparrow>)$$

$$|\Phi^{(\pm)}> = \sqrt{½} \, (|\uparrow>|\uparrow> \pm |\downarrow>|\downarrow>)$$

Here, the "+" sign is allowed since we are focusing on only the spin state, not the spin and spatial states. The Bell-operator basis is four orthogonal states using two two-component systems; e.g. two spin-½ particles. These entangled states are called the Bell states.

The Bell-operator measurement on particles A and X will transmit the quantal part to Bob first. The quantal part is the changing of the state of particle B into one of four states described below. From these four states, Bob can transform the initial state of X onto particle B. This will be further outlined later. When the measurement is performed on particles A and X, it entangles the pair and projects the now two-particle system into one of the four Bell states. The state created by the Bell-operator measurement can be expressed as $|\Phi_{AX}>$.

The Bell-operator measurement produces two results. One result is that Alice now knows which of the four Bell states $|\Phi_{AX}\rangle$ is in. These four states dictate what state particle B is now in. The entanglement causes the state of particle B to be changed since it was in the two-particle state $|\Psi_{AB}\rangle$ before the measurement. The other result is that the state of X is no longer $|\phi_X\rangle$. This change is due to the entanglement since X is now part of a two-particle state. Therefore, the initial state $|\phi_X\rangle$ no longer exists. Similarly, particle A is now part of a two-particle state $|\Phi_{AX}\rangle$. The two-particle state of A and B after the measurement is different from $|\Psi_{AB}\rangle$. Denote the state of particle B after measurement by $|\phi'_B\rangle$; the state of the three particles system is $|\Phi_{AX}\rangle \otimes |\phi'_B\rangle$: see Figure (2). The dashed line indicates that A and B were previously entangled but are no longer. This completes the transmission of the quantal part.

Next comes the transmission of the classical part. After Alice's measurement, particle B has been projected into one of four pure states. The new state of particle B is not necessarily the final state that is wanted, since the Bell-state measurement projects the state $|\Phi_{AX}\rangle$ into one of four Bell-states. These states dictate what state particle B will be projected into. Each of these four states has an equal probability of occurring. The four possible states for particle B to be projected into are (see page 12):

$$\phi'_B = \left\{ \begin{pmatrix} -a \\ -b \end{pmatrix}, \begin{pmatrix} -a \\ b \end{pmatrix}, \begin{pmatrix} b \\ a \end{pmatrix}, \begin{pmatrix} -b \\ a \end{pmatrix} \right\}$$

These states can be transformed into the initial state of particle X by the following transformations:

$$\phi_B = M\phi'_B \qquad \text{where } \phi_B \equiv \phi_X$$

and $$M = \left\{ -\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right\}$$

Note that in the first case, the case where Bob does not need to do a transformation, the state $|\phi'_B\rangle$ is already the same as $|\phi_X\rangle$ except for a minus sign. The transformation is an application of one of the unitary operators above, which correspond respectively to 180-degree rotations around the *z*-, *x*-, and *y*-axes. Since Alice performed the Bell-operator measurement on particles A and X, she knows which Bell state $|\Phi_{AX}\rangle$ is in. This information is sent classically to Bob: see Figure (3). Bob uses this information to decide which transformation to perform on particle B. Once this transformation is complete, the state of B is the initial state of X. Particles A and X are left in one of the Bell states and particles A and B are no longer entangled due to the transformation. This means that at Alice, there is no evidence of the initial state of X. Now both the classical and quantal elements of the unknown state have been transmitted. Therefore, the state $|\phi_X\rangle$ can be said to have been teleported from Alice to Bob. Now that the transformation has occurred, the state of the system can be expressed as the pure product state $|\Phi_{ABX}\rangle = |\Phi_{AX}\rangle \otimes |\phi_B\rangle$ since now, $|\phi_B\rangle = |\phi_X\rangle$. See Figure (4).

Since the transformation information can be sent at no faster than the speed of light then the laws of relativity are still upheld. Therefore, within a time $\Delta t \geq d_{AB}/c$, where d_{AB} is the distance between Alice and Bob, the state $|\phi_X\rangle$ has been teleported.

Note that throughout the teleportation the state $|\phi_X\rangle$ has not been measured. Therefore, when the particle B is in its new state $|\phi_X\rangle$, it is still an unknown state to both Alice and Bob.

Quantum Teleportation Calculation

Two particles, A and B, are prepared in the entangled state

$$|\Psi^{(-)}{}_{AB}\rangle = \sqrt{½}\,(|\uparrow_A\rangle|\downarrow_B\rangle - |\downarrow_A\rangle|\uparrow_B\rangle).$$

The initial state $|\phi_X\rangle$ is the unknown state that is to be teleported from Alice to Bob. This state can be expressed as

$$|\phi_X\rangle = a|\uparrow_X\rangle + b|\downarrow_X\rangle,$$

where $|a|^2 + |b|^2 = 1$. Before the state $|\phi_X\rangle$ is teleported, the three-particle system is in a pure product state where

$$|\Psi_{XAB}\rangle = |\,\phi_X\rangle \otimes |\,\Psi^{(-)}{}_{AB}\rangle$$

$$= (a/\sqrt{2})\,(|\uparrow_X\rangle|\uparrow_A\rangle|\downarrow_B\rangle - |\uparrow_X\rangle|\downarrow_A\rangle|\uparrow_B\rangle) + (b/\sqrt{2})(|\downarrow_X\rangle|\uparrow_A\rangle|\downarrow_B\rangle - |\downarrow_X\rangle|\downarrow_A\rangle|\uparrow_B\rangle).$$

The teleportation is achieved through the Bell-operator measurement of particles X and A. This measurement projects the two particles into one of four orthogonal Bell states:

$$|\Psi^{(\pm)}{}_{XA}\rangle = \sqrt{½}\,(|\uparrow_X\rangle|\downarrow_A\rangle \pm |\downarrow_X\rangle|\uparrow_A\rangle);$$

$$|\Phi^{(\pm)}{}_{XA}\rangle = \sqrt{½}\,(|\uparrow_X\rangle|\uparrow_A\rangle \pm |\downarrow_X\rangle|\downarrow_A\rangle).$$

Note that the form of $|\Psi^{(-)}{}_{XA}>$ is the same as the form of $|\Psi^{(-)}{}_{AB}>$. In addition, these four states are a complete orthonormal basis for particles X and A. Before Alice's measurement, the state $|\Psi_{XAB}>$ can be expressed using the Bell states since each direct product of X and A can be expressed in terms of the Bell operator basis vectors, $|\Psi^{(\pm)}{}_{XA}>$ and $|\Phi^{(\pm)}{}_{XA}>$:

$$|\Psi_{XAB}> = |\,\phi_X> \otimes |\,\Psi^{(-)}{}_{AB}>$$

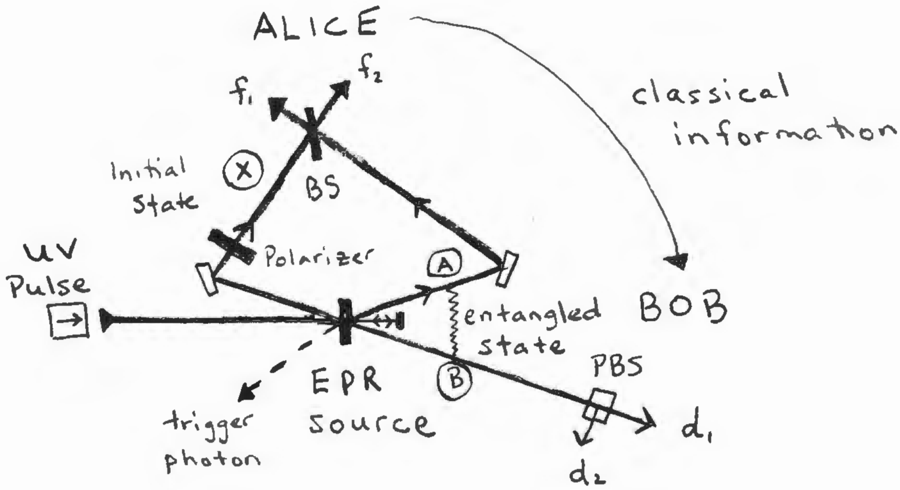
$$= (a/\sqrt{2})\,(|\uparrow_X>|\uparrow_A>|\downarrow_B> - |\uparrow_X>|\downarrow_A>|\uparrow_B>) +$$
$$(b/\sqrt{2})\,(|\downarrow_X>|\uparrow_A>|\downarrow_B> - |\downarrow_X>|\downarrow_A>|\uparrow_B>)$$

$$= \tfrac{1}{2}\,[\,|\Psi^{(-)}{}_{XA}>(-a|\uparrow_B> - b|\downarrow_B>) + |\Psi^{(+)}{}_{XA}>(-a|\uparrow_B> + b|\downarrow_B>) +$$
$$|\Phi^{(-)}{}_{XA}>(a|\downarrow_B> + b|\uparrow_B>) + |\Phi^{(+)}{}_{XA}>(a|\downarrow_B> - b|\uparrow_B>)\,]$$

Alice then performs the Bell-operator measurement; entangling particles A and X and projecting them into one of the four Bell states. Particle B is projected into one of the four pure states corresponding to the Bell state that A and X are in. Since particles A and X have an equal probability of being projected into any one of the four Bell states then the probability that the new state $|\phi'_B>$ is the same as the initial state $|\phi_X>$ to be teleported is 25%. With each of the other Bell states, a transformation must be done in order that $|\phi'_B> = |\phi_X>$. (see pages 9, 10) After this transformation, the state of B is the initial state of X and particles A and X are left in one of the states $|\Psi^{(\pm)}{}_{XA}>$ or $|\Phi^{(\pm)}{}_{XA}>$. Thus, the state of X is said to have been teleported to B.

Experimental Quantum Teleportation

As an example of experimental quantum teleportation, we can look at the experiment from the University of Innsbruck [3]. This paper was one of the first experimental verifications for quantum teleportation. Here the polarization state of a photon is teleported from one photon to another.



Here, photons A and B produced as an entangled pair by sending a pulse of UV-light through a nonlinear crystal. The UV-pulse then gets reflected back through the crystal. This creates another pair of entangled photons. One of these photons will be the photon X, the photon whose state will be teleported. Photon X can be prepared into any polarization that is chosen. The photon that is entangled with photon X is used as a trigger to know that photon X has been created and sent.

Alice then looks for coincidences behind a beam splitter (BS). This is where photon X and photon A are entangled by the Bell-operator measurement. Once Alice finds a coincidence at f1 and f2, which indicates that she has photons A and X in the $|\Psi^{(-)}{}_{XA}>$ state, Alice sends the classical information to Bob that she has this state. Bob then knows that the photon B is now in the initial state that photon X was in. Bob can check this using polarization analysis with the polarizing beam splitter (PBS) and the detectors, d1 and d2.

Findings

Although experimental teleportation has been achieved, several criteria must be realized during the experiment in order to have "true" teleportation. One is that the initial state $|\phi_X>$ to be teleported must be unknown to both Alice and Bob. Another is that the entanglement must be verifiable. This allows that the teleportation has actually been achieved through the entanglement and having that information sent through the classical channel, and not through an accidental measurement of the state $|\phi_X>$.

However, one paper [4] suggests that the current methods of teleportation cannot achieve 100% probability of success in the teleportation of an unknown state of an external quantum system. In [4] it is shown that by not allowing interaction between quantum particles, the complete nondegenerate Bell-operator measurements cannot be performed. Without interactions between the quantum particles, a Bell-operator measurement can only be a degenerate measurement. A complete nondegenerate Bell-operator measurement allows for each of the four Bell states to be distinguished from the other: $|\Psi^{(-)}>$, $|\Psi^{(+)}>$, $|\Phi^{(-)}>$, and $|\Phi^{(+)}>$. If each Bell state is not distinguishable, e.g. only $|\Psi>$ and $|\Phi>$ can be distinguished from each other, then the Bell-operator measurement is said to be degenerate. In [4] it is also claimed that interaction between two quantum particles resulting in conditional state changes allows for a complete nondegenerate Bell-operator measurement.

An example of an interaction between quantum particles would be teleporting an arbitrary electronic state using trapped ions [5]. There would be two traps, trap A (Alice) and trap B (Bob). Ions A and B would initially be in trap B and ion X with the state to be teleported would be in trap A. Ion A would then be transferred to trap A where it would interact with ion X. This would result in a complete Bell-operator measurement.

Another example of an interaction would be using photons, which have nonlinear interactions using optical Sum Frequency Generation (SFG) [6]. Here, four SFG nonlinear crystals are used in the measurement of the Bell-operator measurement and to distinguish the four Bell states. There are two SFG crystals of each type, type-I and type-II. How the Bell states are made is by how the photons are sent into each crystal. Photons A and X can interact either in the type-I crystals or in the type-II crystals. This generates another higher frequency photon whose projection measurements correspond to the four Bell states for photons A and X. Interactions within the type-I crystals correspond to the Bell states for $|\Phi^{(\pm)}>$ and interactions within the type-II crystals correspond to the Bell states for $|\Psi^{(\pm)}>$.

Since the Bell-operator measurement is a significant factor in the process of quantum teleportation then many methods of teleportation do not achieve reliable teleportation. Therefore, according to the authors of [4] complete quantum teleportation can only be achieved by allowing interaction between quantum particles.

It will be shown in this project that, for distinguishable particles, bosons, and fermions, if there is no interaction between the quantum particles involved in the process, true quantum teleportation cannot be achieved. Methods similar to those in [4] are used for distinguishable particles. For bosons, it is shown using the polarization state of photons and using a method similar to what is used in [4]. A different proof from [4] is used for the spin-½ fermions. Thus, interaction between the quantum particles is shown in this project to be a necessary condition for true teleportation, although it may or may not be a sufficient condition.

Bell Operator Measurement Without Interaction Between Quantum Systems

In this section, using the results from [4], it will shown that it is impossible to perform a complete nondegenerate Bell-operator measurement without using interaction between quantum systems. Since the Bell-operator measurement is only performed on two particles, e.g. particles X and A, we will only be focusing on two particles during the proofs. The four distinct orthogonal single-particle Bell states, resulting from the Bell-operator measurement, are defined by the experimental apparatus used in the teleportation scheme. The apparatus uses two channels, left (L) and right (R), and two particles enter into these two channels. The particles have a two-component system which is dependent upon what is being measured; in our case for a spin- ½ particle, spin-up ($\uparrow$) and spin-down ($\downarrow$) are the two states. The Bell-operator measurement can be separated into two stages: the unitary linear evolution and the local detection. The unitary linear evolution is the behaviour defined by equations that describe the transformation that maps the state of a system at some initial time into some later time. We can write the general form for the unitary linear evolution for the four single-particle states as

$$
\begin{aligned}
|\uparrow_L\rangle &= \Sigma_i\, a_i\, |i\rangle \\
|\downarrow_L\rangle &= \Sigma_i\, b_i\, |i\rangle \\
|\uparrow_R\rangle &= \Sigma_i\, c_i\, |i\rangle \\
|\downarrow_R\rangle &= \Sigma_i\, d_i\, |i\rangle
\end{aligned}
\tag{1}
$$

Here the $|\, i >$ represent the orthogonal single-particle local states of each of the particles going through the teleportation apparatus. The linearity implies that the evolution of a single particle in one channel is independent of the state of a particle in another channel.

Distinguishable Particles

We can now define the four eigenstates of the Bell operator for distinguishable particles. Using the four distinct single-particle states from the teleportation apparatus the Bell states are:

$$|\Psi^{(-)}\rangle = 1/\sqrt{2}\,(|\uparrow_L\rangle \otimes |\downarrow_R\rangle - |\downarrow_L\rangle \otimes |\uparrow_R\rangle)$$

$$|\Psi^{(+)}\rangle = 1/\sqrt{2}\,(|\uparrow_L\rangle \otimes |\downarrow_R\rangle + |\downarrow_L\rangle \otimes |\uparrow_R\rangle) \qquad (2)$$

$$|\Phi^{(-)}\rangle = 1/\sqrt{2}\,(|\uparrow_L\rangle \otimes |\uparrow_R\rangle - |\downarrow_L\rangle \otimes |\downarrow_R\rangle)$$

$$|\Phi^{(+)}\rangle = 1/\sqrt{2}\,(|\uparrow_L\rangle \otimes |\uparrow_R\rangle + |\downarrow_L\rangle \otimes |\downarrow_R\rangle)$$

Since the particles are distinguishable, we can identify the particles as particle 1 and particle 2. During the linear evolution each particle can go through either of the two channels, where one particle goes through the left channel and the other particle goes through the right. This gives us the following eigenstates for the Bell operator:

$$|\Psi^{(-)}\rangle = \tfrac{1}{2}\{(|\uparrow_L\rangle_1 \otimes |\downarrow_R\rangle_2 + |\downarrow_R\rangle_1 \otimes |\uparrow_L\rangle_2) - (|\downarrow_L\rangle_1 \otimes |\uparrow_R\rangle_2 + |\uparrow_R\rangle_1 \otimes |\downarrow_L\rangle_2)\}$$

$$|\Psi^{(+)}\rangle = \tfrac{1}{2}\{(|\uparrow_L\rangle_1 \otimes |\downarrow_R\rangle_2 + |\downarrow_R\rangle_1 \otimes |\uparrow_L\rangle_2) + (|\downarrow_L\rangle_1 \otimes |\uparrow_R\rangle_2 + |\uparrow_R\rangle_1 \otimes |\downarrow_L\rangle_2)\} \qquad (3)$$

$$|\Phi^{(-)}\rangle = \tfrac{1}{2}\{(|\uparrow_L\rangle_1 \otimes |\uparrow_R\rangle_2 + |\uparrow_R\rangle_1 \otimes |\uparrow_L\rangle_2) - (|\downarrow_L\rangle_1 \otimes |\downarrow_R\rangle_2 + |\downarrow_R\rangle_1 \otimes |\downarrow_L\rangle_2)\}$$

$$| \Phi^{(+)} > = \tfrac{1}{2} \{ (| \uparrow_L >_1 \otimes | \uparrow_R >_2 + | \uparrow_R >_1 \otimes | \uparrow_L >_2) \qquad (3)$$

$$+ (| \downarrow_L >_1 \otimes | \downarrow_R >_2 + | \downarrow_R >_1 \otimes | \downarrow_L >_2) \}$$

Since the evolution of the states is linear we can rewrite the Bell states using the general form for the unitary linear evolution of the teleportation procedure (1). The Bell state general form is:

$$\begin{aligned} | \Psi^{(-)} > &= \Sigma_{i,j}\, \alpha_{ij} \, | \, i > \otimes \, | \, j > \\ | \Psi^{(+)} > &= \Sigma_{i,j}\, \beta_{ij} \, | \, i > \otimes \, | \, j > \\ | \Phi^{(-)} > &= \Sigma_{i,j}\, \gamma_{ij} \, | \, i > \otimes \, | \, j > \\ | \Phi^{(+)} > &= \Sigma_{i,j}\, \delta_{ij} \, | \, i > \otimes \, | \, j > \end{aligned} \qquad (4)$$

We want to prove that that it is impossible to have measurability of the nondegenerate Bell operator without using interaction between quantum systems; thus assume that there is no interaction and prove by contradiction. Since there is no interaction, we require only local detectors; consequently, we can examine cases where pure product states $| \, i > \otimes \, | \, j >$ are detected. This means that for a certain pair $\{i', j'\}$ we can have

$$\alpha_{ij} \neq 0 \qquad \text{for } i = i' \text{ and } j = j'$$

$$\alpha_{ij} = 0 \qquad \text{for } i \neq i' \text{ and/or } j \neq j'.$$

First we will prove for $i \neq j$ where i corresponds to one particle and j corresponds to the other particle. In order to perform a measurement of the nondegenerate Bell operator, for any selected pair $\{i, j\}$ at least three out of the four coefficients α_{ij}, β_{ij}, γ_{ij}, and δ_{ij} are equal to zero. This would imply a detection of one of the four states $|\Psi^{(\pm)}>$ and $|\Phi^{(\pm)}>$. This means that if the Bell-operator measurement results in the state $|\Psi^{(-)}>$ then we would have that $\alpha_{ij} \neq 0$ and that $\beta_{ij} = \gamma_{ij} = \delta_{ij} = 0$. We can find the coefficients α_{ij}, β_{ij}, γ_{ij}, and δ_{ij} by inserting (1) into (3) to get the following:

$$
\begin{aligned}
|\Psi^{(-)}> &= \tfrac{1}{2}\ \{ (\Sigma_i\, a_i\, |\, i > \otimes \Sigma_j\, d_j\, |\, j > + \Sigma_i\, d_i\, |\, i > \otimes \Sigma_j\, a_j\, |\, j >) \\
&\qquad - (\Sigma_i\, b_i\, |\, i > \otimes \Sigma_j\, c_j\, |\, j > + \Sigma_i\, c_i\, |\, i > \otimes \Sigma_j\, b_j\, |\, j >) \} \\
&= \tfrac{1}{2}\ \Sigma_{i,j}\ \{ (a_i d_j + a_j d_i) - (b_i c_j + b_j c_i) \}\ |\, i > \otimes\, |\, j > \\
&= \Sigma_{i,j}\ \alpha_{ij}\ |\, i > \otimes\, |\, j >
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
|\Psi^{(+)}> &= \tfrac{1}{2}\ \{ (\Sigma_i\, a_i\, |\, i > \otimes \Sigma_j\, d_j\, |\, j > + \Sigma_i\, d_i\, |\, i > \otimes \Sigma_j\, a_j\, |\, j >) \\
&\qquad + (\Sigma_i\, b_i\, |\, i > \otimes \Sigma_j\, c_j\, |\, j > + \Sigma_i\, c_i\, |\, i > \otimes \Sigma_j\, b_j\, |\, j >) \} \\
&= \tfrac{1}{2}\ \Sigma_{i,j}\ \{ (a_i d_j + a_j d_i) + (b_i c_j + b_j c_i) \}\ |\, i > \otimes\, |\, j > \\
&= \Sigma_{i,j}\ \beta_{ij}\ |\, i > \otimes\, |\, j >
\end{aligned}
$$

$$
\begin{aligned}
|\Phi^{(-)}> &= \tfrac{1}{2}\ \{ (\Sigma_i\, a_i\, |\, i > \otimes \Sigma_j\, c_j\, |\, j > + \Sigma_i\, c_i\, |\, i > \otimes \Sigma_j\, a_j\, |\, j >) \\
&\qquad - (\Sigma_i\, b_i\, |\, i > \otimes \Sigma_j\, d_j\, |\, j > + \Sigma_i\, d_i\, |\, i > \otimes \Sigma_j\, b_j\, |\, j >) \} \\
&= \tfrac{1}{2}\ \Sigma_{i,j}\ \{ (a_i c_j + a_j c_i) - (b_i d_j + b_j d_i) \}\ |\, i > \otimes\, |\, j > \\
&= \Sigma_{i,j}\ \gamma_{ij}\ |\, i > \otimes\, |\, j >
\end{aligned}
$$

$$| \Phi^{+)} > = ½ \ \{ (\Sigma_i \, a_i \, | \, i > \otimes \, \Sigma_j \, c_j \, | \, j > + \, \Sigma_i \, c_i \, | \, i > \otimes \, \Sigma_j \, a_j \, | \, j >)$$

$$+ (\Sigma_i \, b_i \, | \, i > \otimes \, \Sigma_j \, d_j \, | \, j > + \, \Sigma_i \, d_i \, | \, i > \otimes \, \Sigma_j \, b_j | \, j >) \, \} \qquad (5)$$

$$= ½ \ \Sigma_{i,j} \, \{ \, (a_i c_j + a_j c_i) + (b_i d_j + b_j d_i) \, \} \, | \, i > \otimes \, | \, j >$$

$$= \Sigma_{i,j} \, \delta_{ij} \, | \, i > \otimes \, | \, j >.$$

This gives us the following equations for the coefficients:

$$2\alpha_{ij} = (a_i d_j + a_j d_i) - (b_i c_j + b_j c_i)$$

$$2\beta_{ij} = (a_i d_j + a_j d_i) + (b_i c_j + b_j c_i) \qquad (6)$$

$$2\gamma_{ij} = (a_i c_j + a_j c_i) - (b_i d_j + b_j d_i)$$

$$2\delta_{ij} = (a_i c_j + a_j c_i) + (b_i d_j + b_j d_i).$$

We have that the two requirements for the measurability of the nondegenerate Bell operator are that there is at least one nonzero coefficient of the kind α_{ij}, β_{ij}, γ_{ij}, and δ_{ij} and that if for a certain pair *i, j* it is not zero, then all others are zero. Therefore, if for a certain pair $\{i, j\}$ we choose $\alpha_{ij} \neq 0$ then $\beta_{ij} = \gamma_{ij} = \delta_{ij} = 0$. So, in order to prove that it is impossible to perform complete nondegenerate Bell operator measurement without using interaction between quantum systems then all we have to show is that either

(i) $\alpha_{ij} = \beta_{ij} = \gamma_{ij} = \delta_{ij} = 0$ for a certain *i, j*

or

(ii) the set of equations for α_{ij}, β_{ij}, γ_{ij}, and δ_{ij} has no solution.

Since the particles are distinguishable, this places strong restrictions on the unitary evolution (1). These restrictions apply because the particles cannot change their identity since they are distinguishable. Therefore, if a particle is in the left channel then it is known that it is not in the right channel. From (1) we get that the restrictions are:

$$a_i \neq 0 \Rightarrow c_i = d_i = 0$$
$$b_i \neq 0 \Rightarrow c_i = d_i = 0 \qquad (7)$$
$$c_i \neq 0 \Rightarrow a_i = b_i = 0$$
$$d_i \neq 0 \Rightarrow a_i = b_i = 0.$$

Note that $c_i = 0$ does not mean that $c_j = 0$ since *i* corresponds to particle 1 and *j* corresponds to particle 2. To prove the impossibility of performing a complete nondegenerate Bell operator measurement, we will assume that $a_i \neq 0$ and $\alpha_{ij} \neq 0$. Using the restrictions from (7) and the requirement for measurability of the Bell operator we get from (6):

$$2\alpha_{ij} = a_i d_j - b_i c_j \neq 0$$
$$2\beta_{ij} = a_i d_j + b_i c_j = 0 \qquad (8)$$
$$2\gamma_{ij} = a_i c_j - b_i d_j = 0$$
$$2\delta_{ij} = a_i c_j + b_i d_j = 0.$$

We can see that this set of equations has no solution. We can show this as follows:

$$2\gamma_{ij} = a_i c_j - b_i d_j = 0$$

or

$$a_i c_j = b_i d_j.$$

Putting the previous equation into the following:

$$2\delta_{ij} = a_i c_j + b_i d_j = 0$$

we get both

$$a_i c_j = 0$$

and

$$b_i d_j = 0.$$

Since $a_i \neq 0$, then c_j must be equal to 0 and either

$$\text{i.} \quad b_i = 0$$

or

$$\text{ii.} \quad d_j = 0.$$

With (i) $b_i = 0$ and given that

$$2\beta_{ij} = a_i d_j + b_i c_j = 0$$

then we find that $d_j = 0$ but from

$$2\alpha_{ij} = a_i d_j - b_i c_j \neq 0$$

we find that this is a contradiction since then $\alpha_{ij} = 0$ and would therefore give no solution.

With (ii) $d_j = 0$ we have that

$$a_i d_j - b_i c_j = 0$$

since $d_j = 0$ and $c_j = 0$. However, this also gives the result of $\alpha_{ij} = 0$ and so once again gives a contradiction. Therefore, the set of equations (8) has no solution. This is enough to prove that it is impossible to measure the Bell operator without using interaction between quantum systems for $i \neq j$.

Now to prove for $i = j$. Here equation (4) becomes:

$$| \Psi^{(-)} > = \Sigma_i \alpha_{ii} | i > \otimes | i >$$

$$| \Psi^{(+)} > = \Sigma_i \beta_{ii} | i > \otimes | i > \qquad (9)$$

$$| \Phi^{(-)} > = \Sigma_i \gamma_{ii} | i > \otimes | i >$$

$$| \Phi^{(+)} > = \Sigma_i \delta_{ii} | i > \otimes | i >$$

The decomposition of the Bell states are now:

$$| \Psi^{(-)} > = ½ \ \{ (\Sigma_i a_{i1} | i >_1 \otimes d_{i2} | i >_2 + \Sigma_i d_{i1} | i >_1 \otimes a_{i2} | i >_2)$$

$$- (\Sigma_i b_{i1} | i >_1 \otimes c_{i2} | i >_2 + \Sigma_i c_{i1} | i >_1 \otimes b_{i2} | i >_2) \}$$

$$= \Sigma_i \alpha_{ii} | i >_1 \otimes | i >_2$$

$$| \Psi^{(+)} > = ½ \ \{ (\Sigma_i a_{i1} | i >_1 \otimes d_{i2} | i >_2 + \Sigma_i d_{i1} | i >_1 \otimes a_{i2} | i >_2)$$

$$+ (\Sigma_i b_{i1} | i >_1 \otimes c_{i2} | i >_2 + \Sigma_i c_{i1} | i >_1 \otimes b_{i2} | i >_2) \}$$

$$= \Sigma_i \beta_{ii} | i >_1 \otimes | i >_2 \qquad (10)$$

$$| \Phi^{(-)} > = ½ \ \{ (\Sigma_i a_{i1} | i >_1 \otimes c_{i2} | i >_2 + \Sigma_i c_{i1} | i >_1 \otimes a_{i2} | i >_2)$$

$$- (\Sigma_i b_{i1} | i >_1 \otimes d_{i2} | i >_2 + \Sigma_i d_{i1} | i >_1 \otimes b_{i2} | i >_2) \}$$

$$= \Sigma_i \gamma_{ii} | i >_1 \otimes | i >_2$$

$$| \Phi^{(+)} > = ½ \ \{ (\Sigma_i a_{i1} | i >_1 \otimes c_{i2} | i >_2 + \Sigma_i c_{i1} | i >_1 \otimes a_{i2} | i >_2)$$

$$+ (\Sigma_i b_{i1} | i >_1 \otimes d_{i2} | i >_2 + \Sigma_i d_{i1} | i >_1 \otimes b_{i2} | i >_2) \}$$

$$= \Sigma_i \delta_{ii} | i >_1 \otimes | i >_2$$

Here, we denote the particles by particle label "1" and particle label "2" since they can no longer be distinguished by *i* and *j*. This gives us the following equations for the coefficients:

$$
\begin{aligned}
2\alpha_{ii} &= (a_{i1}d_{i2} + d_{i1}a_{i2}) - (b_{i1}c_{i2} + c_{i1}b_{i2}) \\
2\beta_{ii} &= (a_{i1}d_{i2} + d_{i1}a_{i2}) + (b_{i1}c_{i2} + c_{i1}b_{i2}) \\
2\gamma_{ii} &= (a_{i1}c_{i2} + c_{i1}a_{i2}) - (b_{i1}d_{i2} + d_{i1}b_{i2}) \\
2\delta_{ii} &= (a_{i1}c_{i2} + c_{i1}a_{i2}) + (b_{i1}d_{i2} + d_{i1}b_{i2})
\end{aligned}
\tag{11}
$$

Note that the form of these equations is identical to that of the equations for the coefficients above. Therefore, using the proof from above, it is shown that it is impossible to perform complete nondegenerate Bell operator measurement without quantum interactions for $i = j$. Thus, it has been proved for distinguishable particles that there cannot be a complete nondegenerate Bell-operator measurement without interaction between the quantum particles.

Bosons

Having proved that it is impossible to perform a complete nondegenerate Bell operator measurement without using interaction between quantum systems for distinguishable particles, we next do the proof for indistinguishable particles. First, we will show this for bosons. We will look at the case of photons, using the horizontal and vertical polarization states.

The Bell states are given by:

$$
\begin{aligned}
|\Psi^{(-)}\rangle &= 1/\sqrt{2}\,(\,|\updownarrow_L\rangle \otimes |\leftrightarrow_R\rangle - |\leftrightarrow_L\rangle \otimes |\updownarrow_R\rangle\,) \\
|\Psi^{(+)}\rangle &= 1/\sqrt{2}\,(\,|\updownarrow_L\rangle \otimes |\leftrightarrow_R\rangle + |\leftrightarrow_L\rangle \otimes |\updownarrow_R\rangle\,) \qquad (12) \\
|\Phi^{(-)}\rangle &= 1/\sqrt{2}\,(\,|\updownarrow_L\rangle \otimes |\updownarrow_R\rangle - |\leftrightarrow_L\rangle \otimes |\leftrightarrow_R\rangle\,) \\
|\Phi^{(+)}\rangle &= 1/\sqrt{2}\,(\,|\updownarrow_L\rangle \otimes |\updownarrow_R\rangle + |\leftrightarrow_L\rangle \otimes |\leftrightarrow_R\rangle\,).
\end{aligned}
$$

Since we are dealing with bosons, which are identical particles, we have to symmetrize so we must have:

$$
|\updownarrow_L\rangle \otimes |\leftrightarrow_R\rangle \to (1/\sqrt{2})\,(|\updownarrow_L\rangle_1 \otimes |\leftrightarrow_R\rangle_2 + |\leftrightarrow_R\rangle_1 \otimes |\updownarrow_L\rangle_2\,). \qquad (13)
$$

Substituting this form into (12) we get:

$$
\begin{aligned}
|\Psi^{(-)}\rangle = 1/2\,\{\,&(\,|\updownarrow_L\rangle_1 \otimes |\leftrightarrow_R\rangle_2 + |\leftrightarrow_R\rangle_1 \otimes |\updownarrow_L\rangle_2\,) \\
&- (\,|\leftrightarrow_L\rangle_1 \otimes |\updownarrow_R\rangle_2 + |\updownarrow_R\rangle_1 \otimes |\leftrightarrow_L\rangle_2)\,\}
\end{aligned}
$$

$$
\begin{aligned}
|\Psi^{(+)}\rangle = 1/2\,\{\,&(\,|\updownarrow_L\rangle_1 \otimes |\leftrightarrow_R\rangle_2 + |\leftrightarrow_R\rangle_1 \otimes |\updownarrow_L\rangle_2\,) \\
&+ (\,|\leftrightarrow_L\rangle_1 \otimes |\updownarrow_R\rangle_2 + |\updownarrow_R\rangle_1 \otimes |\leftrightarrow_L\rangle_2)\,\}
\end{aligned}
$$

$$
\begin{aligned}
|\Phi^{(-)}\rangle = 1/2\,\{\,&(\,|\updownarrow_L\rangle_1 \otimes |\updownarrow_R\rangle_2 + |\updownarrow_R\rangle_1 \otimes |\updownarrow_L\rangle_2\,) \qquad (14) \\
&- (\,|\leftrightarrow_L\rangle_1 \otimes |\leftrightarrow_R\rangle_2 + |\leftrightarrow_R\rangle_1 \otimes |\leftrightarrow_L\rangle_2)\,\}
\end{aligned}
$$

$$
\begin{aligned}
|\Phi^{(+)}\rangle = 1/2\,\{\,&(\,|\updownarrow_L\rangle_1 \otimes |\updownarrow_R\rangle_2 + |\updownarrow_R\rangle_1 \otimes |\updownarrow_L\rangle_2\,) \\
&+ (\,|\leftrightarrow_L\rangle_1 \otimes |\leftrightarrow_R\rangle_2 + |\leftrightarrow_R\rangle_1 \otimes |\leftrightarrow_L\rangle_2)\,\}.
\end{aligned}
$$

We can again use the following equations:

$$| \updownarrow_L > = \Sigma_i \, a_i \, | \, i >$$

$$| \leftrightarrow_L > = \Sigma_i \, b_i \, | \, i > \qquad (15)$$

$$| \updownarrow_R > = \Sigma_i \, c_i \, | \, i >$$

$$| \leftrightarrow_R > = \Sigma_i \, d_i \, | \, i >$$

to find a general form for the eigenstates of the Bell operators.

From the treatment given above for distinguishable particles, we see that this can be written as:

$$| \Psi^{(-)} > = \Sigma_{i,j} \, \alpha_{ij} \, | \, i > \otimes \, | \, j >$$

$$| \Psi^{(+)} > = \Sigma_{i,j} \, \beta_{ij} \, | \, i > \otimes \, | \, j > \qquad (16)$$

$$| \Phi^{(-)} > = \Sigma_{i,j} \, \gamma_{ij} \, | \, i > \otimes \, | \, j >$$

$$| \Phi^{(+)} > = \Sigma_{i,j} \, \delta_{ij} \, | \, i > \otimes \, | \, j >.$$

We have, putting (15) into (14) and letting i correspond to particle label "1" and j correspond to particle label "2":

$$| \Psi^{(-)} > = 1/2 \, \{ \, (\, \Sigma_i \, a_i \, | \, i > \otimes \, \Sigma_j \, d_j \, | \, j > + \, \Sigma_i \, d_i \, | \, i > \otimes \, \Sigma_j \, a_j \, | \, j >)$$

$$- \, (\, \Sigma_i \, b_i \, | \, i > \otimes \, \Sigma_j \, c_j \, | \, j > + \, \Sigma_i \, c_i \, | \, i > \otimes \, \Sigma_j \, b_j \, | \, j >) \, \} \qquad (17)$$

$$= 1/2 \, \Sigma_{i,j} \, \{ \, (a_i d_j + a_j d_i) - (b_i c_j + b_j c_i) \, \} \, | \, i > \otimes \, | \, j >$$

$$= \Sigma_{i,j} \, \alpha_{ij} \, | \, i > \otimes \, | \, j >$$

$$| \Psi^{(+)} > = 1/2 \{ (\Sigma_i a_i | i > \otimes \Sigma_j d_j | j > + \Sigma_i d_i | i > \otimes \Sigma_j a_j | j >)$$

$$+ (\Sigma_i b_i | i > \otimes \Sigma_j c_j | j > + \Sigma_i c_i | i > \otimes \Sigma_j b_j | j >) \}$$

$$= 1/2 \Sigma_{i,j} \{ (a_i d_j + a_j d_i) + (b_i c_j + b_j c_i) \} | i > \otimes | j >$$

$$= \Sigma_{i,j} \beta_{ij} | i > \otimes | j >$$

$$| \Phi^{(-)} > = 1/2 \{ (\Sigma_i a_i | i > \otimes \Sigma_j c_j | j > + \Sigma_i c_i | i > \otimes \Sigma_j a_j | j >) \qquad (17)$$

$$- (\Sigma_i b_i | i > \otimes \Sigma_j d_j | j > + \Sigma_i d_i | i > \otimes \Sigma_j b_j | j >) \}$$

$$= 1/2 \Sigma_{i,j} \{ (a_i c_j + a_j c_i) - (b_i d_j + b_j d_i) \} | i > \otimes | j >$$

$$= \Sigma_{i,j} \gamma_{ij} | i > \otimes | j >$$

$$| \Phi^{(+)} > = 1/2 \{ (\Sigma_i a_i | i > \otimes \Sigma_j c_j | j > + \Sigma_i c_i | i > \otimes \Sigma_j a_j | j >)$$

$$+ (\Sigma_i b_i | i > \otimes \Sigma_j d_j | j > + \Sigma_i d_i | i > \otimes \Sigma_j b_j | j >) \}$$

$$= 1/2 \Sigma_{i,j} \{ (a_i c_j + a_j c_i) + (b_i d_j + b_j d_i) \} | i > \otimes | j >$$

$$= \Sigma_{i,j} \delta_{ij} | i > \otimes | j >.$$

We can see that these expressions have the same form as those for the distinguishable particles, except for the factor of 2. This is also true for $i = j$.

$$| \Psi^{(-)} > = 1/2 \{ (\Sigma_i a_i | i > \otimes d_i | i > + \Sigma_i d_i | i > \otimes a_i | i >)$$

$$- (\Sigma_i b_i | i > \otimes c_i | i > + \Sigma_i c_i | i > \otimes b_i | i >) \} \qquad (18)$$

$$= 1/2 \Sigma_i \{ (a_i d_i + a_i d_i) - (b_i c_i + b_i c_i) \} | i > \otimes | i >$$

$$= \Sigma_i \alpha_{ii} | i > \otimes | i >$$

$$| \Psi^{(+)} > = 1/2 \{ (\Sigma_i a_i | i > \otimes d_i | i > + \Sigma_i d_i | i > \otimes a_i | i >)$$

$$+ (\Sigma_i b_i | i > \otimes c_i | i > + \Sigma_i c_i | i > \otimes b_i | i >) \}$$

$$= 1/2 \Sigma_i \{ (a_i d_i + a_i d_i) + (b_i c_i + b_i c_i) \} | i > \otimes | i >$$

$$= \Sigma_i \beta_{ii} | i > \otimes | i >$$

(18)

$$| \Phi^{(-)} > = 1/2 \{ (\Sigma_i a_i | i > \otimes c_i | i > + \Sigma_i c_i | i > \otimes a_i | i >)$$

$$- (\Sigma_i b_i | i > \otimes d_i | i > + \Sigma_i d_i | i > \otimes b_i | i >) \}$$

$$= 1/2 \Sigma_i \{ (a_i c_i + a_i c_i) - (b_i d_i + b_i d_i) \} | i > \otimes | i >$$

$$= \Sigma_i \gamma_{ii} | i > \otimes | i >$$

$$| \Phi^{(+)} > = 1/2 \{ (\Sigma_i a_i | i > \otimes c_i | i > + \Sigma_i c_i | i > \otimes a_i | i >)$$

$$+ (\Sigma_i b_i | i > \otimes d_i | i > + \Sigma_i d_i | i > \otimes b_i | i >) \}$$

$$= 1/2 \Sigma_i \{ (a_i c_i + a_i c_i) + (b_i d_i + b_i d_i) \} | i > \otimes | i >$$

$$= \Sigma_i \delta_{ii} | i > \otimes | i >.$$

From (17) and (18), we get the following equations:

$$2\alpha_{ij} = (a_i d_j + a_j d_i) - (b_i c_j + b_j c_i)$$

$$2\beta_{ij} = (a_i d_j + a_j d_i) + (b_i c_j + b_j c_i) \qquad (19)$$

$$2\gamma_{ij} = (a_i c_j + a_j c_i) - (b_i d_j + b_j d_i)$$

$$2\delta_{ij} = (a_i c_j + a_j c_i) + (b_i d_j + b_j d_i)$$

and

$$\alpha_{ii} = a_i d_i - b_i c_i$$
$$\beta_{ii} = a_i d_i + b_i c_i \qquad (20)$$
$$\gamma_{ii} = a_i c_i - b_i d_i$$
$$\delta_{ii} = a_i c_i + b_i d_i.$$

We will now show that it is impossible to perform complete nondegenerate measurements without quantum interaction for photons where $i = j$. Since the particles entering the two channels are identical, there are no similar restrictions as there are for distinguishable particles. Using the requirement for the measurability of the nondegenerate Bell operator, that for any given i, at least three out of the four coefficients α_{ii}, β_{ii}, γ_{ii}, and δ_{ii} are equal to zero. Looking at the equations in (20) and assuming $\alpha_{ii} \neq 0$ we have the following:

$$\alpha_{ii} = a_i d_i - b_i c_i \neq 0 \qquad (20i)$$
$$\beta_{ii} = a_i d_i + b_i c_i = 0 \qquad (20ii)$$
$$\gamma_{ii} = a_i c_i - b_i d_i = 0 \qquad (20iii)$$
$$\delta_{ii} = a_i c_i + b_i d_i = 0 \qquad (20iv)$$

From (20iii) we get that

$$a_i c_i = b_i d_i$$

Putting this into (20iv) we get

$$a_i c_i = 0 \qquad \text{and} \qquad b_i d_i = 0$$

Thus, we get that $\gamma_{ii} = 0$ and $\delta_{ii} = 0$. This means that either a_i or c_i is equal to 0, and that either b_i or d_i is equal to 0. If $a_i = b_i = 0$ or $d_i = c_i = 0$ then $\alpha_{ii} = 0$ and this contradicts our assumption. If $a_i = d_i = 0$, then from (20ii) we have that $b_ic_i = 0$ and $\alpha_{ii} = 0$, which contradicts our assumption. If $b_i = c_i = 0$ then (20ii) gives $a_id_i = 0$, and again we have that $\alpha_{ii} = 0$. Finally, $c_i = d_i = 0$ gives $\alpha_{ii} = 0$, a contradiction once again. Thus, we find that

$$\alpha_{ii} = \beta_{ii} = \gamma_{ii} = \delta_{ii} = 0$$

Thus, the requirement for measurability has been proven to be impossible for $i = j$ for identical bosons.

Now we can prove the impossibility of the Bell-operator measurement without quantum interactions for $i \neq j$. Once again we will assume that $\alpha_{ij} \neq 0$ and, using the conditions from above, we can assume $a_i = b_i = 0$. We have from (19) the following:

$$\begin{aligned} 2\alpha_{ij} &= a_jd_i - b_jc_i \neq 0 \\ 2\beta_{ij} &= a_jd_i + b_jc_i = 0 \\ 2\gamma_{ij} &= a_jc_i - b_jd_i = 0 \\ 2\delta_{ij} &= a_jc_i + b_jd_i = 0 \end{aligned} \tag{21}$$

We can see that these equations have the same form as (8) and thus we have no solution for (21) and have proven the impossibility of measurement for $i \neq j$. Therefore, we have proved for identical bosons, in the case of photons, that it is impossible to perform complete nondegenerate Bell operator measurement without using interaction between quantum states.

Fermions

Now we can look at the fermions. We again start with the Bell states:

$$|\Psi^{(-)}> = 1/\sqrt{2}\,(\,|\uparrow_L> \otimes |\downarrow_R> - |\downarrow_L> \otimes |\uparrow_R>\,)$$

$$|\Psi^{(+)}> = 1/\sqrt{2}\,(\,|\uparrow_L> \otimes |\downarrow_R> + |\downarrow_L> \otimes |\uparrow_R>\,) \tag{22}$$

$$|\Phi^{(-)}> = 1/\sqrt{2}\,(\,|\uparrow_L> \otimes |\uparrow_R> - |\downarrow_L> \otimes |\downarrow_R>\,)$$

$$|\Phi^{(+)}> = 1/\sqrt{2}\,(\,|\uparrow_L> \otimes |\uparrow_R> + |\downarrow_L> \otimes |\downarrow_R>\,).$$

Since we have fermions, we must antisymmetrize, thus replacing $|\uparrow_L> \otimes |\downarrow_R>$ with:

$$|\uparrow_L> \otimes |\downarrow_R> \rightarrow (1/\sqrt{2})\,(|\uparrow_L>_1 \otimes |\downarrow_R>_2 - |\downarrow_R>_1 \otimes |\uparrow_L>_2\,). \tag{23}$$

Substituting this form into (22) we get the antisymmetrized Bell states as follows:

$$|\Psi^{(-)}> = 1/2\,\{\,(\,|\uparrow_L>_1 \otimes |\downarrow_R>_2 - |\downarrow_R>_1 \otimes |\uparrow_L>_2\,)$$
$$-\,(\,|\downarrow_L>_1 \otimes |\uparrow_R>_2 - |\uparrow_R>_1 \otimes |\downarrow_L>_2)\,\}$$

$$|\Psi^{(+)}> = 1/2\,\{\,(\,|\uparrow_L>_1 \otimes |\downarrow_R>_2 - |\downarrow_R>_1 \otimes |\uparrow_L>_2\,)$$
$$+\,(\,|\downarrow_L>_1 \otimes |\uparrow_R>_2 - |\uparrow_R>_1 \otimes |\downarrow_L>_2)\,\}$$

(24)

$$|\Phi^{(-)}> = 1/2\,\{\,(\,|\uparrow_L>_1 \otimes |\uparrow_R>_2 - |\uparrow_R>_1 \otimes |\uparrow_L>_2\,)$$
$$-\,(\,|\downarrow_L>_1 \otimes |\downarrow_R>_2 - |\downarrow_R>_1 \otimes |\downarrow_L>_2)\,\}$$

$$|\Phi^{(+)}> = 1/2\,\{\,(\,|\uparrow_L>_1 \otimes |\uparrow_R>_2 - |\uparrow_R>_1 \otimes |\uparrow_L>_2\,)$$
$$+\,(\,|\downarrow_L>_1 \otimes |\downarrow_R>_2 - |\downarrow_R>_1 \otimes |\downarrow_L>_2)\,\}.$$

We can use the general form of the unitary linear evolution of the teleportation procedure for the four states

$$| \uparrow_L > = \Sigma_i \, a_i \, | \, i >$$

$$| \downarrow_L > = \Sigma_i \, b_i \, | \, i > \qquad (25)$$

$$| \uparrow_R > = \Sigma_i \, c_i \, | \, i >$$

$$| \downarrow_R > = \Sigma_i \, d_i \, | \, i >$$

to find a general form for the eigenstates of the Bell operators. This can be written in general form as:

$$| \Psi^{(-)} > = \Sigma_{i,j} \, \alpha_{ij} \, | \, i > \otimes \, | \, j >$$

$$| \Psi^{(+)} > = \Sigma_{i,j} \, \beta_{ij} \, | \, i > \otimes \, | \, j > \qquad (26)$$

$$| \Phi^{(-)} > = \Sigma_{i,j} \, \gamma_{ij} \, | \, i > \otimes \, | \, j >$$

$$| \Phi^{(+)} > = \Sigma_{i,j} \, \delta_{ij} \, | \, i > \otimes \, | \, j >.$$

We have, putting (25) into (24) and letting *i* correspond to particle label "1" and *j* correspond to particle label "2":

$$| \Psi^{(-)} > = 1/2 \, \{ \, (\, \Sigma_i \, a_i \, | \, i > \otimes \, \Sigma_j \, d_j \, | \, j > - \, \Sigma_i \, d_i \, | \, i > \otimes \, \Sigma_j \, a_j \, | \, j >)$$

$$- \, (\, \Sigma_i \, b_i \, | \, i > \otimes \, \Sigma_j \, c_j \, | \, j > - \, \Sigma_i \, c_i \, | \, i > \otimes \, \Sigma_j \, b_j \, | \, j >) \, \} \qquad (27)$$

$$= 1/2 \, \Sigma_{i,j} \, \{ \, (a_i d_j - a_j d_i \,) - (b_i c_j - b_j c_i) \, \} \, | \, i > \otimes \, | \, j >$$

$$= \Sigma_{i,j} \, \alpha_{ij} \, | \, i > \otimes \, | \, j >$$

$$| \Psi^{(+)} > = 1/2 \{ (\Sigma_i a_i | i > \otimes \Sigma_j d_j | j > - \Sigma_i d_i | i > \otimes \Sigma_j a_j | j >)$$

$$+ (\Sigma_i b_i | i > \otimes \Sigma_j c_j | j > - \Sigma_i c_i | i > \otimes \Sigma_j b_j | j >) \}$$

$$= 1/2 \Sigma_{i,j} \{ (a_i d_j - a_j d_i) + (b_i c_j - b_j c_i) \} | i > \otimes | j >$$

$$= \Sigma_{i,j} \beta_{ij} | i > \otimes | j >$$

$$| \Phi^{(-)} > = 1/2 \{ (\Sigma_i a_i | i > \otimes \Sigma_j c_j | j > - \Sigma_i c_i | i > \otimes \Sigma_j a_j | j >) \tag{27}$$

$$- (\Sigma_i b_i | i > \otimes \Sigma_j d_j | j > - \Sigma_i d_i | i > \otimes \Sigma_j b_j | j >) \}$$

$$= 1/2 \Sigma_{i,j} \{ (a_i c_j - a_j c_i) - (b_i d_j - b_j d_i) \} | i > \otimes | j >$$

$$= \Sigma_{i,j} \gamma_{ij} | i > \otimes | j >$$

$$| \Phi^{(+)} > = 1/2 \{ (\Sigma_i a_i | i > \otimes \Sigma_j c_j | j > - \Sigma_i c_i | i > \otimes \Sigma_j a_j | j >)$$

$$+ (\Sigma_i b_i | i > \otimes \Sigma_j d_j | j > - \Sigma_i d_i | i > \otimes \Sigma_j b_j | j >) \}$$

$$= 1/2 \Sigma_{i,j} \{ (a_i c_j - a_j c_i) + (b_i d_j - b_j d_i) \} | i > \otimes | j >$$

$$= \Sigma_{i,j} \delta_{ij} | i > \otimes | j >.$$

This gives us:

$$2\alpha_{ij} = (a_i d_j - d_i a_j) - (b_i c_j - c_i b_j)$$

$$2\beta_{ij} = (a_i d_j - d_i a_j) + (b_i c_j - c_i b_j) \tag{28}$$

$$2\gamma_{ij} = (a_i c_j - c_i a_j) - (b_i d_j - d_i b_j)$$

$$2\delta_{ij} = (a_i c_j - c_i a_j) + (b_i d_j - d_i b_j).$$

When $i = j$ we have $\alpha_{ii} = \beta_{ii} = \gamma_{ii} = \delta_{ii} = 0$. This is due to the antisymmetrization in accord with the Pauli principle since the product state vanishes when the two quantum numbers are equal, i.e. $i = j$. Therefore, for $i = j$, the impossibility of measuring the complete nondegenerate Bell operator is proven.

Now to prove for $i \neq j$. Let's consider the case where the detection of a certain product state $|\, i > \otimes \,|\, j >$ signifies finding $|\, \Psi^{(-)} >$ or $|\, \Psi^{(+)} >$. This means that for that pair $\{i, j\}$, we have that $\alpha_{ij} \neq 0$ or $\beta_{ij} \neq 0$ and also have:

$$2\gamma_{ij} = (a_i c_j - a_j c_i) - (b_i d_j - b_j d_i) = 0 \qquad (29)$$

$$2\delta_{ij} = (a_i c_j - a_j c_i) + (b_i d_j - b_j d_i) = 0.$$

Here γ_{ij} and δ_{ij} must be equal to 0 since the detection signifies finding $|\Psi^{(\pm)}>$ and not $|\Phi^{(\pm)}>$. From (29), we find the following:

$$a_i c_j - a_j c_i = b_i d_j - b_j d_i$$

and

$$a_i c_j - a_j c_i = -(b_i d_j - b_j d_i).$$

This results in both

$$a_i c_j - a_j c_i = 0$$

and

$$b_i d_j - b_j d_i = 0.$$

This gives

$$(\, a_i / a_j \,) = (\, c_i / c_j \,)$$

and (30)

$$(\, b_i / b_j \,) = (\, d_i / d_j \,)$$

Also we have that

$$2\alpha_{ij} = (a_i d_j - a_j d_i) - (b_i c_j - b_j c_i) \neq 0$$

or

$$2\beta_{ij} = (a_i d_j - a_j d_i) + (b_i c_j - b_j c_i) \neq 0.$$

Therefore, this means that we have either of the following:

$$(a_i d_j - a_j d_i) \neq (b_i c_j - b_j c_i) \qquad \text{(i)}$$

or

$$(a_i d_j - a_j d_i) \neq -(b_i c_j - b_j c_i). \qquad \text{(ii)}$$

In the first case, we can divide (i) by $a_j b_j c_j d_j$:

$$\frac{a_i/a_j - d_i/d_j}{b_j c_j} \neq \frac{b_i/b_j - c_i/c_j}{a_j d_j}$$

Using the equalities from (30) we have that:

$$\frac{c_i/c_j - b_i/b_j}{b_j c_j} \neq \frac{b_i/b_j - c_i/c_j}{a_j d_j}$$

$$\Rightarrow \qquad \frac{c_i/c_j}{b_j c_j} - \frac{b_i/b_j}{b_j c_j} \neq \frac{b_i/b_j}{a_j d_j} - \frac{c_i/c_j}{a_j d_j}$$

$$\Rightarrow \qquad \frac{c_i/c_j}{b_j c_j} + \frac{c_i/c_j}{a_j d_j} \neq \frac{b_i/b_j}{a_j d_j} + \frac{b_i/b_j}{b_j c_j}$$

$$\Rightarrow \qquad (c_i/c_j)\left(\frac{1}{b_j c_j} + \frac{1}{a_j d_j}\right) \neq (b_i/b_j)\left(\frac{1}{a_j d_j} + \frac{1}{b_j c_j}\right)$$

$$\Rightarrow \qquad (b_i / b_j) \neq (c_i / c_j).$$

Similarly, for the second case, we get:

$$(b_i / b_j) \neq -(c_i / c_j).$$

Here, the (-) sign does not matter due to the choice in constants from (16).

Similar results can also be obtained to find that

$$(a_i / a_j) \neq (d_i / d_j).$$

Using the above results and the results from (30) we get that:

$$(a_i / a_j) = (c_i / c_j) \neq (b_i / b_j) = (d_i / d_j). \tag{31}$$

The same can be shown for the detection of $| i > \otimes | j >$ signifying the finding of $|\Phi^{(-)}>$ or $|\Phi^{(+)}>$, i.e. $\gamma_{ij} \neq 0$ or $\delta_{ij} \neq 0$ giving:

$$(a_i / a_j) = (d_i / d_j) \neq (b_i / b_j) = (c_i / c_j). \tag{32}$$

Equations (31) and (32) are valid providing there are no vanishing denominators.

Now we want to prove that there cannot be a "common" state in the product states corresponding to finding $|\Psi^{(\pm)}>$ and $|\Phi^{(\pm)}>$, i.e. we want to prove that there are no states in the two product states that are the same. We can prove this by contradiction: suppose $| i > \otimes | j >$ corresponds to finding $|\Psi^{(\pm)}>$ and $| k > \otimes | j >$ corresponds to finding $|\Phi^{(\pm)}>$. This would give us, from (31) and (32):

$$(a_i / a_j) = (c_i / c_j) \neq (b_i / b_j) = (d_i / d_j); \tag{31a}$$

$$(a_k / a_j) = (d_k / d_j) \neq (b_k / b_j) = (c_k / c_j). \tag{32a}$$

From $(a_i / a_j) = (c_i / c_j)$ and from $(a_k / a_j) \neq (c_k / c_j)$, dividing these two we get:

$$(a_k / a_i) \neq (c_k / c_i).$$

This means that, according to (31), $| k > \otimes | i >$ cannot be identified with finding $|\Psi^{(\pm)}>$. From $(b_i / b_j) \neq (c_i / c_j)$ and from $(b_k / b_j) = (c_k / c_j)$, dividing these two we get:

$$(b_k / b_i) \neq (c_k / c_i).$$

This means that, according to (32), $| k > \otimes | i >$ cannot be identified with finding $|\Phi^{(\pm)}>$. Therefore, $| k > \otimes | i >$ cannot be identified with finding any state. Since $| k > \otimes | i >$ can be any arbitrary product state, then this is a contradiction; therefore, there cannot be a common state in the product states corresponding to finding $|\Psi^{(\pm)}>$ and $|\Phi^{(\pm)}>$.

We have shown that if detecting $|\, i > \otimes \,|\, j >$ signifies finding $|\Psi^{(\pm)}>$ while $|\, k > \otimes \,|\, m >$ signifies finding $|\Phi^{(\pm)}>$ then $|\, i >, |\, j >, |\, k >, |\, m >$ are all different states. So we have

$$(a_i / a_j) = (c_i / c_j) \neq (b_i / b_j) = (d_i / d_j), \qquad \text{for } |\Psi^{(\pm)}>, \text{(31b)}$$

$$(a_k / a_m) = (d_k / d_m) \neq (b_k / b_m) = (c_k / c_m), \qquad \text{for } |\Phi^{(\pm)}>. \qquad \text{(32b)}$$

For the same types of coefficients (eg. a and c) we can always find an equality and an inequality, e.g.

$$(a_i / a_j) = (c_i / c_j)$$

and (33)

$$(a_k / a_m) \neq (c_k / c_m).$$

Since all four states $|\, i >, |\, j >, |\, k >, |\, m >$ are different then at least one out of the following inequalities is true:

$$(a_i / a_m) \neq (c_i / c_m)$$

or (34)

$$(a_j / a_k) \neq (c_j / c_k).$$

We can prove this by assuming that the equations in (34) are both equalities. This would give us:

$$(a_i / a_m) = (c_i / c_m)$$

and

$$(a_j / a_k) = (c_j / c_k).$$

Dividing these two equations we get:

$$(a_i / a_j)(a_k / a_m) = (c_i / c_j)(c_k / c_m).$$

From (33), we know that $(a_i / a_j) = (c_i / c_j)$ and so we are left with

$$(a_k / a_m) = (c_k / c_m)$$

This however, contradicts (33) so we know that at least one of the inequalities in (34) must be true. This would mean that either $| i > \otimes | m >$ or $| k > \otimes | j >$ corresponds to the detection of $|\Phi^{(\pm)} >$. This is a contradiction since $| i > \otimes | j >$ corresponds to finding $|\Psi^{(\pm)} >$. Therefore it is impossible to measure a nondegenerate Bell operator for fermions without interaction between the two quantum systems.

Review of Literature

The first experimental verification of quantum teleportation was performed at the University of Innsbruck in Austria. Bouwmeester *et al.* [3], [7] produced pairs of entangled photons using the process of parametric down-conversion and used two-photon interferometry to transfer the polarization state of one photon to another. In their experiment, teleportation was shown for a chosen basis. For the polarization states, this consisted of horizontal and vertical polarization. Therefore, a superposition of the horizontal and vertical polarizations was chosen for the experiment. Also, the teleportation had to be shown to work for superpositions of those base states. This was shown using teleportation for circular polarization. However, true teleportation could only be achieved 25% of the time. Braunstein and Kimble [8] also suggested that this still wasn't proper teleportation since Bouwmeester *et al.*'s procedure necessitates the destruction of the state at Bob in some of the cases. This results in the teleported state not being available for further examination. Bouwmeester *et al.* responded that they believed that their teleportation procedure did involve all the properties required for true teleportation.

Using a procedure similar to [3], Koniorczyk *et al.* [9] investigated the possibility of teleporting superpositions of one- and two-photon states. Here, instead of two photons emerging from the parametric down-converter, two intersecting light cones would emerge where one is in the horizontal polarization state and the other is in the vertical polarization state. At the intersection of the two cones would be the superposition state to be teleported. When the paper was published, the process still needed to be found in order for the experiment to be realized.

Boschi *et al.* [10] also used photons for the teleportation procedure but used two photons instead of three. Here, the state to be teleported is prepared on one of the entangled photons and therefore cannot be prepared outside. This resulted in Alice having the unknown state to be teleported and not an outside party. Ideally, this could result in 100% teleportation although it is not feasible for the teleportation of an unknown state of an external particle.

These papers focus on the teleporting the state of a finite-dimensional system. However, the teleportation of continuous variables corresponding to infinite-dimensional systems has also been realized. Furusawa *et al.* [11] at the California Institute of Technology used the experimental setup described by [12]. Here the teleportation process uses states of an electromagnetic field. This procedure involves a third party, Victor, who produces the initial input in the form of a coherent state of the electromagnetic field. Once the teleportation is complete, Victor verifies that the teleportation has happened. Victor evaluates the amplitude and the variance of the field produced by Bob and compares it with the initial input. This brings about fidelity in teleportation, or the overlap between the input and output states. True teleportation for continuous variables in infinite-dimensional systems is realized when the input states are sufficiently equal to the output states, i.e. the fidelity is correct. Unlike [10] where no outside physical state can enter the device and [6] where the teleported state is destroyed at Bob, this can be considered a more correct version of teleportation according to the original paper [1].

One paper [13] suggests several criteria for ideal continuous-variable teleportation. These are:

1. "The states should be unknown to Alice and Bob and supplied by an actual third party Victor."
2. "Entanglement should be a verifiable used resource, with the possibility of physical transportation of the unknown states blocked at the outset. There should be a sense in which the output is 'close' to the input – close enough that it could not have been made from information sent through a classical channel alone."
3. "Each and every trial, as defined by Victor's supplying a state, should achieve an output sufficiently close to the input. When this situation pertains, the teleportation is called *unconditional*."

4. "The number of bits broadcast over the classical channel should be 'minuscule' in comparison to the information required to specify the 'unknown' states in the class from which the demonstration actually draws."

5. "The teleportation quality should be good enough to transfer quantum entanglement itself instead of a small subset of 'unknown' quantum states."

6. "The sender and receiver should not have to know each other's locations to carry the process through to completion." [13]

The focus of [13] is how to choose the fidelity such that criterion 2 is sufficiently met since ideal teleportation occurs when an unknown state is teleported from Alice and the exact same state occurs at Bob. Since fidelity allows the states only to be sufficiently close to each other, then this is a major concern for whether true teleportation has been achieved or not. Here, true teleportation should not be achieved unless fidelity of 1 occurs.

In a continuation of the work from [11], [14] uses the fact that a squeezed vacuum state is also entangled in number and phase, not just the amplitude and variance. By making joint number and phase measurements, this entanglement is used for the teleportation procedure. [14] shows that a given source of entanglement could yield more than one means of teleportation.

It has been shown that the methods above do not give 100% reliable teleportation according to the original teleportation paper [1]. This was proven for the finite-dimensional teleportation by showing that true teleportation could not be achieved with no quantum-quantum interaction. For the infinite-dimensional teleportation, it can be said that unless the input state is exactly the same form as the output state then true teleportation has not been achieved. However, the above experiments agree that the output state must be only sufficiently close to the input state in order to have true teleportation, which can be considered a contradiction to the original teleportation paper [1]. However, the recent work [5] suggest that by using ions in a finite-dimensional system; the authors of [5] claimed that true teleportation could be achieved due to the quantum-quantum interaction between the two ions. This proposal shows that it is possible to have reliable teleportation for the internal states of trapped ions.

Conclusion

Although teleportation has been experimentally verified, it has been shown that true teleportation cannot be achieved without interactions between quantum particles. Here, it has been proven for distinguishable and indistinguishable particles, the case of photons for bosons and the case of spin-½ particles for fermions, by showing that a complete nondegenerate Bell-operator measurement cannot be performed without interaction between the quantum particles to be teleported. This was shown by assuming that there was no interaction between the quantum particles and then proving by contradiction that the four Bell states could not be identified from each other. Without the complete Bell-operator measurement, teleportation, as proposed in the original paper, cannot be achieved.

The results of this project indicate that interactions between quantum particles are required for true quantum teleportation. Thus, we can conclude that interactions between quantum particles is a necessary condition for true quantum teleportation; the work in this project does not, however, show that such interactions between quantum particles is a sufficient condition for teleportation.

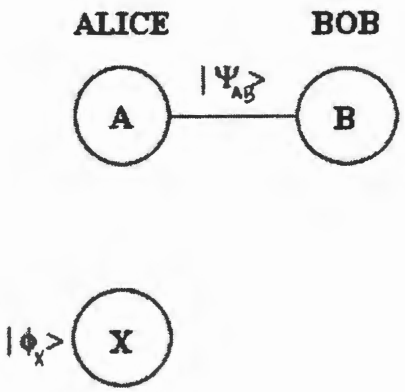


Figure 1. Alice and Bob share particles A and B in a two-particle state $|\Psi_{AB}>$. Alice has access to particle X which is initially prepared in the state $|\phi_X>$ which is unknown to Alice and Bob.

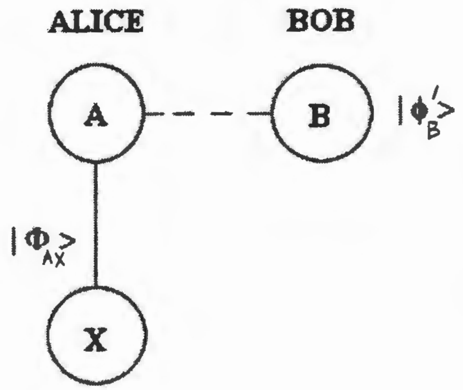


Figure 2. Alice performs a Bell-operator measurement on particles X and A projecting them into the state $|\Phi_{XA}>$ which is one of the four Bell states. The state of particle B has been changed to $|\phi_B'>$.

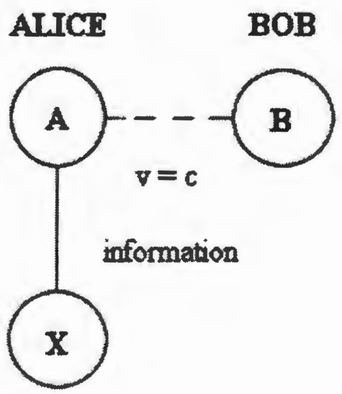


Figure 3. Alice sends the information to Bob of what Bell-state particles X and A are in.

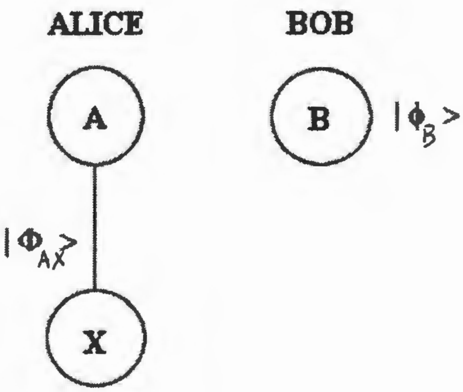


Figure 4. Bob uses the information that Alice sent to perform a local unitary transformation on particle B to change the state of particle B from $|\phi_B'>$ to $|\phi_B>$ which is the same form as the initial state of particle X.

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