

TEACHING MATHEMATICS FOR UNDERSTANDING: DEVELOPING
TEACHER KNOWLEDGE THROUGH CLASSROOM INSTRUCTION

by

Mary Josephine Kerrigan

B. A., University of Manitoba, 1979

Ed. Cert., University of Brandon, 1982

THESIS SUBMITTED IN PARTIAL FULFILMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF EDUCATION

in

CURRICULUM AND INSTRUCTION

THE UNIVERSITY OF NORTHERN BRITISH COLUMBIA

August 2005

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Your file Votre référence

ISBN: 978-0-494-28376-9

Our file Notre référence

ISBN: 978-0-494-28376-9

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Abstract

This study examines the process of implementing a constructivist approach to teaching mathematics at the early elementary school level. Using action research methodology, the investigation highlights the mutual learning experiences of both the students and teacher as the concept of multiplication is introduced into a combined grade one and two classroom. Using the NCTM Principles and Standards for School Mathematics (2000) as a guide, the study provides a model of how one teacher developed a unit of study that addresses the provincial curriculum, but also provides opportunities for the students to explore and develop their own mathematical knowledge. Through a process of reflection and analysis the teacher developed her professional knowledge about the way students think about the multiplication process, and in turn learned to plan for more effective instruction. This study highlights the vital role that discourse plays in the classroom. Attentiveness to the students' ideas and thinking processes enabled the teacher to assist the students in making the connections necessary to develop a deeper understanding of the subject matter. The study illustrates how effective professional development may be teacher generated and focus on nurturing the unique learning opportunities that arise in individual classrooms as students and teachers interact and explore new ideas and concepts.

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Acknowledgements

I would like to express my sincere appreciation to the Grade One and Grade Two students who were my partners in this action research endeavor. My external committee member was Dr Kyle Matsuba and I thank him for his probing questions that helped to strengthen this work. I wish to express my strong appreciation to Dr. Willow Brown as her keen editorial eye aided my writing work and her thoughtful suggestions challenged me to develop my skills in critical thinking and analysis. I'm especially grateful to Professor Cathy McGregor for her consistent positive support and encouragement throughout this process. Finally I'd like to thank Glen, Anna Marie, and Lisa who came along on this journey with me and provided their unconditional love and support.

Chapter One

Background to the Research

In the fall of 2003 a new mathematics program was introduced to the elementary schools of School District. 57 (Prince George). The decision to implement this pilot program was the result of a district initiative started in October, 2001 by the Curriculum and Instruction Department. One of the major goals of the Curriculum and Instruction Department was to develop a plan to improve student achievement in mathematics in the Prince George School District. An initial step toward this goal was the establishment of a leadership team called the Numeracy Task Force. Its mandate was to develop a comprehensive long-term plan for improving mathematics education in the district. The new pilot program represented the first phase of a district plan to revitalize and improve mathematics education in the elementary schools in Prince George.

The initiative by the Curriculum and Instruction department was influenced in part by the results of the provincial Foundation Skills Assessment (FSA) and also by a requirement of the Provincial Ministry of Education that each school district devise a yearly accountability plan. The FSAs are a series of provincially developed tests, administered each spring throughout the province to students at the grade four, seven, and ten levels. These tests assess reading comprehension, writing, and numeracy. Results from these tests are published annually and provide provincial average scores and school district averages throughout the province. Year to year comparisons within each district provide information on academic achievement that can be used to assess the effectiveness of district programs. In the three years that the results from the FSA tests have been published, average scores in numeracy in the Prince George School District have fallen

below the provincial averages at all three grade levels (Foundation Skills Assessment, 2003). Consistently poor performance in the FSA numeracy tests is a major concern for the district and a significant factor in the numeracy initiative.

The District Plan for Student Success is an accountability contract developed by the school district and presented to the Ministry of Education, outlining the district's goals, objectives, and strategies for improving academic achievement for the school year. In the current School District 57 document, (School District No. 57 Prince George, 2004) student improvement in mathematics is listed as one of the five major goals to be pursued. Under the numeracy goal, the district plan identifies five objectives with specific strategies designed to meet these objectives. One of the key strategies is the creation of the Numeracy Task Force to act as a leadership team, to select core materials, develop a scope and sequence structure, and support the implementation of best practices within the elementary mathematics program.

A key decision made by the Numeracy Task Force was to select and promote a base text to assist in the delivery of the provincial mathematics curriculum. The Addison Wesley *Math Makes Sense* program, (2004) presents a specific philosophical and practical approach to teaching mathematics. It requires teachers to examine the processes of learning mathematics and challenges them to adjust their teaching methods to incorporate alternative ways of learning. In choosing this resource, the Task Force was attempting to provide elementary math teachers with a different approach to teaching mathematics; one that they believed was more effective and reflected the current research on learning mathematics.

With the introduction of this resource, a series of professional development initiatives that focused on the processes of teacher learning were also developed and implemented. These initiatives were designed to reflect the philosophy and values of the new resource and model the types of learning strategies promoted in the new resource. The Task Force recognized that changes in teacher behaviour take place gradually and are only sustained over time if teachers are given the opportunity to take ownership of their learning. This ownership does not happen overnight, but is a gradual process where teachers are given the opportunity to work with new ideas, share experiences with their colleagues, and problem solve together, adapting the program to suit the needs of their students. This type of gradual implementation is in itself a reflection of some of the current research into teacher change and teacher development (Fullen & Hargreaves, 1992). The aim of the district initiative is to improve the articulation of the present mathematics curriculum, and to promote positive models of best practices in mathematics education. Ultimately, the long-term goal is to improve the mathematical performance of all students within the school district.

The purpose of this study was to explore an alternative approach to teaching mathematics at the early primary level. The focus was on how to develop student understanding of mathematical concepts consistent with the new District 57 initiative. This initiative reflects the goals of the mathematics reform movement, championed by the National Council of Teachers of Mathematics (NCTM), which has been highly influential in directing the process of change in mathematics education in North America over the past two decades (Van De Walle, 2001).

Problem

Rationale

“Teachers are the key to children’s math learning, the conduits between the child and the math curriculum.” (Burns, 1999)

The success of any educational initiative is dependent on the participation and willingness of teachers and administrators at the school level to effect change. In order for change to occur educators need to examine their beliefs and values, and be prepared to adjust to new expectations and conditions. This is not an easy task, as beliefs and values develop over a lifetime of experiences and shape the way educators perceive their roles and responsibilities.

The struggle for change in mathematics education has been an ongoing process since 1989, after the publication of the Standards for Mathematics Education developed by NCTM. This document proved to be highly influential in its challenge of the traditional practices of mathematics teaching (Hedden & Speer, 2001; Van De Walle 2001). NCTM set in motion an ongoing process of reform throughout North America. Adapting to the new demands of mathematics teaching has not been an easy process and the struggle continues. It requires mathematics teachers to re-examine their roles in the mathematics classroom and make substantive changes in the way that mathematics education is incorporated into the total curricula. The initiative taken by the local school district accepts the philosophy of the NCTM Standards and acknowledges the need to re-evaluate how mathematics is taught in the local elementary schools.

Problem Statement

In this study I will explore the ways in which the concept of multiplication can be taught to seven and eight year old children using the goals and guidelines of the current mathematics reform movement. Teaching mathematics according to the Standards developed by the National Council of Teachers of Mathematics (1989) requires that the mathematical concepts be taught in relation to their function within the child's everyday world. This approach to teaching would broaden the definition of multiplication to include much more than the traditional procedures of learning the standard algorithms and mastering the times tables. The Standards emphasize the importance of students being given opportunities to explore mathematical concepts through problem-solving investigations, in real world situations, to allow them to construct their own understanding of multiplication. To this end, students need to be provided with experiences that challenge their thinking and reasoning skills, encourage them to develop and test their own theories, and justify their reasoning and solutions to problems with other members of the group. It necessitates making connections between other strands of mathematics such as patterning and geometry, and through classroom discussion and negotiation; students are encouraged to articulate their understanding, thus making connections to the broader numeracy and literacy goals of the primary curriculum.

My goal for this research was to examine possible ways of teaching multiplication for understanding. A major focus was on methods that can be employed to connect student learning to prior knowledge of mathematical concepts and the inter-relationship between these concepts. The research examines the methods of instruction that allow students to make connections between numeracy and literacy, with the intent that the

students will develop a sound understanding of the multiplication process. Evidence of this understanding was determined through a variety of classroom activities that required students to recognize problems that call for multiplication solutions. These included identifying appropriate procedures for carrying out the calculations, student evaluation of procedures, and applying the results in other problem solving situations. Students were encouraged to justify their solutions to problems both orally and in writing, thus developing the notion of mathematical literacy. These classroom activities were designed to reflect the constructivist approach to learning described in the NCTM Curriculum and Evaluation Standards (1989), or what Van de Walle (2001) refers to as the *developmental approach*. This study not only focuses on student learning, but also on the process of teacher learning and professional development through reflective analysis of classroom practice.

The method I chose for this study was action research, and therefore I had a dual role as the teacher and researcher. As the teacher I developed a general plan for the multiplication unit and created a learning environment conducive to the goals of the project, providing activities that were designed to support learning for understanding. I was also attentive to the provincial curriculum goals for this particular age group of students. However, the direction of the unit was guided by the interactions and discussions that emerged as both teacher and students engaged in the study. This method of teaching reflects the constructivist or developmental approach to learning, and is compatible with action research methodology. As the researcher I was continuously and deliberately reflecting on the teaching and learning episodes, and making teaching decisions based on my evaluation of the learning environment. This cycle of action,

reflection, evaluation, and revised action is consistent with the methods of action research (Altricher, Posch, & Somekh, 1993; Elliott, 1991)

Historical Overview

The current mathematics curriculum in British Columbia was shaped to a large extent by the recommendations made in the Mathematics Assessment Technical Report (1990). This report, commissioned by the Ministry of Education, was the result of province wide mathematics testing of all students in Grades One, Seven, and Ten and included information gained from questionnaires completed by these students and their teachers. The report identified four major areas of concern: low participation rates of women in mathematics education, incomplete coverage of curriculum content, rigid teaching methods, and negative student attitudes towards mathematics. The report did not give specific directives but suggested possible directions for improvement. These suggestions were incorporated into the revised mathematics curriculum, which was introduced in 1995 as the Integrated Resource Package (IRP): Mathematics K-7. The mathematics IRP is a comprehensive document that lists the provincially prescribed learning outcomes for each grade level, suggests instructional strategies for achieving the outcomes, provides assessment strategies, and recommends learning resources approved by the Ministry. The integration of learning outcomes, instructional strategies, and assessment criteria are key features of all the provincial IRP documents and emphasize the responsive aspect of teaching and learning that is embraced in the Ministry of Education's K to 12 Education plan (1992).

The goals and objectives for each grade level are clearly laid out in the Mathematics IRP K-7 and suggestions for teaching strategies are systematically aligned.

However, there is limited information about the underlying research and philosophy on which the program is based. The introduction clearly states the principles of learning to be (a) active participation by the student, (b) the recognition that people learn in a variety of ways and at different rates, and (c) that learning is both an individual and a group process (BC Ministry of Education 1995, p. 1). These points also reflect the NCTM philosophy. However, the rationale is brief, and does not clearly articulate the wholesale changes in teaching methods and classroom organization needed to implement the curriculum according to its stated principles and philosophical premise. Undoubtedly, the Mathematics IRP is heavily influenced by the NCTM Standards; however it lacks the necessary discussion regarding the constructivist approach to learning mathematics.

A more thorough discussion of educational philosophy is provided in the Primary Program: A Framework for Teaching (BC Ministry of Education, 2000). This document provides a clear philosophical grounding for the provision of primary education and promotes the establishment of a constructivist, integrated approach to learning. It highlights the important link between literacy and numeracy and uses examples from the Curriculum and Education Standards (NCTM, 1989), to highlight the types of mathematical learning experiences children should have.

Clearly, the provincial elementary mathematics curriculum reflects the core recommendations of the NCTM standards documents both in content to be taught and the processes to be developed. Since this is the approach adopted in British Columbia it will provide the premise for this investigation. As mentioned in the preceding section, adoption of the NCTM standards requires significant shifts in thinking about the delivery of elementary school mathematics programs. The kind of teaching envisioned in the

Standards (1989) and reiterated in the BC Mathematics IRP (1995) is significantly different from what many teachers have themselves experienced in mathematics classes.

One of the purposes of this investigation is to examine the extent to which the new approach to mathematics can be incorporated into a primary classroom in Prince George. The mathematics initiative being promoted by the Curriculum and Instruction Department at School District 57 suggests that district leaders have some concerns regarding teaching practices and student outcomes. The specific choice of a base text for all primary teachers steers a clear path towards a particular teaching philosophy that is consistent with NCTM Standards. This choice could present challenges for some teachers and may require shifts in attitudes and thinking regarding daily classroom practice in the teaching of mathematics. This research examines both student and teacher learning in a constructivist mathematics classroom.

Research Question

How does teaching to the NCTM Curriculum and Professional Standards enhance student and teacher learning in the elementary mathematics classroom?

This question will be answered by exploring the following:

1. How do primary aged students develop an understanding of the concept of multiplication using a constructivist approach to learning?
2. How do primary aged students demonstrate their understanding of the multiplication process?
3. How can teachers become more aware of their students' mathematical development in relation to the multiplication process?
4. How can teachers use their knowledge of student understanding to plan for responsive

and effective instruction when teaching the concept of multiplication?

Definitions of Terms

Primary Education in this thesis refers to educational programs being delivered in elementary classrooms from Kindergarten to Grade 3.

Change in the context of this study refers to the movement from a teacher centered mathematics classroom to a more student centered mathematics classroom.

Literacy is the ability to construct meaning and articulate thinking through the integrated processes of reading, writing, speaking and listening.

Mathematical Literacy is articulated in the NCTM Standards (1989) as the capacity to value mathematics, having confidence in the ability to do mathematics, using mathematics to problem solve, communicating mathematically, and the ability to reasoning mathematically.

Numeracy is mathematical literacy. According to the BC Ministry of Education (2000) it has a broad definition, which encompasses the understanding of mathematics in personally meaningful terms. Numeracy is a way of thinking mathematically that helps people make sense of their world and allows them to communicate their ideas to others (BC Primary Program, 2000).

Constructivism views learning as an active process in which individuals are continuously making sense of new information by relating it back to what they already know. Through a personal process of experimentation, discussion, and negotiation individuals construct new knowledge and understanding (Parkay & Hass, 2000; Van De Walle 2001).

The Standards refers to the National Council of Teachers of Mathematics (NCTM) Curriculum and Evaluation Standards (1989), the NCTM Professional Standards for

Teaching Mathematics (1991), and the NCTM Principles and Standards for School Mathematics (2000).

Summary

Mathematics education is in a state of change. Calls for change have come from many spheres, most notably from mathematics educators who wish to tailor mathematics education towards the needs of an increasingly technological society. Changes in curriculum have been far reaching but the questions raised in this study focus on how teaching practices can adapt to the demands of the mathematics reform movement. The *NCTM Professional Standards for Teaching* (1991) emphasized the key role teachers play in the change process. However, it also noted, “change takes time as teachers learn about and develop new teaching practices”. The study will examine the process of how teachers develop knowledge about their teaching practices and how they can apply this knowledge in their classrooms. Using an action research method of inquiry, the study focuses on one primary classroom and explores the teaching and learning episodes that evolve during a unit of mathematical study investigating the concept of multiplication.

My Personal Journey

This research presents a personal journey of learning and professional development. I have taught at the elementary level for over 20 years in a variety of teaching positions. My particular interest in math education began in 1992 when I was appointed to a provincial committee in Manitoba that was developing a new mathematics curriculum for kindergarten to grade four. The committee’s work was heavily influenced by the NCTM Standards publications and the new Manitoba mathematics curriculum reflected the philosophy of NCTM. Many teachers on the committee were moving

towards a constructivist style of teaching mathematics. However, their experiences did not reflect the reality of how math was being perceived and taught in regular classrooms. As a result, the promotion of the new curriculum was met with some hostility, as educators were unsure of how to implement the new approach of teaching mathematics.

My journey with constructivist style teaching methods began during this time. I had experimented with certain aspects of constructivism in isolated ways in order to present alternative lesson formats. I had also attempted to develop a classroom culture that would support a constructivist approach to learning that integrated subject matter. Since moving to British Columbia in 1994 I have worked at many teaching positions in the elementary schools, both in the classroom at a variety of levels, and as a support teacher and learning assistance teacher. Through these experiences I have come to realize that although much of the teaching processes championed in the various British Columbia elementary curriculums are based on constructivist philosophy this is not reflected to a large degree in practice.

The purpose of this study is not to ask why constructivist teaching has not been more widely adopted generally, but rather to examine my own journey. My renewed focus on mathematics teaching began when I took a position teaching a combined grade one and two class in 2003, and was informed that I would be piloting a new mathematics program for the Prince George School District. The program, *Math Makes Sense*, (Pearson, 2004) is based on the reform principles of the NCTM Standards but is packaged to present a more palatable and familiar format of math instruction. In reviewing the program and the initiatives taken by the School District, I became aware that the struggle to change and improve mathematics education was continuing and that

practice had changed little from my experiences in Manitoba in the early nineties.

Despite all the literature to support reform, school districts were still having difficulty promoting change. With this realization, and my participation in the Prince George district mentor program, I decided to examine my own practice and document my learning journey in implementing a Standards based approach to teaching mathematics in my classroom. I focused on one unit of mathematics study that the students would not have previously experienced. My goal was to examine the process of how my students learned the concept of multiplication and how I as a teacher responded and adapted to the evolving learning needs of the students. In this process I was also participating in the learning process as I critically examined my role as teacher and guide in the learning process.

The following literature review will examine some of the current research that has taken place in mathematics education. It will focus in particular in the area of teacher change and professional development as it relates to implementation of the NCTM Standards.

Chapter Two

Literature Review

In this chapter I will examine some of the current literature that focuses on reform in elementary mathematics education. I will start by highlighting the evolution of the reform movement and its theoretical foundations. I will then present literature that discusses the challenges of implementing reform curricula in the mathematics classroom. I will conclude by discussing the conditions and processes deemed necessary to implement reform based mathematics education.

The Impetus for Change

The publication of the *Curriculum and Evaluation Standards for School Mathematics* in 1989 and the *Professional Standards for Teaching Mathematics* in 1991 by The National Council of Teachers of Mathematics (NCTM) signaled the beginning of a new era in mathematics education. The Standards documents were produced in response to increasing concerns from educators and the general public that traditional mathematics education programs were not sufficiently preparing students to meet the challenges of the modern technological world. Educators had also become increasingly concerned with reports of declining mathematics achievement across North America. The Standards documents provided a powerful case for reform, and established a clear vision for mathematics education.

Since publication, the Standards have been adopted by education authorities throughout the United States and Canada as a guide in the development of reform curricula in mathematics. However, the reform movement has met with varying degrees of success as education authorities and school districts have struggled to implement new

approaches to teaching and learning. The Standards challenge many traditional teaching practices in mathematics education and require significant changes to be made in the way mathematics classrooms are structured. The recommended approach to mathematics focuses on the understanding of mathematical processes rather than learning rote computational skills. Traditional learning outcomes that primarily measured competence in computational skills are insufficient and do not adequately assess mathematics taught for understanding. The unique nature of the reform initiatives places great emphasis on the relationships developed between the teacher and students during units of mathematical study. As a result, the classroom teacher is charged with the responsibility of enabling and promoting the reform agenda. Much of the literature on the reform movement requires teachers to make significant shifts in their thinking about the nature of mathematics education, (Cooney & Shealy, 1997; Franke, Fennema & Carpenter, 1997; Thompson, 1993) and consequently changes in applications and practice. Ultimately, these researchers would argue that the success of the reform movement is dependent on the skill and adaptability of the classroom teacher in responding to the reform agenda.

With any new education program new teacher learning is required. However, the reform of math education not only requires new learning but also a systematic change in the pedagogical structures that support learning. It requires a change in ideas about the nature of learning and how learning evolves within the classroom, not only for the students but also for the teacher. In this chapter I will identify the key theories underlying the mathematics reform movement and the implications for classroom

practice. I will then examine the learning challenges presented to teachers charged with implementing the reform curricula.

Research into the change process continues to identify the teaching and learning environments that support reform. Teachers are regarded as pivotal in creating a climate of change. However, much depends on their ability to adapt and assimilate new ideas about learning. This chapter will identify the various perspectives of teacher change and the resulting expectations placed on teachers. Administrative support is crucial if teachers are to change their practice in order to support reform. On-going research (Brendefur & Foster, 2000; Wisconsin Centre for Educational Research, 2002) has identified systems of structural support that have contributed to effective teacher change. These will also be examined in order to determine conditions necessary to assist the reform process. The current reform movement supports change in educational practice based on students developing a greater understanding of mathematical processes. By examining and evaluating emerging models of practice, data accumulates that informs and contributes to the development of a theory of mathematics education.

Theoretical Basis For Reform

The notion of what progressive mathematics education should look like has been evolving over the last three decades and continues to receive considerable attention in the academic world (NCTM, 2003). This change was spurred in part by the rapid technological advances that occurred in society during the last two decades (NCTM, 1989). These changes require different mathematical skills than the ones that were traditionally taught. New information from the cognitive and social sciences has provided greater insight about how children think and learn, and a new, more

encompassing definition of literacy has been evolving in educational theory (Pappas, Kiefer & Levstik 1999). These societal changes and the increasing body of knowledge regarding learning theory questioned the effectiveness of the traditional style of mathematics education and provided the NCTM with its mandate to assume leadership in the reform movement.

The Standards documents of 1989 and 1991 and the Assessment Standards (1994) have recently been revised and combined into one comprehensive package entitled *Principles and Standards for School Mathematics* (2000). The Standards have provided the mathematics education community with a clearly articulated vision of a new approach to mathematics education. They detail the essential elements of curriculum content and the fundamental learning processes believed necessary for school mathematics programs. These Standards have had a tremendous impact on the mathematics education community across North America and have become the benchmark by which education authorities across the continent evaluate their programs (Van de Walle, 2001; Sgroi, 2001). The documents have guided the reform movement and have provided a structure for the development of new mathematics curricula within North America and beyond (Heddens & Speer, 2001; Van de Walle, 2001).

The *Curriculum and Evaluation Standards* (1989), describe the societal changes that have taken place requiring a new and more comprehensive view of mathematics education:

All industrialized countries have experienced a shift from an industrial to an information society, a shift that has transformed both the aspects of mathematics that need to be transmitted to students and the concepts and procedures that they

must master. Information is the new capital and the new material, and communication is the new means of production. (NCTM, 1989, p. 3)

The authors of the document assert that the mathematical skills needed in the workforce today go far beyond basic mathematical competence. They suggest that mathematical principles have permeated all areas of society and guide many of the decisions made about what we do and how we live. Rapid accumulation of knowledge and information makes it difficult to predict what the future will hold. However, in order to deal effectively with this accumulation of knowledge, workers need the skills to process and assimilate large amounts of unfamiliar information. This requires flexibility in the workforce and the ability to solve problems and adapt to new situations. In order to cope effectively with this new technological future the *Curriculum and Evaluation Standards* (1989) require that all students have an opportunity to become mathematically literate. To this end the standards articulate five general goals for all students: (a) that they learn to value mathematics, (b) that they become confident in their abilities to do mathematics, (c) that they become mathematical problem solvers, (d) that they learn to communicate mathematically, and (e) that they learn to reason mathematically (NCTM, 1989, p. 5).

This notion of developing mathematical literacy requires a fundamental change from the way mathematics had been taught in the past. Traditional practice was linear and sequential where the students gradually accumulated facts and process skills. The focus was on the teacher, who disseminated the facts and showed the students how to apply the processes and rules. The students practiced until they could apply the processes proficiently. The Standards not only changed the emphasis on what should be taught, but

more importantly, focused on how mathematical content should be delivered. In fact, specific mathematical processes are now considered to be crucial content by NCTM (1991), and necessary for the development of mathematical literacy. Not only are students expected to become competent using mathematical processes, but also, that they become effective in communicating their ideas and reasoning to others. Specific mathematical processes considered content by NCTM Standards (1991) include:

Examining patterns, abstracting, generalizing, and making convincing mathematical arguments...definitions, examples, and counterexamples and the use of assumptions, evidence and proof. Framing mathematical questions and conjectures, constructing and evaluating arguments, making connections and communicating mathematical ideas. (p. 133)

These ideas imply that mathematics education should no longer be viewed as the lone silent practice of computation (Heaton, 2000), but instead it should be regarded as a group process in which learners work together to make sense of and understand mathematical situations. This is not to say that proficiency in computation is not valued, but rather, the focus of mathematics education should change to meet the demands of a technological society. Tasks that can be performed easily by technological tools such as calculators and computers should be de-emphasized in favour of developing higher thinking and problem solving skills.

The *Professional Standards for Mathematics Teaching* (1991) lists five major shifts in the teaching and learning of mathematics:

- towards classrooms as mathematical communities away from classrooms as simply a collection of individuals;

- towards logic and mathematical evidence as verification – away from the teacher as the sole authority for right answers;
- towards mathematical reasoning – away from merely memorizing procedures;
- towards conjecturing, inventing and problem solving – away from an emphasis on mechanistic answer-finding;
- towards connecting mathematics, its ideas, and its application - away from treating mathematics as a body of isolated concepts and procedures (p. 3).

The classroom environment envisioned in the Principles and Standards for School Mathematics (2000), is shaped by constructivist theories of learning that have been steadily incorporated in educational philosophy since the mid 1980s. A constructivist view of learning focuses on how learners make sense of new information and how they construct meaning based on what they already know (Parkay & Hass, 2000). This theory suggests that students develop new knowledge through a process of active construction. They do not passively receive or copy input from teachers or textbooks. Instead, “they actively mediate it by trying to make sense of it and relate it to what they already know (or think they know) about a topic” (Good & Brophy, 1997, p. 398). As they work with new ideas, their active engagement leads them to fit new learning within an established framework of understanding. The more ideas a student connects, the more solid their understanding will be of a particular topic. In the context of mathematics knowledge, this suggests a greater emphasis on the understanding of procedures: when students understand mathematical procedures, they can recall them easily and apply them in a variety of settings, relying more on reasoning than on rote memory. This conceptual

understanding provides the basis for acquiring new knowledge and solving unfamiliar problems.

Social constructivism, advanced in the theories of the Russian scientist Lev Vygotsky (1896-1934), emphasizes the importance of social interaction and a child's use of language to articulate and refine new learning constructs. Interacting and the negotiating of meaning within the learning community are regarded as instrumental in testing and refining new knowledge and developing deeper understanding (Carlson, Buskist, Enzle & Heth 2000; Hoff, 2001; Rice, 2001). This social aspect of learning is central to the type of learning environment envisioned in the Standards (Forman, 2003).

Applying constructivist theories to mathematical education requires a classroom structure quite different from traditional arrangements. Since children are encouraged to explore and experiment with ideas, and then discuss their learning with others, the focus is on the student and the student's interaction and communication with peers. Rather than disseminate facts and procedures, the teacher is expected to present meaningful problem solving situations that are connected to the students' real life experiences. The teacher's role is to provide guidance as the students collaborate and explore possible solutions to mathematical problems. The use of real life problem solving situations allows the child to make connections with previous knowledge and view mathematics as a vital and integral part of daily life.

As this approach suggests, a key component of constructivist mathematics education is communication. This entails learning the language of mathematics and being able to use that language to explain processes and negotiate meaning with others.

This expression of literacy is a fundamental goal underlying all the content strands listed in the *Curriculum and Evaluation Standards* (1989):

To understand what they learn, they must enact for themselves verbs that permeate the mathematics curriculum: “examine,” “represent,” “transform,” “solve,” “apply,” “prove,” “communicate.” This happens most readily when students work in groups, engage in discussion, make presentations, and in other ways take charge of their own learning (NCTM, 1991, p. 2).

This scenario of classroom activities is quite different from the traditional approach to mathematics education, and challenges teachers of mathematics to make fundamental changes in their approach to teaching and learning. It requires a significant change in thinking about the nature of mathematics and the application of mathematical ideas. The NCTM recognized this challenge when it produced the *Professional Standards for Teaching Mathematics* (1991). The goal of this document was to provide guidance for those involved in changing mathematics education. It acknowledges the key role that teachers play in the change process and details the teacher education and training that is necessary to fully implement this vision of change. It also provides examples of classroom interactions that demonstrate the principles of this new mode of educational practice.

Despite the clearly articulated vision of the NCTM Standards, its authors do not provide a guidebook for change. The processes by which these standards are incorporated into provincial curricula have been wide-ranging and have met with varying degrees of success. What has become apparent to researchers analyzing the change process is the crucial role the classroom teacher plays in orchestrating change. As such,

educational researchers have focused much of their attention on examining the prerequisites necessary for teacher change and the knowledge base required by teachers to support the reform agenda (Fullen & Hargreaves 1992; Wells 1994).

Challenges For Teachers

In order for teachers to change their teaching practices there needs to be a fundamental change in their beliefs about the nature of mathematics education (Fennema & Nelson, 1997). The Standards are based on a constructivist approach to learning that requires the teacher to focus on the individual student, and recognize the unique skills and knowledge that each student brings to the learning environment. How effectively teachers adjust to this philosophical premise determines the success of a Standards based curriculum. Nelson (1997) identifies a set of interconnected changes in beliefs that mathematics teachers need to adopt if they are to base their teaching on a constructivist view of learning. First, teachers need to view students as learners who are intellectually generative and can direct the process of learning. The second required change is that instruction can be based on the development of students thinking rather than relying predominantly on text based instruction. Thirdly, teachers need to accept the idea that text should not be the focus of intellectual authority but rather, authority for learning is negotiated by the teachers and students, through discussion and debate generated in the classroom. The final shift is for teachers to understand that they and their students can use the mathematical modes of reasoning to generate and validate mathematical knowledge.

Shifting from a transmission model of learning to a constructivist view is not an instant process. It occurs gradually as new knowledge and information is acquired and

assimilated. Ideas about mathematical knowledge, and beliefs concerning mathematical education are developed over time and are greatly influenced by one's own personal experiences. Teachers' beliefs have been described as filters (Ball, 1988; Lubinski & Jaberg, 1997) through which teachers interpret and ascribe meaning to their experiences. The interpretation and implementation of curricular is significantly influenced by teachers' knowledge and beliefs about mathematics education (Lubinski & Jaberg, 1997). If a change in attitudes towards mathematics education is to occur, teachers need to be exposed to new experiences that challenge previously held beliefs. What these particular experiences should be and what type of new knowledge teachers need in order to change their beliefs, and ultimately their teaching practice, is an ongoing debate in the field of educational research (Ball, 2001; Nelson, 1997).

Three Perspectives on Teacher Change

The study of teacher beliefs and teacher change is a relatively new area of research (Nelson, 1997; Thompson, 1997). It recognizes the close link between teachers' conceptions of mathematics education, and how those conceptions shape the way in which mathematics is taught. There is no consensus on what type of new knowledge teachers need in order to effect changes in belief. Researchers have examined the issue from various disciplinary perspectives (Nelson, Warfield, & Wood, 2001) and have identified three distinct disciplines that have influenced thinking on teacher change. Although the perspectives are similar, in that they accept the principles of constructivist learning, they take a different stance on what type of knowledge will generate change in belief and practice. The psychological perspective places great emphasis on teachers' knowledge of student thinking and cognitive development (Carpenter, Fennema, &

Franke, 1996; Fennema & Carpenter, 1996). The mathematical perspective suggests teachers need a greater knowledge about mathematical concepts and ideas so they can make connections between the strands and create a positive mathematical culture in the classroom (Schifter, 2001; Warfield, 2001). The sociological perspective places great value on the ability of teachers to create learning environments that promote discursive processes where meaning is negotiated and mathematical understanding is the product (Wood & Turner-Vorbeck 2001). I will briefly outline these in turn.

The Psychological Perspective

The psychological perspective examines the mathematics teaching process in relation to the teachers' own learning about students' mathematical development. It emphasizes the importance of the teachers' understanding of their students' cognitive development in relation to mathematics. A constructivist approach to teaching requires teachers to be keenly aware of the students' previous knowledge and experiences so that instruction can be structured to access this prior learning. In classroom practice, the student becomes the centre of the learning process and the teacher plans instruction based on the perceived needs and cognitive development of the student. For teachers to take this approach to mathematics learning they would need to have specific knowledge of their students' cognitive processes especially in regard to their mathematical development. This focus on teacher knowledge has been used to develop the Cognitively Guided Instruction (CGI) project that has been very successful in providing a learning framework for teachers (Carpenter, Fennema & Franke, 1996; Fennema, & Carpenter, 1996).

The CGI program, which is funded by the National Science Foundation and operates out of the Centre for Education Research at the University of Wisconsin, was designed to improve elementary mathematics teachers' knowledge of their students' cognitive development. The program is based on the assumption that children enter school with a great deal of informal or intuitive knowledge of mathematics. Teachers need to be able to recognize this knowledge and use it as the basis for developing the formal mathematics of the curriculum. The CGI teacher-training program is not a specific method or formula for instruction, but rather a training process that helps teachers to assess their own level of mathematical understanding and their pedagogical content knowledge. The underlying belief is that students construct knowledge from experiences provided in the classroom rather than merely assimilate what has been taught. CGI supports teachers' understanding of their students' mathematical thinking. CGI helps teachers construct models of the development of children's thinking in well-defined content domains. Students' thinking provides the context for teachers to enhance their own understanding of mathematics, and the models act as guides for developing instructional practices. The teachers' role in the learning process is to acknowledge their students' intuitive problem solving strategies and assist them in connecting new ideas to previously existing knowledge.

As part of the CGI project, a longitudinal study was conducted to examine changes in beliefs and instruction of 21 primary grade teachers who took part in the CGI program over a four-year period. During the course of the program fundamental changes in the beliefs of 18 of the teachers were noted. Their role in the classroom evolved from demonstrating procedures to helping children build on their mathematical thinking by

engaging them in a variety of problem-solving situations and encouraging them to talk about their mathematical thinking (Fennema & Carpenter, 1996). For every teacher who participated in the project, class achievements in concepts and problem solving were higher at the end of the year, with no overall change in computational performance. By taking the focus off computational skills students continued to learn the basic procedures but also gained a greater understanding of mathematical processes.

Teachers involved in this study reported that they became more aware of their understanding of children's thinking and that this cognitive knowledge continued to be refined as they applied their skills in the classroom. The lessons learned about teacher education demonstrate that just as students construct their knowledge based on experiences in the classroom, so teachers expand and refine their knowledge in a process of practical enquiry focused on their own teaching. Ultimately the classroom becomes a dynamic learning environment for both the teacher and the student as each participant engages in a knowledge building process.

The CGI perspective on teacher change has met with widespread success and has developed into a well-established program of professional development that is being adopted in many school districts across North America. It continues to evolve as researchers test and evaluate its effectiveness as a teaching model and a vehicle for teacher change. However, despite its success in providing a framework for change, others would argue (Schifter, 2001) that in order to teach mathematics for understanding at the elementary level there needs to be greater emphasis on developing teachers' general mathematical knowledge.

The Knowledge Perspective

Schifter (2001) argues that to improve mathematics education teachers must enter classrooms with stronger mathematical backgrounds. She disagrees with the routine assumption that elementary mathematics is so simple that any educated person can teach it. In fact, she suggests that teachers often lack basic understanding of mathematical processes because they were never given opportunities to develop this type of mathematical knowledge in school, or in their own teacher education program. Traditional instructional methods provided structures and formulas but did not focus to a large extent on reasoning and logic. As a result, teachers generally enter the classroom ill equipped to promote the reform agenda or understand its philosophical base. Schifter (2001) believes that elementary school teachers need opportunities to investigate topics such as the structure of the base-ten number system, the meaning of basic operations, the logic of rational numbers, and the properties of geometric shapes. They also need opportunities to explore and make connections between mathematical concepts so that they can relate these ideas flexibly with their students.

Beyond the actual mathematics content, Schifter identifies four important skills mathematics teachers need in order to promote deeper understanding of the mathematics being taught and to develop an appreciation of students' mathematical thinking. The first skill is being able to attend to the mathematics in what the students are saying or doing. This involves analyzing written narratives of interactions or viewing videotaped classes that allow the teacher to focus entirely on the mathematical ideas being presented. The second skill is being able to assess the validity of student ideas and evaluate the soundness of invented student procedures. In order to do this, teachers need to be able to

discern the student logic in problem solving. If a teacher determines that a student's strategy is mathematically unsound she needs to be able to identify where the strategy came from and isolate the errors in logic in order to identify strengths as well as weaknesses. This ability gives the teacher clues about the child's level of thinking. The last skill noted by Schifter (2001) is the ability to identify the conceptual issues individual students are grappling with so that instruction can be tailored to individual needs.

Warfield (2001) argues that in order for teachers to plan effectively in a constructivist classroom, they need both a detailed knowledge of students' cognitive processes and a strong mathematical content knowledge. He advocates for the type of training teachers receive in a program such as CGI to allow them to gain valuable insights into the thinking processes being used by their students. However, in order for teachers to pose thoughtful questions and critically examine the validity of their students' thinking they need a sound knowledge of mathematical ideas. If they are to create learning tasks that challenge students to extend their thinking, they need to understand fully the implications of the mathematical concepts they are presenting. Simon (2001) takes this relationship between cognitive and mathematical knowledge a step further. From an examination of classroom processes, he notes that teachers equipped with this dual knowledge base are able to examine learning episodes critically. Through a reflective process they can then develop and test hypotheses that inform and shape practice. However, this scenario is only possible if teachers have confidence in their own mathematical knowledge structures and have an in-depth knowledge of higher mathematical ideas.

Fennema and Franke (1992) examined the issue of teachers' mathematical knowledge at great length. They agree with the notion that elementary teachers generally lack a detailed knowledge of broad mathematical principles, but they believe there is little evidence to support the idea that the level of teacher knowledge in mathematical principles is directly related to student performance in mathematics. Although knowledge of mathematics is important, it is the way in which that knowledge is passed on to students that is the key to student learning. Fennema and Franke (1992) argue that the way teachers organize their own mathematical understandings, and the manner of knowledge transmission to students through classroom structures and activities, provides the basis for student understanding. What is important for teachers is to be aware of their own knowledge base and be cognizant of their students' knowledge and understanding as they plan for instruction.

Knowledge of the subject matter aids the teacher in establishing agendas and scripts for lessons, particularly in deciding what representations of the mathematical topic to use...however, subject matter itself is not a primary determinant of teaching behaviour. Instead the rich repertoire of agendas and scripts built over time (with reflective practice), determine instructional methods (Fennema & Franke, 1992, p. 158).

The ability of the teacher to interpret and organize mathematical knowledge, and impart this knowledge to students, requires teachers to examine the classroom structure and the learning culture established within the classroom. A constructivist approach to learning mathematics, according to The Standards, places great emphasis on establishing a social climate of cooperative learning, exploration and negotiation. Some researchers

would argue that lack of teacher knowledge and skill in facilitating this type of learning has undermined the effectiveness of the reform movement (Wood & Turner-Vorbeck, 2001).

The Sociological Perspective

Wood and Turner-Vorbeck (2001) extend the notion of teacher change in mathematics education to examine the social learning modes in reform classrooms. They contend that the new teaching methods require more of teachers than merely acquiring content knowledge, or pedagogical content knowledge. They argue that reformed teaching involves creating modes of social interaction where students engage in the process of inquiry, share information and ideas, explain thinking, and present challenges. Through this group process, meanings are negotiated and validated and a common ground of mathematical thinking is developed. The authors believe that the skill with which the teacher is able to orchestrate this social learning process influences the type and depth of mathematical knowledge the students will acquire. This theoretical perspective is based on three central tenets from psychological and sociological theory. The first is that to understand teaching one must examine it in conjunction with students' activity in the form of social interaction. Secondly, teaching has to do with the development of meaning, both individual and collective. Finally, teaching is about enabling students to acquire accumulated knowledge of the culture.

Wood and Turner-Vorbeck's (2001) theoretical framework is based on the outcome of research conducted in early primary classrooms over a period of ten years. The classrooms were highly interactive in nature and used constructivist theory to develop instructional activities. The goal was to develop mathematics programs in

accordance with the NCTM Standards (1989). From observations of student and teacher interactions, Wood and Turner-Vorbeck (2001) developed a theoretical framework identifying three distinct social interactions or discussion contexts that take place during a math class problem-solving activity. These three contexts, *Report Ways*, *Inquiry*, and *Argument* (p. 190) reflect the reasoning skills advocated in the Standards. Within each context two dimensions were noted, one of thinking and the other of participation. The role of the teacher is to set the context and then, through careful direction and questioning techniques, encourage participation and stimulate thinking. From the classroom examples, Wood and Turner-Vorbeck (2001) demonstrate the process of activity and thought taking place in elementary mathematics classrooms. In their framework the teacher plays a key role as facilitator, encouraging participation, directing and evaluating thinking activities, and providing feedback to participants. How well this is handled affects the type and depth of mathematical learning within the classroom.

Despite its current acceptance as a theory of learning, constructivism presents great challenges for classroom teaching and organization. Much of the research on constructivist models of teaching and learning has focused on small group situations and has not acknowledged the practical application and the current realities of teaching in a large multi-dimensional classroom. Also, determining a balance between what knowledge students are given and what they should “discover” has been recognized as a fuzzy area needing further research and discussion (Richardson, 2001). These are two very important areas in the constructivist approach to teaching mathematics that still need to be addressed and they are issues that probably create the greatest tension between the

reform movement philosophy and the desire to retain a traditional transmission approach to mathematics education.

An Integrated Model of Teacher Change

Clearly, viewing teacher change from simply a psychological, mathematical, or sociological perspective is insufficient in determining the type of knowledge teachers need in order to alter their beliefs and practices. It is useful to examine them in isolation to fully understand the demands placed on mathematics teachers if they are to engender change. However, each type of knowledge base does not operate in isolation. Having a broad mathematical knowledge does not override the need for grounding in cognitive theory and pedagogical content knowledge. Together these are insufficient if the teacher is struggling with creating a social context for learning. The reforms envisioned in the Standards require an integrated knowledge base (Fennema, 1993) that is not static but is constantly evolving and adjusting to the unique learning environments developed in individual classrooms.

Accepting the idea that knowledge is the vehicle for teacher change, and that the type of knowledge teachers need is multifaceted, one must then ask how this knowledge is acquired. Traditionally professional development for teachers has been behavioral in orientation. The goal was to help teachers assimilate new techniques into an existing system of ideas about pedagogy and content knowledge. The system of ideas was rarely in question. Research conducted in mathematics education was largely quantitative in nature, comparing teacher input to student performance in the form of standardized test scores. This has been labeled the process-product paradigm (Nelson, 1997; Richardson, 2001) and was based on a transmission model with the teacher being the vehicle for the

transmission of knowledge. Professional development focused largely on teacher behaviours while transmitting a set body of mathematical ideas. It attempted to identify the most effective set of procedures for the transmission of knowledge and skills. The reform movement not only calls for dramatic changes in the way mathematics is taught but also advocates changes in the way professional development and teacher training is conducted. This advocacy was made clear with the publication of the *Professional Standards for Teaching Mathematics* (1991) that presented six standards for the professional development of mathematics teachers and provided a vision of teacher education in the field of mathematics:

Teaching mathematics is a complex endeavor. It demands knowledge of mathematics, students and teaching as well as opportunities to apply this knowledge in a variety of field base settings. It requires an understanding of the impact that socioeconomic background, cultural heritage, attitudes and beliefs, and political climate have on the learning environment. Above all, it entails a developing personal knowledge of oneself that combines sensitivity and responsiveness to learners with the knowledge, skills, understandings, and dispositions to teach mathematics (p. 123).

The NCTM vision of teacher education represents a paradigm shift away from the process-product examination of teacher behaviour. Instead it focuses on teacher thinking, examining the content of teachers' mathematical ideas and beliefs, and how those ideas relate to the decision making processes in the classroom (Nelson, 1997). The classroom is not viewed as a static entity where input can be controlled and output is measured reciprocally, but rather as a dynamic collection of individuals engaged in an ever-

evolving process of learning. The teacher provides the direction but is also directed in her decision making by the learning process. In this type of teaching situation the teacher brings knowledge and skills to the classroom, but the teaching decisions made are based on the unique learning situations created by the collection of individuals in the classroom. Consequently, there are no guidebooks for this type of practice, the teacher makes pedagogical decisions based on her knowledge of the subject matter, knowledge of students' abilities, and her skills in orchestrating a classroom culture for learning. This approach requires the ability to reflect on practice and use those reflections as a basis for further classroom direction.

Richardson (2001) explored this shift from a behaviourist perspective on teacher education to one that reflected the constructivist approach to student learning in the classroom. Instead of teachers being “taught” constructivist methods that they can apply in the classroom, a recent trend in the professional development literature is advocacy for teachers to learn within a constructivist model themselves (Hargreaves & Fullan, 1992; Kochendorfer, 1994; Ritchie & Wilson, 2000). This requires teachers to use their classroom environment to create a knowledge base to examine and refine their teaching practice. It is based on reflective practice in a community of learners, who share their ideas and develop knowledge within their particular educational frameworks.

This idea of professional development is radically different from the traditional approach. It not only challenges teachers to examine and change their learning processes but it also challenges educational authorities to alter their practices of teacher education and evaluation. It places greater autonomy in the hands of teachers but it also requires teachers to become actively and continuously involved in their own learning, and

encourages teachers to take ownership of their learning situations. Knowledge is not only received from external authorities but is constructed as it is applied in the teacher and student learning environment. This process of reflective learning forces teachers to examine their own knowledge, beliefs, and practices, which ultimately reaches the heart of the reform movement. If teachers are not given the opportunity and freedom to examine the nature of their teaching, or given the tools and skills to create new models of teaching, then the process of reform will be severely curtailed (Darling-Hammond & Sykes, 1999; Kochendorfer, 1994; Wells, 1994).

The research community has provided the Professional Standards that can guide the teacher change process (NCTM, 2000; NCTM, 2003). However, there is no simple guidebook for change, due to the very nature of the change that is being required. Change in mathematics education requires not only a personal redefinition of belief structures but also a structural one. If teachers are to make changes they need to be knowledgeable about the issues of reform but they also need the administrative supports that will value and foster change.

There is a growing body of research literature that has examined the way in which educational authorities and individual teachers interpret the reform standards (Fennema & Nelson, 1997; NCTM, 2003). Models for teacher change are being developed that provide guidance in how reform principles in mathematics education can be addressed and implemented. One such model is the CGI teacher-training program mentioned earlier, that focuses on the understanding of students' mathematical thinking (Carpenter et al. 2000). From its research base, it has evolved into a successful program used to assist teachers in understanding the reform process. The Wisconsin Centre for

Educational Research continues to support the development of this teacher education program, and its training facilities have been used by a large number of school districts in the area to train teachers in its methods. Participating teachers and school districts have also become part of the ongoing research and evaluation of the program. Researchers provide classroom support and feedback to the participants. With the aid of a mentorship program, participants are encouraged to develop learning centres within their own schools to promote continuous teacher learning. The program is not a plan of action but provides a framework of knowledge about student learning that assists teachers in making instructional decisions.

A regular newsletter published by the centre provides a forum for on-going dialogue between practitioners, administrative personnel, school authorities, and parents who are engaged in the reform process. From the various accounts of practice experience (Cognitively Guided Instruction & Systematic Reform, 2000), it is evident that the reform movement is a process that is continually evolving as educators examine, evaluate, and refine their practice. Its success is also dependent on a commitment by the participants to develop collegial support networks to foster mutual growth and development. Through the centres' publications school authorities are able to share their experiences regarding teacher change initiatives and this serves to inform the research community as the reform movement progresses.

An example of a change initiative outside the North American experience is the Early Numeracy Project (ENP) currently sponsored by the Ministry of Education in New Zealand. This professional development program is similar to the CGI program and is designed to provide teachers with "an effective means to assess students' current levels of

thinking and provide guidance for instruction” (Gill, Tagg, & Ward, 2002). After a three-year implementation period that included 3300 teachers and 64000 students, an impact study was undertaken to assess the effectiveness of the program. Information was gathered through questionnaires sent to 246 participating teachers at 50 randomly selected schools regarding the usefulness of the training and the effects on teaching practice and students’ mathematical abilities. Student achievement was also measured using an assessment tool developed for use in the ENP. Feedback from participating teachers was deemed very positive. Ninety-six percent of the teachers believed that their knowledge of how children learned mathematics had been developed because of their participation in the project. They credited this knowledge with a belief that their teaching had become more effective with 92% of responding teachers indicating changes in the way they teach mathematics.

Student achievement in all areas of the early mathematics program showed significant improvements and this was consistent across gender, ethnic, and socio-economic groups. These very positive results suggest the program of professional development undertaken by the Ministry of Education was successful in accomplishing its goal of improving student performance in mathematics. The report concludes that by empowering teachers with knowledge and understanding of students’ mathematical thinking, students are the ultimate beneficiaries. A cornerstone of the program was the provision of mentors who were available to provide materials, advice, and demonstrations to schools and classrooms. There were also regular feedback sessions where teachers could report on their classroom progress and discuss challenges within practice. This aspect of professional collaboration and support during the change process

is a theme running through much of the literature on successful reform programs. It highlights the socio-cultural element of a constructivist approach to learning, and it appears to have as much relevance to teacher learning as it does to student learning in the classroom.

The importance of developing collegial support networks to promote mathematics reform is highlighted in the research conducted by Gamoran (2002). In his multi-year study with colleagues at the National Centre for Improving Student Learning and Achievement in Mathematics (NCISLA), he identifies the factors that contribute to a school's capacity to advance and sustain reform in mathematics education. The study was based on the premise that professional development is the engine of change, and that in order to sustain reform ideals, there is a need for ongoing professional development. The researchers followed the progress of 102 teachers as they participated in professional development programs conducted over three years. Forty-two district and school administrators were also included in the study.

Information gathered by Gamoran (2002) provided the basis for a new model of professional development that differs substantially from the traditional resource input and achievement output model, which Gamoran (2002) called the "black box" model. This process was essentially linear in nature and examined the quantity of input and the correlating student output. In Gamoran's new model professional development is regarded as dynamic and multi-directional. It does not exist in isolation as a separate entity, but is constantly evolving due to a complex series of relationships between teachers, professionals, administrators, and teachers' perceptions of student learning and understanding.

Gamoran's (2002) notion of teaching resources is at the core of this new model and he divides these into three types. He identifies the traditional resource materials necessary to assist in change, but he emphasizes the importance of human and social resources in the change process. Human resources include teachers with particular expertise in certain areas of math education that can share their knowledge with their colleagues. This resource also includes outside professionals who can be used for referral and assistance in implementing new ideas. The social resources referred to by Gamoran provide the essential link in the professional development model and are perhaps the most difficult to cultivate. They represent the communication structures between teachers, between teachers and other professionals, and between teachers and administrators. A well-developed communication structure built on trust and collaboration creates a sense of community and results in the exchange of human and material resources. This exchange allows groups to negotiate a common purpose and develop shared norms. Thus, professional development becomes the product of active professional communities working collectively to refine and enhance the teaching and learning process, both for themselves and their students.

Franke and Kazemi (2001) also acknowledge the importance of developing communities of practice to support on-going reform. Their research focuses on how teachers sustain generative growth after they have participated in training and professional development programs. They define generative growth as "on-going practical enquiry". Their study follows a group of elementary school teachers during their four-year training session with Cognitively Guided Instructional (CGI) methods and the following four years of their practice. Franke and Kazemi characterized teachers

demonstrating generativity as having detailed knowledge of their students' mathematical reasoning, and capacity to constantly test and revise their knowledge. These teachers use their knowledge to create connections across students thinking, mathematics, and pedagogy.

The authors also note the socio-cultural aspect of generativity. Teachers work within a community of practice and shape their identity as learners as they interact with others in this community. Thus generativity is not just an individual process but one that takes place within a group, as teachers share, discuss and analyze their private acts of teaching. As a result, teacher knowledge is characterized as constantly evolving and adapting as new information is collected, applied, evaluated, and refined on a personal level and also within a community of learners. The community supports teachers but the teachers must be willing to continuously challenge themselves as learners in their own classrooms in what Franke et al. (1997) term practical inquiry.

The shift in thinking, outlined in the Standards, from a teacher-centered mathematics classroom to a student centered learning environment requires teachers to continuously reflect on their practice and make decisions based on knowledge generated within the classroom. What this looks like in practice has been a particular focus of research in recent years as teachers and researchers have struggled to articulate this mode of change. Teachers in effect have become researchers in their own classrooms, collecting data and interpreting information to develop a better understanding of student learning that in turn supports teacher learning.

Summary of Teacher Change

The research on teacher change provides many vignettes from teachers learning in and from practice (Fennema & Nelson, 1997; Wood et al., 2001). The CGI initiative at the University of Wisconsin has been particularly active in promoting this area of investigation. Ruth Heaton (2000), a respected teacher educator and classroom teacher, provides a detailed account of her attempts to change the way she teaches elementary mathematics. She describes her own learning process while she examines the student learning taking place in the classroom. Heaton demonstrates how learning mathematics in the context of practice is different from learning methods of teaching mathematics. It involves the complex interplay of the teacher, students, and the subject matter. The script is developed through reflective practice and is a continuous state of change.

These stories of teacher change demonstrate the difficulties of implementing the reform curricula envisioned in the NCTM Standards. The very nature of the reforms and the accompanying constructivist philosophy preclude the development of an instruction manual or road map. As the Standards are adopted, educational authorities are faced with interpreting the vision of change individually and applying the principles of the reform movement to the local situation. Teachers entrusted to carry out the reform agenda in their classrooms need new types of knowledge in order to understand the goals of reform and participate in it. This new knowledge is multifaceted and requires greater understanding of content knowledge, knowledge of children's cognitive processes, and an understanding of socio-cultural aspects of learning.

The research indicates that teachers play a pivotal role in creating the environment necessary for developing the thinking skills students need in order to understand

mathematical processes. As such, the reform movement depends heavily on teachers acquiring the knowledge and skills that foster Standards based mathematics learning. Such changes do not happen automatically; they require learning. Heibert (1999) notes that just as students require an opportunity to learn, teachers need similar opportunities. Unless such opportunities are provided, teachers are asked to do the impossible – teach in new ways without being given the opportunity to learn them. Heibert (1999) summarizes the research on teacher learning and shows that fruitful opportunities to learn new teaching methods share several core features:

- (a) ongoing (measured in years) collaboration of teachers for purposes of planning with (b) the explicit goal of improving students' achievement of clear learning goals, (c) anchored by attention to students' thinking, the curriculum, and pedagogy, with (d) access to alternative ideas and methods and opportunities to observe these in action and to reflect on the reasons for their effectiveness (Heibert, 1999, p. 15).

By providing teachers with such a learning environment, educational authorities would be well on their way to developing the human and social resources identified by Gamoran (2002) as crucial in sustaining the reform process in mathematics education.

Educational Change and Teacher Development

Research on teacher development in the mathematics community reflects the trend towards a more holistic approach to educational change and teacher development in the wider educational community. Research into educational change (Fullan & Hargreaves, 1992; Hargreaves & Fullan, 1992), suggests that in order for any lasting educational change to take place, school authorities need to alter the way in which

professional development programs in general are initiated and implemented (Wells, 1994). The term “teacher development” is now often viewed as having negative connotations, referring to something done to teachers to address deficits in skills and knowledge (Jackson, 1992). In the past, it has generally been a process imposed on teachers from above, relying heavily on the certainty of educational research findings (Fullan & Hargreaves, 1992). This process, although efficient in disseminating information, has often ignored the individual needs of teachers who were attempting to implement new programs. Fullan & Hargreaves (1992) contend that much time, energy, and resources were put into this form of professional development but the expected results were not realized or sustained over time. They suggest the reasons for the failures were that teachers were not given a voice in the process. They were expected to carry out reforms in a uniform manner but were given little opportunity for input, and their expertise, gained through experience was not recognized as valid data.

Action Research and Teacher Development

Research into constructivist learning suggests that sustained improvement is more likely to occur when teachers are reflective learners (Kochendorfer, 1994; Patterson et al. 1993). This entails careful self-examination of teaching practice and instructional decision-making based on the knowledge gained during daily classroom interactions with students. This form of teacher development is a very personal process and requires the teacher to take both a subjective and an objective view of classroom events. Through examination of classroom events the teacher not only gains understanding of the students’ knowledge, but also develops her own knowledge of student and teacher behavior. When teachers collaborate and share the accumulated knowledge of their personal learning

processes, they are informing others and constructing pedagogical knowledge. In the field of mathematics, collaborating teachers gain insight into children's understanding of mathematical processes, which informs and supports decisions made in educational practice.

The purpose of this research is to examine the process of interactive learning that can develop between the teacher and students in an elementary mathematics classroom using a constructivist approach to learning. I will examine the connection between student learning and teacher learning using an action research method. The study takes place over a three-week period during an introductory unit on the multiplication process at the grade two level. The content of the unit is based on the requirements of the BC Mathematics Curriculum. However, the processes used to develop student knowledge attempt to incorporate a constructivist approach to learning. These processes call for the teacher to be responsive to student learning and constantly adapt instruction to meet the needs of the students. Through reflection, adaptation, and modification the teaching experience becomes a learning process for the teacher and an integral part of her professional development. The NCTM Professional Standards (1991) place great emphasis on the reflective teaching model and an active, inquiry-based approach towards professional development. The successful implementation of a Standards based classroom requires the teacher to actively engage in the learning process together with the students. This study will focus on this dual learning process.

Chapter Three

Research Method

In this chapter I will discuss the research method used in the study. I will describe the research site, the participants involved, and the general procedural steps in the study. The final part of the chapter will explain the data collection process and how the data was organized for analysis. A more detailed description of the procedural steps will be presented in Chapter 4 where the individual action steps and continuous analysis will be discussed as part of the action and reflection cycle.

Research Design

This is a qualitative study using an action research method of inquiry. It is exploratory and descriptive with the researcher being both observer and participant. It takes place in a naturalistic setting within the researcher's own classroom and is presented as a case study. Exploratory research is used to gather information on a topic or question that is relatively new or unstudied in a particular context. It is not intended to provide definitive conclusions but is useful in finding solutions for local problems and identifying further research questions (Rubin & Babbie, 1997). Action research is used to explore an evolving social situation and it requires active participation of the researcher.

The origins of action research as a method of inquiry can be traced back to the 1930s, and the work of the social psychologist Kurt Lewin (Elliot, 1991; Gold, 1999; Lewin, 1997). From his studies of early childhood behaviour, Lewin, developed and applied action research methods to help identify and provide solutions to complex social problems. Lewin argued that it was not always possible to draw accurate conclusions about an individual's behaviour simply by observing that overt behaviour. In order to

truly understand individual behavior within a group, Lewin believed, the researcher should be immersed in the group, and become attuned to the inner organization and nuances of the social relationships within. He was concerned with the process of social change and group problem solving, and through this he developed the idea of *life space*. He theorized that human behaviour is a product of positive and negative forces competing within the individuals' environment. In order to understand an individual's behaviour within the group at a given time, one needs to understand these competing forces and use them to enact positive social change (Gold, 1999). Lewin's study of social change did not separate investigation from the action needed to solve the social problems being investigated. Lewin's action research methodology and his approach to social change are grounded in the value of the democratic process and notions of egalitarianism. In order for effective change to take place within the group, all members need to be informed about the forces competing within the group and must be given equal opportunity to effect positive change. The researcher then is not only an observer of behaviour but also a catalyst for change.

Elliott (1991) describes action research as "the study of a social situation with a view to improving the action within it...In action research 'theories are not validated independently and then applied to practice...they are validated through practice' (p. 69). When applied in the educational field, action research is not only an observational and analytical tool but also a vehicle to initiate change in the area of inquiry. As such, action research has been used with increasing effectiveness in the last few decades as a process to promote educational reform and to support and encourage teacher development (Elliott, 1991; Hargreaves & Fullan, 1992).

The action research format fits well with investigations into teaching methods and student learning, as it provides a process through which individual practitioners study their own teaching as a means of increasing knowledge, and using that knowledge to improve teaching and learning. Action research is characterized by cycles of problem identification, systematic data collection, reflection, analysis, further action, and problem redefinition (Altrichter, Posch, & Somekh, 1993; Elliott, 1991; McKernan, 1991). The initial problem presented by the researcher is constantly being redefined as the participants interact within their learning environment and new knowledge and understandings are created. This cycle of action, reflection, and action has been referred to as the development of ‘practical theory’ (Altrichter, Posch, & Somekh, 1993; Elliott, 1991). Within this form of inquiry, theoretical abstraction plays a subordinate role to the knowledge developed through reflective examination of practical experiences.

Action research carried out by teachers can contribute significantly to personal professional development. It can also be used to assess more general curriculum development initiatives in practical situations. By putting individual teaching practices under scrutiny the professional teaching community as a whole broadens its knowledge base and thus contributes to the further development of educational theory. In doing so, the action research method allows the teaching profession to reconcile “the strange position of being simultaneously both the subject and agent of change” (Sikes, 1992, p. 36). “It democratizes research by bringing those who are usually ‘subjects’ of research to a position where they have equal rights and responsibilities. In doing so, it ensures the practical relevance of educational theory” (Altrichter, Posch, & Somekh, 1993). This definition corresponds to Lewin’s original conception of action research.

Although the process of change in primary mathematics education has been examined extensively in recent years, the particular conditions relating to teachers and students in Prince George are unique. The local administration has developed an action plan for change that recognizes past experiences within the district and focuses on the current perceived needs of the students in the district. Within this framework each teacher is given the responsibility to interpret the curriculum changes and develop an action plan for implementation within her unique social learning environment. As such, the change process for each teacher will have unique characteristics relating to the social and cultural milieu of each individual classroom.

Data collected from this study, although unique to the particular setting, adds to the body of knowledge already being generated in the field of instructional change in mathematics education. It will not only be valuable to me in my capacity as a teacher from a planning and instructional perspective, but also, in my role as a member of the Math Mentor team for School District 57. This team was appointed in March of 2004 to provide leadership to teachers in implementing the new mathematics program promoted by the district. In my capacity as a Math Mentor, I am responsible for providing programming advice and teaching strategy supports to teachers in the primary grades. Knowledge and expertise gained through my own action research will be invaluable to me in carrying out this role.

Participants

The selection of participants in this action research project was purposive as the research was specific to a particular group of students and a teacher in a pre-determined learning environment. This research took place in my combined grade one and grade two

elementary classroom in College Heights Elementary School in Prince George in the spring of 2004. The class was made up of ten grade one students, and twelve grade two students who ranged in age from six years eight months to eight years three months. There were eighteen boys and four girls, which is unusual, but reflects the general gender imbalance within that particular age group in the school. The district policy of creating heterogeneous class groupings is evident in this particular class as there is a cross section of abilities and aptitudes at the grade two-level. However, the grade one students selected to be in the class were chosen on the basis of their ability to work independently and their higher level of social maturity. Performance indicators (Curriculum Based Measurement), conducted in the fall of 2003 and the spring of 2004 suggested that eight of the ten grade one students were performing well above expectations for their grade level. All the grade two students were performing within the widely held expectations for math at the grade two-level according to the BC curriculum. However, three of the grade two students were performing below grade level in Language Arts and were receiving reading instruction from the learning assistance teacher for three periods each week.

Class composition remained fairly constant for the eight months prior to the research being conducted. There were no changes in grade two, but two students were added in grade one. The first arrived in November of 2003 and the second student registered in April of 2004. The addition of the new students presented some difficulties to the teaching environment as they were less independent and less capable than their peers, and needed more individual assistance from me.

College Heights Elementary is located in a predominantly middle class neighbourhood in a western suburb of Prince George. The school is approximately thirty-two years old and was built during a rapid expansion of the town during a boom period in the local forestry industry. The neighbourhood has been fairly stable for the last twenty years and the school population has remained constant despite the aging of the population. A significant factor for stability was the creation of a dual track French Immersion program in 1986. Children who enter kindergarten in the French track have the opportunity to continue their elementary education in French at all grade levels up to grade seven. This structure has attracted enrollment outside the regular catchment area and sustained the student population numbers. This study took place in an English speaking classroom although two of the members were originally enrolled in the French program.

Research Environment

In my role as teacher and researcher I was both a participant and observer in this project. As the teacher, I was responsible for providing the context of the study and designing and shaping the physical environment. The site development evolved over the previous eight months that I had worked with the students. Some of the schedules that were followed were pre-determined by general school timetables. Classroom routines were based on my own principles of classroom organization and were shaped by curriculum demands of the two separate grade levels. However, patterns of interaction within the classroom emerged over the course of the year and were based on the unique personalities and needs within the classroom.

My teaching style incorporated both large group formal teaching activities and small group, specific skill teaching. During academic activities small group teaching was the norm since there was a wide range of abilities, particularly in the area of reading. During arts and crafts, social studies, science, and some math activities students often worked with partners or in groups of three or four and were often cross-graded. The students enjoyed working with their peers and were accustomed to sharing tasks and working cooperatively. This flexibility of structure that emerged over the year proved to be particularly useful as the students easily adapted to the variety of learning situations developed for this mathematics study.

The research took place in the regular classroom setting during the predetermined periods of mathematics inquiry. The classroom was a simple square shape measuring approximately 30 meters by 30 meters. The student desks were arranged in pairs in the centre of the room with various thematic learning centres along the outer walls. The classroom was on the main floor of the building with two exit doors, one to the main hallway and one to the outside. The only window in the classroom was the one on the upper half of the outer door. In one corner of the room there was a large carpeted area measuring four meters square. The students gathered in this area every morning for opening activities, which included daily school information and an overview of the day. It was also used for calendar activities and at various times throughout the day for group meetings and discussions. The students generally sat on the carpet in a circular arrangement. Models of appropriate social interaction such as turn taking, practicing good listening skills, and responding respectfully to questions and comments had been practiced throughout the year and were constantly being reviewed. This social skill

training was also developed as part of the BC Personal Planning curriculum and proved to be compatible with, and supportive of the Professional Standards for Teaching Mathematics (NCTM 1991).

In my role as researcher it was necessary to collect data during instructional time, and the students were fully aware of the purpose of my project. I explained to them that tape-recording our conversations would help me understand how we learn new ideas in math and consequently help me plan more effective classroom activities. The students were excited about the prospect of being taped and were keen to hear recordings of our classroom interactions. Their eagerness to listen to themselves and their comments about their performances in the recorded discussions became an interesting dynamic in the data collection process that was not anticipated at the outset of the investigation. The students were also aware that their parents had to give consent for their participation in the research project, and I discussed the contents of the consent form with the students.

Procedures

This research took place over approximately a three-week period during the daily allocated 45 minute mathematics period, which amounted to approximately 15 lessons. However, as the sessions progressed classes were extended or modified as the activities developed. One of the goals of the research was to demonstrate to students how math principles are evident in real-world contexts, and to highlight the connections between numeracy and literacy. Despite provincial requirements regarding specific daily time allotments for core curriculum subjects, the philosophy of the British Columbia Primary Program (2000) also recognizes the overlap in primary curricula and encourages the integration of subject matter. Thus, specific time allocations were not followed rigidly

and lessons were developed daily according to the students' needs and the outcomes of the previous lessons. Having clear lesson goals is important, but flexibility is also necessary during lesson development as it allows the teacher/researcher to assess the learning situation and make planning decisions based on unique learning events that take place during classroom discourse.

The unit plan for introducing multiplication at the grade two level was developed based on the requirements of the BC Mathematics K-7 Curriculum (Appendix B). The plan also reflects the philosophy of the BC Primary Program (2000), which provides a framework for instruction at the primary level. This framework is based on the view that mathematical knowledge is much more than the ability to perform mathematical skills and procedures. It requires that students become numerate by developing "a sense of number, space, probability, pattern, and relationship that enables them to see mathematics in all aspects of their lives" (p. 147). A model for introducing the concept of multiplication, designed by Marilyn Burns (1991), was used to develop the unit plan. This model employs a combination of whole group class activities and discussions as well as small group exploration. In addition to these activities, principles of Cognitively Guided Instruction (Carpenter et al., 1999) were applied to the instructional methods together with strategies developed by Van De Walle (2001). The unit plan provided opportunities for students to engage in hands on activities using manipulatives that were expected to promote the understanding of the multiplication process. It also provided opportunities for students to apply their knowledge of multiplication in familiar problem solving situations. The unit culminated in a language arts activity where students collaborated to create a class book of multiplication word problems. The collection of

problems served to demonstrate student understanding of the multiplication process and also provided the literacy connection for numeracy skills. The activities in the unit plan also represents the initial actions in the action research model.

I developed an organizational framework in the form of a data matrix modeled after Sagor (1992). The matrix is a visual organizational tool that represents the relationship between the action steps, the data sources, and the themes that emerged from the analysis (Appendix A). A sample of the matrix is shown in Figure 1.

Figure 1. Action Plan and Data Collection Matrix

ACTION STEPS	DATA COLLECTION
<p>AS 1. WHAT IS MULTIPLICATION?</p> <ul style="list-style-type: none"> • elicit student information • class discussion • introduction to journal writing • journal assignment <p>Curriculum Standards 2, 3, 6, 13</p>	<p><u>Tape 1A</u></p> <ul style="list-style-type: none"> - class discussion <p><u>Student Journals</u></p> <ul style="list-style-type: none"> - individual accounts of mathematical knowledge <p><u>Class Chart</u></p> <ul style="list-style-type: none"> - student generated ideas <p><u>Teacher Journal</u></p> <ul style="list-style-type: none"> - after class notes and general impressions
<p>THEMES FROM ANALYSIS</p> <ul style="list-style-type: none"> • communication • oral / written • using language ‘versus’ knowledge of language 	

Each large box in the matrix is divided into three sections. The top left section identifies the topic of each lesson and the action steps (AS) or activities that took place in the classroom during the implementation of the unit plan for teaching multiplication. The topics are numbered in the matrix AS 1 – AS 10. The long section to the right highlights the sources of data that were collected during each lesson. The bottom left section lists the themes that emerged as the data were reviewed and analyzed.

The use of italics in the action steps highlights areas of revisions or additions to the original action plan after reflection and analysis. These revisions tended to increase in frequency as the cycle of action and reflection progressed. For each of the action steps indicated, specific NCTM Standards were identified, numbered and used as guiding principles in the lessons (Appendix C). However, not all the Standards were applicable to this unit of study and therefore are not referred to in the action plan matrix. For this unit of study I chose to focus on certain key Standards as I believed they represented the underlying philosophy of the whole NCTM mathematics reform movement. These are: Standard 1: Mathematics as Problem Solving, Standard 2: Mathematics as Communication, Standard 4: Mathematical Connections, and Standard 7: Concepts of Whole Number Operations.

Data Collection

Data was collected from four principal sources: a daily teaching journal, student journals, students' daily work samples, and audiotapes of teaching episodes. The audiotapes were replayed later and notes were made to assist in analysis and evaluation. Teacher field journal entries recorded highlights of teaching events, impressions of how the teaching episodes progressed, and reasons for the decisions made during the emerging research. The teacher's field journal was also supplemented with observational notes and anecdotal records when the classroom situation allowed. This was only possible when students were engaged in independent small group activities and the teacher moved between the groups to offer advice and give direction. The large group lessons were audio taped along with less formal mathematics sessions, where students were beginning to use their mathematical knowledge to apply their skills in other daily activities. The

decision to audiotape classroom discussions other than specific math lessons was made during the course of the study. This was an unplanned use of the audiotape but proved to be an additional source of data that supported and highlighted the math lesson objectives and mathematical applications in other curriculum areas. Students also kept journals to record their ideas as they learned new skills. Daily work samples and students' written work were also collected in order to evaluate student understanding of the mathematical principles as they were introduced.

Operationally, data collection from multiple sources is known as triangulation. This method of data collection addresses issues of reliability and allows the researcher opportunities to analyze the data from a variety of perspectives (Altrichter et al 1993; Elliott 1991; Miles & Huberman, 1998; Sagor 1992). Denzin (2000) states "credibility, validity, and reliability in action research is measured by the willingness of the local researchers to act on the results of action research" (p.96). McMillan & Schumacher (1997) define reliability in action research as "the extent to which the results approximate reality and are judged to be trustworthy and reasonable (p. 608). Thus, the collection and analysis of data from multiple sources increases the level of trustworthiness, and decisions about further action steps can be made with greater confidence.

By collecting and comparing different types of data on a particular theme, the researcher is able to provide a more objective analysis of the events and this reduces the possible effects of researcher bias. The need to collect multiple sources of data in a teacher initiated action research project is of the utmost importance because systematic records are created. Decisions made throughout the process are based on the

reconciliation between the various sources of data rather than spontaneous impressions made by the teacher and researcher.

The process for collecting multiple sources of data was developed in conjunction with the general teaching plan. This is detailed in Appendix A (Action Plan and Data Collection Matrix). The bold print identifies the sources of data followed by general comments on the content of the data. The classroom recordings were labeled Tape 1A, Tape 1B up to and including tape 9B. After the teaching sessions, notes were made from the contents on the tapes and pertinent details, themes and ideas were referenced with the counter number on the tape recorder.

Analysis of Data

The nature of action research requires that data is constantly being collected and examined, and subsequent actions are based on the findings and interpretation of that data (Elliott, 1991; McKernan, 1991; Patterson et al., 1993). However, as the data accumulated over time, my analysis focused on identifying themes in the data. These themes became apparent as the tapes were analyzed and compared with the students' individual work and the teacher's impressions that were recorded daily. As each theme emerged it was charted in the data matrix and evidence was recorded in the form of references to specific tapes segments, student work samples or teacher's notes. A sample of this method of analysis is provided in Figure 1.

The data matrix evolved as the research progressed, and it proved to be a useful organizational tool for analysis. For each action step, the sources of data were identified on the matrix and the information could then be analyzed from a variety of perspectives, thus addressing issues of trustworthiness. After reviewing the tape recordings, specific

numerical references were recorded identifying recurring themes and pertinent classroom discourse. Italics were used in the matrix to identify changes to the original action plan and highlight informal classroom discussion that demonstrated student understanding in the context of the study. Overall the matrix functioned as an organizational instrument outlining my curriculum goals, identifying the Standards that I wanted to focus on and recording the data sources for analysis. After examination of the data I was then able to identify and record recurring themes on the matrix for each action step.

The complete Action Plan and Data Matrix can be found in Appendix A. It demonstrates the triangulation of data collection and data analysis.

Ethics

This research took place in the regular classroom setting during instructional activities designed to deliver specific learning objectives as required by the Provincial Mathematics Curriculum. However, since the data collected was to be reported outside the regular school district evaluative process, it was necessary to obtain several levels of approval in order to address ethical issues.

All classroom research in School District 57 requires the approval of the senior school district administration for programming. In order to receive this approval an application was made using the standard district application form, outlining the goals and objectives of the research. I then participated in a formal interview with the senior administrative officer, Bonnie Chappell. Evidence of curriculum compatibility was assessed and a copy of the parental permission form was approved (Appendix D). Approval from the school principal was then necessary in order to contact the parents of

the students and receive permission to use student data beyond the normal scope of the classroom evaluative process (Appendix E).

A letter explaining the nature of this research and a consent form was then sent to the parents of all the students involved. This letter explained how confidentiality would be maintained (Appendix F). For reporting purposes, all student participants were assigned pseudonyms known only to this researcher. All audio taped recordings of the teaching sessions were kept in a locked cupboard at my home and were erased after transcription. Data collected in the form of student journals and work samples were analyzed, and student samples used in the reporting of the results were used with pseudonyms. They became the property of this researcher until they were returned after analysis, to the students participating in the research.

The Ethics Review Committee of the University of Northern British Columbia granted approval for this research in May of 2004 (Appendix M).

Chapter Four

Results

Due to the nature of action research with its cycles of action, reflection and further action, a description of specific procedures in the research will contain continuous analysis and reflection. All actions in this type of research are dependent on previous reflection and analysis, and information gathered from these activities determines the next set of action steps (Elliott, 1991; McKernan, 1991). In this chapter of reflections on the data collected, I will examine the process of teacher planning and student and teacher learning that occurred as I introduced the concept of multiplication to a group of grade one and grade two students. The action plan that is detailed in Figure 1 (Appendix A) was developed to address the learning objectives required in the BC mathematics curriculum, outlined in Appendix B. The methods and activities implemented to meet these learning objectives were designed to incorporate, as much as possible, the NCTM Curriculum Standards for Teaching Grades K-4 Mathematics and the NCTM Professional Standards for Teaching Mathematics (Appendix A and Appendix B respectively). However, with any action research initiative there is an overall goal, and as the research progresses, modifications are made to the action plan based on information gained during the procedures. This cycle is compatible with the constructivist approach to teaching, which recognizes the highly personal and individual process of knowledge acquisition. The discussion relating to the data gathered for this research will demonstrate the cyclical nature of the learning process for both the teacher and the student.

The inclusion of the grade one students in this research project was based on previous knowledge of their general abilities and aptitudes. As mentioned previously, the

majority of the grade one students in this particular class were performing well above expectations in all academic areas, and were often integrated into cross grade activities. Even though the curriculum does not expect a formal introduction to multiplication algorithms at the grade one level, topics they had experience with, such as patterning and counting in multiples, provided background knowledge and allowed them to participate in the group activities. The quality of work samples from a number of these students equaled and sometimes surpassed the expectations for the grade two students.

This chapter systematically describes the series of action steps that evolved during the teaching of the multiplication unit. These action steps are identified in the Action Plan and Data Collection Matrix (Appendix A) and are labeled AS 1 through to AS 10. The discussion provides the objectives for each action step and examines the responses and reactions of the students involved. The analysis of student performance provides the rationale for the development and implementation of subsequent action steps by the teacher. The analysis of the classroom episodes is included to provide insight into the teacher learning taking place during the cycles of action and reflection.

Before embarking on this narrative description of the teaching and learning process, it is important to highlight an aspect of daily classroom activity that was on-going, but not directly planned in relation to this unit. This activity, known as the Morning Meeting, covered a variety of topics and was an integral part of my classroom organization. During this study it proved to be a valuable indicator of student performance and mathematical knowledge.

The Morning Meeting

The Morning Meeting was a daily activity that evolved over the course of the school year. It served a variety of housekeeping functions related to various aspects of the school and classroom organization. It was conducted for about twenty to thirty minutes at the beginning of each school day. This type of forum is a fairly common component of the primary program in many of the elementary schools in Prince George, but its significance varies depending on teaching styles and educational philosophy. I had purposely developed this forum to incorporate various goals of the BC Primary Program (2000), and to promote the integration of a cross section of curriculum objectives. These included speaking and listening skills, social discourse, and general problem solving skills related to student concerns. In addition, I developed this session as a time to review math skills and apply them in specific meaningful ways, to promote NCTM curriculum standards and provide the students with opportunities to communicate mathematically and make connections between different topics in mathematics. It was through this forum that I was able to determine students' initial level of understanding about the multiplication process before it was formally introduced.

The Morning Meeting took place in the meeting corner at the back of the classroom. All the activities during this session were conducted orally with some of the counting activities performed in unison. Some activities were conducted in a question and answer format with individual students taking turns to solve the problems posed by the teacher. All the activities used large visuals that all the students could see and manipulate easily. The students sat in four rows on the carpet facing the bulletin board,

which had a large calendar with detachable numbers. Each day a student would clip the correct date to the calendar and the class would recite the date.

I also used the large calendar to develop student knowledge regarding aspects of time. Students would often be asked questions such as “What will the date be on the third Thursday?” or “How many days/weeks until...?” The calendar was used to find patterns in the number lay-out, in particular the students were informally introduced to groups of seven and the number sequence 7, 14, 21, 28. Next to the calendar the selected student added to a tally that recorded the number of days already passed in the month. As each tally of five was completed the students had an opportunity to count in fives. During the month, bundles of five were combined to create tens, and every month the students were able to count up to thirty in fives and tens. Each day the students were asked to come up with as many ways as they could to create the number on the date. If the date, for example was the 15th, they would give answers such as $7+8$, $6+9$ or $16-1$. The students would use the large visual calendar to assist in their calculations. This particular activity became very popular with several of the grade two students and they started a competition amongst themselves at the start of each day to see who could be the first to write out ten ways of making the daily number.

Next to the large calendar was a hundred chart on which the assigned student recorded the total number of days in school. Each day a new square was coloured to represent the number of days in the year that the students had attended school. Base ten blocks were used to represent the number concretely. As the year progressed, the numeration system was demonstrated showing the tens and hundreds place both concretely and symbolically. The hundred chart was also used to demonstrate the

concept of odd and even numbers. Early in the year the students completed an activity using counting blocks to determine whether a number was odd or even. If the blocks could be separated into two equal groups then the number was classed as even. Using the numbers from 1-30 the students were able to recognize the pattern of even numbers and create a rule for the pattern. This rule was applied to the hundreds chart each day to determine if the number was odd or even. If the number was determined to be even then it was circled on the chart. Students would periodically check their prediction using counters or blocks. The idea of *even splits* and *doubles* laid some of the groundwork for introducing multiples of two in the multiplication unit.

Next to the hundred chart was a large weather pictograph on which the assigned student recorded the type of weather for that day. This not only gave the students practice in developing and reading graphs but it also provided me with an opportunity to teach the students how to make comparisons. Questions such as, “How many more sunny days than rainy days have we had?” were posed, and the students used various strategies such as counting on and counting back to find the answer. Eventually this led to the term *finding the difference* and the development of a subtraction sentence to solve the comparison problem.

Another activity added periodically to the Morning Meeting was a review or an extension of telling the time. Using a large demonstration analog clock, the students would practice identifying time to the minute and relating specific times to important events in the daily school schedule. This activity provided further opportunities for counting in fives and the informal introduction of numbers that are multiples of 5.

As the students responded to questions I posed during the Morning Meeting, I generally countered with the question, “How do you know that?” This was intended to give the students practice in justifying their answers and explaining their thinking to their peers. My goal throughout these morning activities was to apply mathematics in meaningful ways to daily living situations and thus promote understanding. It also served to develop the social and communication structures for a math program that I believed was more reflective of the NCTM curriculum standards.

The Action Steps

1. What Does Multiply Mean?

I introduced the study by explaining to the students that we would be starting a new unit in math and that I was going to record how they demonstrated their learning to me. They were aware that I would be tape-recording our activity periods and replaying them so that I could conduct my research. I explained that one of the tools that I was going to use in this process was the math journal. This was a new component of the math program. The students were quite familiar with journal writing because they were assigned a specific time each day to write on selected topics. At various intervals through the month we had an *author's corner* where the students chose their favourite pieces of writing to share with the class. However, math journaling was a new venture and needed some explanation. The math journal was to be less structured than the daily writing journal and was meant to provide students with a variety of ways to express their math knowledge. I explained that the format for completing the journal was up to them and they could use words, pictures, or symbols to communicate their ideas. The intent was for the students to use the journal at specific times during the math unit to express their

level of knowledge concerning the multiplication unit. This knowledge could be displayed in whatever format they found the easiest or most compatible with their learning style. Each page in the journal was divided in half with the top half of the page blank and the bottom half lined. The students were instructed to use the pages however they wished.

With most new writing activities at this developmental level, I provided several examples of possible ways to complete the activity on the chalkboard. I drew a picture showing a set of three flowers and a set of four flowers and asked the class to tell me how I could show that with numbers. Most students could provide an addition algorithm, and with some prompting a number of students were able to develop a subtraction algorithm. I recorded the family of addition and subtraction facts generated on the board and then I wrote out the algorithms in words to demonstrate that numbers and symbols were shortened ways of describing the picture mathematically. The students were familiar with the process because they had previous exposure to it when they studied the addition and subtraction process. I then wrote the addition algorithm $6 + 3$ on the board and asked what other ways this could be shown. Immediately several students wanted to give me the answer rather than look at the symbols and provide alternative ways of expressing them. It took several attempts before the students became comfortable drawing pictures of sets that could be combined, or using words to express the symbols. After several attempts the students became more comfortable with the idea that their mathematical knowledge could be shown in a variety of ways and that the algorithm was not the only right way to demonstrate understanding.

This initial activity had a two-fold purpose. The introduction of math journals was to be an extension of the student discussion sessions. In doing so it was addressing the NCTM curriculum standard requiring students to demonstrate and communicate their mathematical knowledge in a variety of ways and explain their reasoning. This journal activity was also going to provide me with information to assess the level of background knowledge that the students were bringing to the multiplication unit, and help me plan a focus for the initial set of activities.

The word *multiply* had come up at various times through the year and several students had asked me when we were going to learn to multiply. They had obviously heard the term from other students, older siblings, or adults. A number of the grade two students had told me some of the *times* that they knew. On certain occasions in the Morning Meeting students had given me multiplication algorithms to express the date. These algorithms included 3×3 , 2×2 , 3×5 , 2×10 , and were mostly linked to counting in multiples of 2, 5 and 10. For most of these algorithms they were able to use the tally sheet or number charts to help with their formulations. From this information I had concluded that most of the grade two students had some initial exposure to the concept of multiplication and were familiar with some aspect of language or calculation. As a result, I decided that my opening activity was to find out what they could tell me about multiplication.

In a large group discussion format I explained to the students that I wanted to find out as much as they could tell me about multiplying and we were going to have a brain storming session. As they volunteered their answers I recorded them on a large class chart. The following are samples of responses that were recorded:

- Zach: when you put a times in the question the first one is how much of the last one you need
- Ian: (grade 1) – when you're doing like five times five it's like you have five, five times, and if you count up it gives you the answer
- Peter: (grade 1) its like when you double something
- Mike: it's like a hundred times ten is a thousand – you just add the hundreds together
- Ian: (grade 1) – because ten groups of ten make a hundred
- Leo: times is like if you count – like if you say twelve times twelve you count twelve, twelve times and its hundred and two, no I mean hundred and four
- Ian: (grade 1) – times is like a plus but you say it over and over again 'cos like if it said on my math sheet ten times ten I would count ten, twenty, thirty... 'cos ten groups of ten is a hundred
- Tony: (grade 1) – times is like eleven times eleven and you keep adding a one and a ten over and over
- David: (grade 1) – two plus two equals four and two times two equals four

Clearly the students demonstrated that they had some understanding of the process of multiplication. The idea that multiplication was like repeated addition came up in several of the responses. Also, it was evident that the students were using their ability to count in multiples of ten and five to explain multiplication.

After this discussion I wrote the following question on the board: “What does multiply mean? Tell me or show me what it means when you multiply?” I then asked the students to respond in numbers, words, or pictures (similar to the previous board work with adding and subtracting) and show me anything they knew about multiplying. They were to write this on the first page in their journals. I wanted to know if they could write an algorithm and explain its meaning in pictures or words, or how they could figure out the answer to an algorithm. I was expecting some students to be able to do this based on their previous knowledge of counting in multiples of twos, fives, and tens and their responses during the discussion period.

Even though the students were excited to be finally “doing” multiplication they found getting started on the activity difficult. Their difficulties may have been due in part

to the new format of math journals, which was my initial assessment after having circulated through the room observing and reminding students that they did not just have to use numbers, but pictures and words were also acceptable for their explanations. I referred frequently to the examples I had demonstrated on the board. I was expecting a number of the grade two students to be able to demonstrate the idea of repeated addition, as they appeared to be using this strategy in the morning meetings to provide alternative ways to express a number. However, these particular students were having difficulty starting the journal assignment and I noticed they were looking through their books in order to find multiplication algorithms that they had previously written, and were copying these into their math journals.

I explained that I did not want to see how many problems they knew but how they would work out a multiplication problem to which they did not know the answer. After an examination of the journals it was obvious that despite the students' frequent use of the terms *multiply* and *times* and their knowledge of certain multiplication algorithms, they had difficulty generalizing the process of multiplication or explaining how it would work with numbers that were not multiples of five or ten. Half of the grade two students gave clear explanations of multiplying with tens and hundreds. Several of them used the multiplication symbol correctly and provided several examples of the algorithm. Their explanations all involved the idea of repeated addition. Only one student used tallies to represent multiplying in fives. This was a little surprising since many students had used the word *times* in reference to the number of tallies we counted during the month.

It had seemed from informal conversations about multiples that the students had a deeper understanding of the meaning of multiplication than they were able to express in

their journals. Using visuals the students had demonstrated knowledge of the multiplication process but were not at the stage of generalizing the process without these visual aids. I had expected a higher level of thinking from these students, based on their previous performance during problem solving situations. Sara in grade one was the only student who was able to generalize the multiplication process beyond multiples of two, five and ten. She chose to explain the meaning of seven times four. She drew seven circles and placed pictures of four dogs in each. She then crossed off the dogs as she counted them and wrote the algorithm $7 \times 4 = 28$ beside her picture. From the erasing in her picture it was clear she had grappled with the problem of whether she should be drawing seven groups of four or four groups of seven. Somehow she had come to the correct conclusion.

The information generated in the journal samples and the trepidation with which the students approached this activity clearly showed me that although the students were using multiplication terminology and were quite successful in using it in specific situations, their knowledge was informal or intuitive knowledge based on previous skills of counting in multiples. Clearly they had not generalized the skills they were using and had not reached the level of understanding that I thought they had when I initially planned the unit. However, their knowledge of multiples provided the framework for subsequent activities that introduced multiples of less familiar numbers.

The use of the journal format was obviously going to be an evolving process. I believed that the students were struggling with the format and the lack of formal expectations in the way they could express themselves. This was a new way of working and I think that the anxiety to complete the page “correctly” was interfering with the

mathematics they were trying to express. I believed that the format would likely become more effective as the students grew accustomed to the idea that there were many ways to express and communicate their knowledge.

For the next series of activities I used the work of Marylin Burns (1991) *Math By all Means*, and the problem solving activities she designed to provide real world examples of multiplication concepts. The first activity is the chopstick problem and requires the students to apply their knowledge of multiples of two.

2. The Chopstick Problem

I explained the scenario to the class: I was going to take my family to a Chinese restaurant and we were going to eat with chopsticks. The students were able to tell me that each member of my family would need two chopsticks and that both chopsticks would be the same as there were no right or left chopsticks. Once this was established I asked the class how many chopsticks my family of four would need. As I gave time for the students to think I began drawing a series of four faces on the board to help students try and visualize the problem. Most of the students in the class had little difficulty figuring out the problem. They all seemed to agree that we would need eight chopsticks. However, I was interested in how the students worked out the problem and the mathematical skills they used to explain the solution. The following responses were recorded:

Mike: because two plus two plus two plus two equals eight and that's four twos and that equals eight

Sara: (grade 1) OK, I kind of thought that I would put a chopstick on each person's head (she had drawn pictures of four people) and that was four. Then I had to give each person another one and that was four again, so four plus four is eight

Brady: I said two times four

Zach: I used the eyes of the people and pretended they were chopsticks and I

counted in twos

Eddie: I counted in threes up to six and then I added two more

David: I said two times two is four, then times two again, (I think that's how I did it) and you have eight

Leo: two plus two plus two plus one plus one makes eight

From the responses I received, most students were aware that they could count in multiples to find the answer. However, their confidence with counting was limited to the visuals they were using to justify their answer. Edward seemed to pick up the idea that he could count in multiples, and he could circle sets of three on the picture he had drawn. Then he realized that his system had broken down when the remaining group was two. Leo initially used repeated addition, and then decided to count in ones as the number became higher, possibly because he lacked the confidence to continue adding twos. As the students gave their responses I repeated them to the class and demonstrated on the chalkboard how each of the students had solved the problem. My intention was to provide models of how children could have solved the problem, and there were several ways they could use to determine the right answer. I was interested to see how the students who counted in fours would react to the next problem that had a much larger number.

For the following activity I explained to the students that I wanted them to work in pairs and find a solution together that they could then share with the class. They had to figure out how many chopsticks would be needed if our class of twenty-one students was going to the Chinese restaurant. The students were asked to write about or illustrate their solutions in their journals. The immediate reaction of a number of the students was to start looking around the room and counting two for each student. Although the students were usually seated in rows of two many of them had moved their chairs to be seated

with their partners. There was also a fair amount of movement in the class so I could see that this strategy was not going to provide an accurate answer. Eventually most of these students became frustrated with the strategy because of the movement of the students and started to attempt other ways. Some students however, immediately started to draw faces on their work page to represent students in the class. Use of this strategy suggested that their ability to use symbolic representation was more developed. As I observed students at their work I noticed only a few students truly working together to negotiate the solution. Most were working independently drawing pictures to help find a solution. It was only after several prompts that the students started to share their ideas or explain their methods to each other. After a period of approximately twenty minutes I asked the students to stop and each pair was given a chance to share their solutions with the class.

The following solutions were offered;

Sara: (grade 1) I did like, um, 21 plus 21 and I drew kind of like little dots to help me, then I got mixed up so I put numbers underneath them

Eddie: I counted the rows of kids, there are 6 kids in that row so it is twelve then I counted another twelve that was 24 then I counted another twelve and that made 36 then there were three kids left so I had to add on another six

Darcy: I just went around and counted two, four, six, eight and I went around the room like this

Ian: (grade 1) I knew there were like 21 kids in the class and I said one plus one is two and two (tens) plus two is four so there is 24

Peter: (grade 1) No it's 42!

Ian: Yeah I mean 42 - it's like if there were 21 people in the class and they had to get two chopsticks, it's like you get two and two more and two more until you get ten, then you add another ten then you add two.

Mike: He took mine! I counted in twos.

I asked if any students had drawn pictures in their journal, and if they could describe their pictures. I was interested to see if the pictures that had been drawn truly reflected the way in which they had solved the problem. From my observations of the students working, I could see that some of the students were working the problem out in

their heads by counting in multiples, but were having a difficult time explaining their thinking on paper.

Leo: I did - I drew faces, 21 faces then counted two for each one because they would have two chopsticks

Sean: (grade 1) I drew chopsticks but I counted the people in the room and I counted in twos

James: I drew chopsticks but I didn't draw all of them.

Even though most of the students knew that we would need 42 chopsticks, only eight students were able to give an adequate explanation that matched the picture they had drawn in their journals. Of these students, most drew pictures or symbols to represent the chopsticks, drawing a one-to-one correspondence. Even though some of these students said they counted in twos they needed to draw each item to make sure they were correct. Only Leo and Jason were able to draw a symbol for each person and explain that each symbol was worth two chopsticks. I noticed many errors in the diagrams that the students drew even though orally they could tell me how to reach the correct answer. Zach, Ian, and Tom were the only students to demonstrate the answer as two groups of 21.

From this exercise it was apparent that the students had difficulty translating their knowledge from the rote and concrete to the symbolic form and making generalizations. Since the overall goal of this unit was to introduce the symbolic form of the multiplication algorithm and develop an understanding of its meaning, it was necessary to provide students with more real world examples of objects that demonstrated the multiples concept. I continued to use the work of Marilyn Burns (1991) to develop the classroom activities that would help the students develop an awareness of multiples in familiar objects.

3. What Comes in Groups of...?

This next activity was intended to help the students generalize the concept of multiples and to help them recognize that they are surrounded by items in the real world that are usually seen in multiples. I started with the idea of multiples of two since the students were familiar with the idea from the chopstick problem. I asked the class what other things they knew of that always came in twos. I posted a large chart on the chalkboard and recorded their answers. Most of the students thought immediately of body parts and gave answers such as two eyes, two ears, two hands, two feet, elbows, and so on, and then they progressed to clothing such as two mitts or gloves, two socks, legs on a pair of pants. Moving away from the purely personal, they offered such items as wheels on a bike and pairs of skates, pedals on a bike, scissor blades, classroom lights, two dollars in a loonie, and twins. After reviewing the list with them I assigned a homework project that I hoped would engage families in thinking about multiples with the students. I asked the students to make a list of objects around the home that came in multiples of two and I explained that we would share these lists during the Morning Meeting the following day. A note was sent home with the students explaining the project and requesting that family members assist in making students aware of multiples in the home. This was an open-ended activity that was intended to promote a discourse at home and engage parents in the process I was developing in the math class at school.

The following day students brought their various lists to school. Many of them included similar items to lists generated in class such as body parts, clothing items, and so forth. The students had obviously been able to demonstrate to their parents the type of discussion that had taken place in the classroom. This communication was important to

note as it indicated that the students were involved in a discourse at home and were able to explain what we had been doing in class. Several of the students told me they had talked about the chopstick problem with their parents. In doing so, they were communicating their mathematical knowledge and internalizing the solution processes we had discussed in class. This was an important aspect of the exercise as it extended the communication goal beyond the classroom and continued to engaged students in the development of mathematical ideas and awareness beyond the classroom.

I spent some time during our morning meeting giving the students an opportunity to share their lists and I added the new examples to the chart we made the previous day. The chart was posted on the bulletin board along with several other large blank charts. After the sharing session I explained to the students we were going to discuss other objects that came in groups.

4. What Comes in Groups of 3's, 4's, 5's and 6's?

During the planning stage for this unit I sought literature-based examples of mathematical concepts relating to multiplication. One such book that provided the opening for my next activity is titled, *What Comes in 2's, 3's, 4's?* (Aker, 1990). This picture book has little text but visually provides many examples of multiples or groupings familiar to young students. I shared this book with the students at the story corner as an introduction to our new math activity. After reading, and discussing the pictures, I explained to the students that we were going to try and create more class charts similar to the twos chart using items that came in threes, fours, and fives. The students were divided into groups of two and were instructed to work together to come up with as many ideas as they could of objects that generally came in threes. They were given about 15

minutes to work and then there was to be a period of sharing where all ideas would be compiled on a class chart. Students were to decide amongst themselves who was to scribe.

I broke the students into small groups in order to observe how they worked together and how they would judge each other's responses. I also wanted to listen for contributions from students who had remained less engaged the previous day. The twos chart provided the model for the students' independent lists, and examples from the literature selection enabled most groups to attempt an independent list.

The students took to the activity noisily and were motivated by the picture book. Many of the groups started their list with the ideas from the book, and then quickly ran out of ideas. As I circulated I realized I needed to prompt the students to develop their ideas. Another strategy that emerged as I talked to pairs of students was the sharing of ideas between pairs. This tended to generate further ideas for each group. An interesting discussion emerged in one section of the room when one student suggested the three billy goats gruff, which set the students to thinking about other stories in which sets of three occurred. In a short time they had listed the three bears in Goldilocks, the three wishes and genies in Aladdin, the three blind mice, the three musketeers, and so on. Out of this math activity emerged the awareness of a literacy convention and the significance of certain numbers in our cultural heritage.

Once the students had exhausted their lists I brought the class back together so that each pair could share their ideas with the whole group, and we could create a class chart. Systematically I went around the groups asking for contributions to the class list until all new ideas were exhausted. I was attempting to include more students in the

sharing process, because I was becoming aware of certain students who were beginning to dominate during large group sessions when ideas and answers were received randomly. Items generated for the class list included such things as tricycle wheels, traffic lights, sides on a piece of pizza, sides on a triangle, corners on a triangle, primary colours, alien fingers, trio, sets of wheels on an airplane, a tripod, three meals a day, triplets, and hands on a clock. This last idea generated some discussion regarding whether clocks always had three hands. The clock I was using to teach the students time had three hands and was very familiar to them; because I used it to demonstrate the recording of time during the day in seconds, minutes and hours we decided to keep the *hands on a clock* on our list.

After all contributions had been accepted, it became evident that I could introduce another language and math element to the discussion. I asked the students to scan the list of items and see if they could find a pattern in some of the items we had listed. The students' responses did not indicate they were close to the idea I was trying to present. I then asked if anyone noticed a pattern in some of the words. After a brief silence, Brady, a quiet, but very perceptive student, was able to announce that several of the words on our chart began with the same three letters. As he read out the words, I had the rest of the students repeat them, following my belief that vocabulary building and reading are always promoted in the primary classroom. The words tricycle, triangle, tripod, and triplets were identified, which gave me an opportunity discuss the prefix *tri* and explain its significance in language use. From this discussion I knew that I could also highlight the prefix *quad* when we came to our fours chart, and thus provide another language component to the math class. I had noticed that many of the boys had written 'quad' on

their lists of objects in fours. Many of their families had 4x4 recreational vehicles and they often talked about riding their *quads*. By highlighting this word I could develop their knowledge of vocabulary using quad as a prefix and also introduce the term quadrilateral, which would later be used in geometry.

Class charts were generated for objects that came in fours and fives in much the same way as the previous activity. Items generated in the fours list included car wheels, chair legs, desk legs, many types of animal legs, quarters in a dollar, sides or corners on books, sides and corners on a square, food groups, and wheels on a quad. The fives list included toes on a foot, fingers on a hand, finger nails, toe nails, petals on a flower, points on a star, pennies in a nickel, and line strokes in a tally. I had planned to stop at the fives but one student challenged the whole group to see if we could find some things that came in sixes. Since this was student generated and the children were motivated I created another chart for sixes. This appeared to be a tough challenge but after some thought I was offered the following items; juice packs, gum in a pack, some truck wheels, faces on a cube, sides on two triangles, and two rows of six in an egg carton. As I listed these items and scanned all the charts I noticed that the students were demonstrating connections between various aspects of their mathematical knowledge. Not only were they demonstrating their knowledge of geometry, but also their understanding of money and equivalent coins. I used this knowledge to extend the student learning. Since we had identified a three-sided figure and a four-sided figure I asked the students what we called a five-sided figure. No one knew this so I wrote the term pentagon on our chart and drew the familiar pentagon shape of a house on the board. I repeated this activity with the term hexagon and in doing so made the connection between our present study and the later

study of geometric figures. The students were highly motivated at this point, which allowed me to develop a foundation for further mathematical discussion and study.

In order to close this activity I reviewed the charts with the class and asked if there were any additions before posting them on the bulletin board. I explained that we would continue to add to the lists as our study progressed, and asked that the students make a note in their homework planners to enlist the help of family members to generate more ideas.

Although this series of activities proved to be successful in generating examples of multiples familiar to the students, I noted a number of concerns in my journal later in the day. The time devoted to producing the lists and the ensuing discussion was far greater than I had originally intended. The sharing of ideas took a long time and a number of students became distracted once they had their turn to share. Another problem was how to deal with incorrect or questionable answers. From the tape-recorded discussions I noticed that I was generally providing the rationale for why a certain item should go on the list or not. In retrospect, I felt I should have left this decision up to the group and let them provide the rationale for inclusion. This, after all, would promote the use of reasoning skills. I know that I was trying to speed up the process because I was concerned about timelines and the need to get on with the next activity.

I also questioned the students' ability to truly listen to one another and respond to each other's ideas appropriately. Although the NCTM Standards promote this type of sharing, I found that it does not happen spontaneously in the regular classroom setting with the mix of aptitudes and abilities. The sharing forums need to be structured in such a way so that different students get an opportunity to speak, and the time devoted for

listening needs to be monitored and controlled to suit the students' attention level.

Although I was pleased with the math and language connections I was able to highlight from the lists, the process took place at the very end of the activity and I questioned how many of students were actually mentally engaged at this point. Since the idea of making connections is such a fundamental aspect of teaching mathematics according to the Standards, I felt it was essential to note the connections that were made during discussion periods, and make deliberate attempts to review these in subsequent classes. The Morning Meeting was a forum in which I could informally review the connections and assess the students' level of understanding.

Another of my concerns related to how the students worked together, and how each individual was able to contribute to the class discussions. I thought that the alternating pairing situation and the large group discussion was successful because it engaged more students actively in the problem solving process. I was able to circulate in the room and observe the discourse taking place, which allowed me to focus on certain students who were not as vocal in the large group, and to provide assistance individually if required. However, in order to allow each group to have a turn to share, the activity became prolonged and interest waned. I questioned the length of time students at this age are truly able to listen and respond to their peers. Clearly, discussions need to be kept short and I saw the need to plan a way in which all responses could be validated but sharing kept to a level that maintained attention and engagement.

In the preceding sharing activity students were seated at their desks, all facing the board where the items were being recorded on charts. In reflection, I thought that the sharing time might have been more interactive and focused if the students had been

sitting in the meeting corner where the morning activities take place. In this area the students generally sat in a circle and faced one another and were use to engaging in conversation governed by specific listening and responding formats. I noted that I would try to utilize this format in future sharing sessions focusing on math topics.

After further review of the class charts I noticed the connections that could be made between the students' knowledge of money and equivalent values, and the concept of multiples. I had recorded two dollars in a loonie, four quarters in a dollar and five pennies in a nickel on the respective class charts. This expression of student background knowledge could now be connected with the concept of multiples and be used to demonstrate the significance of multiples in the students' daily living experiences. I decided to draw the students' attention to this connection informally at one of our future morning meetings as we continued to add items to our charts and review the contents. We usually practiced counting in twos, fives and tens at that time and counting with money would be another opportunity to make connections between our daily activities.

5. The Circles and Stars Game

The next planned activity was designed to lead the students towards an understanding of the formal multiplication algorithm. It was going to take place over several math periods and students were initially going to be working in pairs. Since I had a very specific goal for this activity, and I had noted some concerns about how the students divided up the tasks when working in pairs, I gave specific instructions on how I wanted the students to proceed. Once again I used the lesson plans of Marilyn Burns (1991) to develop the procedure. Each student was given two dice and a student-recording booklet. The pages in the booklet were divided in half with the top half left

blank and the bottom half with several lines for writing on (Appendix G). The students were told that within their pairs they were to take turns rolling the dice. The roll of the first die indicated the number of large circles each was to draw on the top half of the page. The roll of the second die would indicate the number of stars they were to draw in each circle. On the first line under the picture the students were to write an addition sum and record the total number of stars they had drawn. For example, if the student first rolled a four then he or she would draw four large circles at the top of the page. If the next die rolled produced a three, then the student would draw three stars in each circle, which would lead to the addition sentence $3+3+3+3=12$ being written under the picture. Students were to create their own separate booklets, but they were instructed to correct their partner's work before they went on to the next set of dice rolls (Appendix H).

After a teacher demonstration the students worked quickly. The variety of activities for completion and the provision of a clear structure appeared to be motivating, and the students approached this activity with confidence. The only thing the students had to negotiate was who should go first. One pair immediately chose to do the Rock, Paper, Scissors routine and since it seemed like a good idea I acknowledged their problem solving ability, and the process was quickly imitated by many of the other groups. Some of the grade one students had difficulty with the order of the activities and were not sure when they had to draw circles and when they had to draw stars. Some initially thought they were adding the number of circles and the number of stars. I had to reinforce the idea that the circles were drawn to hold groups of stars. Eventually the classroom became a hive of activity as the students worked together and I was able to circulate through the classroom listening to student conversation and providing individual

direction. After a few minutes Mike and Tom asked why there were so many lines to write on when they only had to write one addition sentence. At this point I stopped the activity and explained to the class that the other lines were going to be used to write other ways to describe the picture. This was my invitation to some of the more advanced students to consider what other ways might be found.

As I circulated in the classroom I was able to observe the methods the students were using to find the answer to their addition sentence. I noted in my journal that most of the students were counting in multiples if they had groups of two or five. Some appeared to be counting in multiples of three. However, I noticed that a few students did not have the confidence to do this at all, and they counted each individual star to find the total. They were not yet able to apply their knowledge of counting in multiples in a problem-solving situation. They would need more practice and guidance in this activity before they were able to generalize the application.

The *circles and stars* activity progressed well and a number of the grade two students finished the booklet very quickly. As they finished I challenged them to think of other ways that the picture could be described besides the notion of repeated addition. A few students noted that when they drew groups of two and five stars they didn't need to count all the stars and that they could skip count. In my observational journal I noted that several of the students were in fact skip counting and others were combining groups, and then adding those combinations together. Leo, for example, had a picture showing six circles and three stars in each circle. When I asked him to explain how he came to the total of 18 very quickly without skip counting, he told me that he knew three plus three was six, then he combined another two groups to make six, and he said that was easy

because it made twelve, then he added on another six to make 18. In demonstrating this process Leo was already developing knowledge of common multiples that would be applied in future years in areas such as fractions and ratios.

I expected that a few of the more advanced students would be able to relate this activity to the idea of multiples and the phrase *groups of* that I had used intentionally during the Morning Meetings when we were counting tallies. I had often interchanged the word tally with the words *bundles of* or *groups of* to prepare the students for the language of multiplication. In fact, many of the students had used a multiplication algorithm when they were providing ways to make up the number in the date (during the Morning Meeting). Many students seemed to know that three times five equaled 15 and four times five was 20. Three times three, and three times four, were also common equations that the students could answer. From these activities I assumed that the students had a good understanding of how simple multiplication algorithms worked. However, this did not seem so apparent now, because they did not connect this previous knowledge with the illustrations in their circles and stars pictures. Thus, my next task was to guide the students to make the connections between their knowledge of counting in multiples, the random facts that they knew, and the pictures they had drawn and described with repeated addition algorithms.

By the end of this period most of the students had completed all ten pages in their booklets, had written addition sentences to describe their pictures, and had totaled the number of stars they had drawn on each page. Some of the students who had finished early were directed to get a calculator and find the total number of stars they had drawn. They were then told to compare their totals to see which student had the largest number.

This was an extra activity not necessarily connected with the multiplication unit but it provided an opportunity for the students to practice using the calculator and a chance to compare large quantities. They were told to work in pairs and check each other's work. In assigning this task I was intentionally looking for all the math possibilities from which the students could benefit in a single activity.

I collected the books and checked the work to make sure the steps had been followed as directed. All the students completed the task well and the pictures they drew were clear and matched the addition sentences they had written. However, as I noted in my journal, I was surprised that none of the students had made the connection to the multiplication algorithms that they seemed to use with confidence during our Morning Meetings. The optimism and confidence in my students' ability to grasp this concept that I had experienced at the beginning of the multiplication unit was beginning to erode, because they did not make the connections that I thought the majority of them would find obvious after the *circles and stars* activity. In order to examine this further I decided to tape the Morning Meeting the following day to explore the students' informal use of multiplication and how they applied their knowledge of multiples.

The morning meeting discussion 1. The date of the morning meeting happened to be the tenth. This meant that on the tally chart (showing the number of days so far in the month), the student helper was going to draw two tallies to illustrate the number ten. My intention during this activity was to connect the two tallies showing bundles of five to the circles and stars activity of the previous day, to demonstrate the use of the language of multiplication. It was Eddie's turn to make the tally and I asked him how many bundles he had made. He told me he had made two. I then asked him how many were in each

bundle, and he told me five. I started the sentence, “two bundles of five make...” and Eddie and many of the students responded with “ten”. I rephrased and explained that we could also say, “two groups of five equals ten.” I then asked the students to think back to the math activity we had completed the previous day. Relating the bundles or *groups of* to the circles, I asked how many circles we would need in order to show the tally. The chorus of responses indicated two circles were needed, and this response was matched by the knowledge that they would have to draw five stars in each circle.

I then decided to further refine the language in preparation for the following math class. I asked the students how many *times* Eddie had to draw a bundle of five. The students had no difficulty telling me he drew the bundle two times. So I rephrased their comments by replying, “Yes, two times five equals ten” and pointed to each bundle as I emphasized the two groups. The connection was clearly made so I asked some “what if” questions to extend the idea. I asked, “What if Eddie had to draw the tally three times. How many would there be altogether?” The students had no trouble following this line of questioning as I extended the idea up to ten times. Many of the students commented that we were just counting in fives and I was just asking them how many times they had to count five. A little later on in this session the students were providing ways of making the number ten, which was a daily activity using the number in the date. Typical answers included three plus seven, four plus six, and so on. Darcy offered ten times one, which I immediately responded to with the question, “How did you know that?” He replied that he just added one, ten times. Another student, Zach, offered five times two. Again when I asked him to explain he told me he was counting in twos and he showed me on his fingers how many times he had to count two. These interactions suggested some of the

students were beginning to internalize the meaning of the word *times*. However, others would clearly have difficulty with it if I removed the structural context.

This short activity at the beginning of the day served to reinforce the skills practiced the previous day, it helped make connections between mathematical strands, and provided me with a context for reviewing and introducing mathematical terminology.

6. Introducing the Multiplication Algorithm

For the next math session, I explained that the students were going to describe the pictures they had drawn in the Circles and Stars booklet in a different way. Referring back to my use of the words *groups of* in place of *tallies*, I asked the students to look at the first picture they had drawn and count the number of groups or circles. They were to write this number on the line below their addition sentence. Next to this they were to write the phrase *groups of* and then they were to write the number in each group followed by the word equals, and the total number of stars they had drawn. Using the addition sentence of the initial example; $3+3+3+3=12$ the new sentence underneath became 4 groups of 3 equals 12. After several examples the students proceeded independently, describing each picture in the format I had demonstrated.

This activity proved to be quite challenging initially for students who had weaker language skills. I noted in my journal that several students had to be guided at the beginning of each step, because they were not sure which number should go first. They seemed intent on putting the number of stars first. Constantly, I had to repeat the questions, “How many groups?” and “How many in each group?” These students had not yet made the connection that what they were doing with their pictures was just the same activity that they found so easy in the Morning Meeting with the tallies. The difference

was that the pictures they had created by rolling two dice were random combinations of factors and did not follow a specific set of multiples. As I circulated and observed in the classroom I noted a number of students were struggling with the steps in the task and I was verbally repeating the steps out loud as they worked. I monitored these students closely so that their sentences would accurately reflect the pictures they had drawn.

The next step in this process was to rephrase the sentence they had just written about each picture and replace the term *groups of* with the term *times*. I asked the students if they could tell me a word that I could use instead of *groups of* that would be shorter. Darcy immediately responded with the word *times*. Using the same example as before on the board, I demonstrated the types of sentences they would have under each picture in their booklets. I modeled: $3+3+3+3=12$, 4 groups of 3 equals 12, 4 times 3 equals 12. The students were then asked to go through their booklets writing the descriptive sentence with the word *times* instead of *groups of*. This transition was quite apparent for some of the students but Zach announced that he was stuck and didn't know what to do. He said he had one circle and one star in it. Even though he wrote *one group of one equals one* he had difficulty using the word *times*. Other students who had one group also said they didn't know what to do. I responded by asking the question, "How many groups do you have?" and explained that they had to ask themselves that question first before they counted the number in the group. I noted in my journal later that I was constantly guiding a number of students by repeating the same question to get them started on each page. I had not expected these steps to be so difficult for the students, especially Zach. Even though he was a grade one student he had a very good understanding of multiples and had demonstrated his knowledge on previous occasions.

My planned sequence of steps that moved from the pictorial to repeated addition sentences, then to *groups of*, and then to *times* was not working as smoothly as I had anticipated. I noted in my journal that perhaps the activity would have worked better if I had completed all the steps with each picture at once, before moving on to the next picture. My intention had been to introduce the change in language gradually and practice its use before refining the idea. After much oral repetition and guided support all the students managed to complete the task. I monitored their work closely during the activity because I was concerned that the sentences they wrote should be accurate reflections of the pictures they had drawn.

Despite the struggles of some of the students, many of the grade two students had asked if they could stop writing the words and put the symbol \times for *times*. It was apparent that they had made the connections between the words *groups of*, *times*, and the multiplication symbol \times . Others were still having difficulty with the process. I assigned the more confident students to help the ones struggling, and listened to their conversations as a way to assess their understanding. It was clear that a number of students were confidently representing repeated addition as a multiplication algorithm while others were not yet ready to make that transition. I could see that there was a split beginning to form between the students who had made the connection and those that had not. This split did not divide the grades but rather cut across the grade groups fairly evenly. I continued to complete the activity but noted in my journal that I needed to assess individual students to determine their level of understanding. There was much animated discussion during the class period and I could sense the rising anxiety level of a number of students so I decided to leave the introduction of the multiplication algorithm

until the following day when I could begin the session with some whole group guided practice and review.

I started the following day by drawing a picture of six circles on the board with three stars in each and asked the students how I could describe the picture using numbers. Several of the students knew the multiplication algorithm right away. However, I wanted to emphasize the aspect of repeated addition, so I asked for an addition sentence first. I then took the students through the series of steps they had used previously to introduce the word *times*. The final step was to write the multiplication algorithm replacing the words *times* and *equals* with their mathematical symbols. I completed several examples on the board and the general impression I got was that the students appeared comfortable with the process. However, once I assigned the students their task of using a multiplication algorithm to describe the pictures in their booklet a number of students were still struggling. Clearly, not all the students were as enlightened as I had expected them to be after this process. Even though many of them had used multiplication terminology and had applied the multiplication algorithm correctly in specific areas such as the tallies, they were not generalizing the skill of creating *groups of* to multiples other than the familiar twos and fives. Reflecting on the previous lesson and listening to the tapes of the interactions I thought that I was verbally guiding the students too much, and that the connections between the pictures, language, and the symbols were not being made independently. These students obviously needed more practice creating equal groups, and relating the multiplication algorithm and the words that describe the algorithm. This had been accomplished successfully with the tallies in the morning meeting so I made a conscious decision to incorporate *multiplication talk* during the

morning meetings, introducing discussions of multiples other than twos and fives. The charts that had been created to demonstrate objects in multiples could be employed to develop the visual cues to support the students thinking.

Morning meeting discussion 2. The following day was the 11th. I completed the opening activities and the assigned student drew the tally for the morning. He had drawn two bundles and one more to start a new bundle. I wanted to make the connection between the circles and stars activity and the counting of tallies. I also wanted to emphasize the connection between repeated addition and the multiplication algorithm. The following discourse started a rather prolonged exercise in mental arithmetic that the students found challenging and motivating. I pointed to the two bundles and asked;

Teacher: How many groups of five are there shown on the tally sheet?

Leo: Two

Teacher: Remember our circles and stars booklet? If you were showing two groups what would you do? If I rolled the dice and I got a two what would I do?

Jason: Draw two circles.

Teacher: What would I put inside the circles if I was using this tally?

Jason; Five.

Teacher: So what would the addition sentence be?

Jason: two...two, er no five er...

Teacher: What are we adding? How many will be in the circle?

Jason: Five plus five

Leo; it's ten, its ten!

Teacher: Or I could say two groups of..

Leo: two groups of five – equals ten

Jason: Yeh, two groups of five is ten

Teacher: Mmm, good, lets see if we can try another one. If I had two groups of four, what would I have? Two groups of four?

(silence)

Teacher: Tony, what do you think?

Tony: It's eight.

Teacher: If I had two groups of seven what would I have?

Vicky: 14

This discourse continued in a question and answer format as I randomly picked

two groups of a number. Jason and Leo were beginning to dominate the session so I told the students they needed a break and asked for other volunteers. I posed the question and gave some thinking time before asking volunteers to give their answer. The speed of the interaction increased as the students became comfortable with the process. I wanted to find out if they could explain their thinking in order to get a sense of the mathematical connections they were making. I asked the question, “What are you doing to get the answer?” Several students responded that they were just doubling. Doubling was a term we had used frequently early in the year when the students were learning addition facts. The doubles were a group of facts that the students tended to memorize early and we had used the knowledge of doubles to learn other facts. From this information I could see that the students were thinking in terms of repeated addition to answer my *groups of* questions, as well as their ability to rote count in multiples.

I decided to extend the activity to see if the students could apply the repeated adding strategy to three groups. I asked, “How many would I have altogether if I had three groups of two?” This question was met with silence initially and as the students were thinking I demonstrated on my fingers, two, plus two, plus two. Immediately there was a chorus of students with the answer six. Again I repeated the “three groups of” question with random numbers below five and demonstrated on my fingers the number in each group. When I asked for three groups of five there was no wait time as most students knew that was 15 from their knowledge of rote counting. I then wanted to see if they would make the connection with the morning tallies and asked, “Where have you seen three groups of five before.” Sara immediately responded that it was like the tallies

on the calendar. Several students agreed with her, which indicated to me that connections were being made between the various mathematical applications.

Even though this was a mental arithmetic activity and the students were seated in a large group on the floor with few visuals to help, they were very motivated and eager to figure out the answer to the next question. I had gone beyond the time limit I usually put on the morning meeting, but the exercise was proving to be very productive so this was my opportunity to continue and extend the learning. I next moved on to *four groups* and continued with the questioning. When I came to four groups of three, twelve was provided as the answer, and Jason announced that it was just the same as three times four.

This comment was significant in two respects. First, he had made the commutative connection, meaning that the order of the factors does not affect the product. Secondly, he had used the word *times* which I had consciously avoided so far during this session because of the confusion it had generated for certain students the previous day. It was surprising that this information came from Jason because he had been having difficulty composing the multiplication sentences in his circles and stars booklet. Clearly, incidental learning was taking place as I approached the topic in various ways in structured and less structured formats. The questioning technique I was using was fairly systematic and the students were forming the notion of multiplication as repeated addition. The commutative property of multiplication was also noted again when I started questioning about groups of five:

Teacher: Five groups of one?

Ian: Five

Teacher: Five groups of two

Zach: ten

Peter: It's just like two, four, six, eight, ten

Teacher: It is just like two, four six, eight...

Zach: It's just like five plus five

Teacher: Yes, it is just like five plus five

Zach: No, I mean it's two times five, and five times two, it's the same

When I got to *five groups of five* I picked Vicky, a grade one student who was not participating as well as the others, and told her we would work it out together. I raised five fingers and said we could add in fives together as I pointed at each finger. When we got to 25 Darcy responded with, "It's five times five!" Again the commutative property was recognized and the word *times* was being used instead of *groups of*.

I had not planned to take this activity as far as I did, but it seemed to be a teachable moment. My intention was to connect this adding process with the circles and stars activity of the previous day, and in doing so, reinforce the language of *groups of* before replacing it with the word *times*. I wanted to make sure the students had the notion of a number designating a quantity of groups, and a number designating the amount in each group. This was further reinforced later in the day when one of the students reminded me that I had promised to replay some of the audiotapes I had made during our math classes. I replayed morning session and the students listened intently for their voices. When the tape came to the *groups of* question and answer activity the students started to join in and try to give the answers before the answers were given on tape. As a result, this impromptu listening activity became a highly motivating review of the math skills I was teaching earlier in the day. Not only that, some students expressed concern about how little they heard themselves on the tape. Their comments made me aware that certain students were beginning to dominate the large group sessions and that I should find ways to draw less vocal students into the discussions. I noted later in my

journal names of students that I should observe and monitor more closely, and provide opportunities for them to contribute.

7. Group Review of the Concrete to the Symbolic

I had two goals for the following math class. First, I wanted to review the steps the students had followed in creating their Circles and Stars booklet, and reinforce the symbolic form of the multiplication algorithm. Secondly, I wanted to revisit the class charts of items that came in multiples. These charts had been posted in the classroom for several days and students had been adding to them as ideas had surfaced. I started the lesson by having two students play the circles and stars game in front of the class as another student recorded the picture on the board. With each picture I guided the students through the steps of writing an addition sentence, a *groups of* sentence and a *times* sentence finally ending with the symbolic algorithm. I stressed that the numerals, multiplication symbol, and the *equals* symbols were merely short ways of describing the picture that had been drawn. My intention was to show the students that the long repeated addition sentence could be replaced with a much shorter multiplication sentence and the meaning would remain the same. To reinforce this idea, I had the students work in pairs and read through their Circles and Stars booklet. One of the students was to read a multiplication sentence from their book and the partner, without looking, was to say how many circles were drawn and how many stars were in the circle. This was not a previously planned activity but one I thought of during the morning as I observed a few students playing this 'game' informally. It proved to be a fun activity for the students but it gave me an opportunity to observe individual students and assess their understanding of the multiplication process. The activity also gave the students more informal practice

using the multiplication language. This definitely was a case of students taking charge of their own learning and my input in designing this activity was minimal.

I ended this session by bringing the class back together in the large group and focusing their attention on the charts we had made showing objects that come in multiples. I wanted to make the connection between the work we had been doing with *groups of* and how we might use that knowledge in everyday situations. This was going to be the focus for the following set of lessons so I was preparing the students for the next independent activity. After reading and discussing the new items added to the charts I picked the example of a tricycle. I asked the students how many tires I would need for a tricycle. There was no hesitation with their response of three. I continued the questioning with two tricycles, then three. Very quickly, the students realized that what I was doing was counting in threes, much like the mental arithmetic activity we had completed at the morning meeting. Normally I would not have approached the mental arithmetic until the students had practice with the concrete or pictorial. In this case however, the motivation of the students in the morning set the groundwork for what I had planned to introduce later with the charts. I noted in my journal that this was a clear case of the students directing their own learning and I felt it was important to switch plans and take advantage of the learning opportunity.

This activity concluded the first week of the multiplication unit. In my journal I had noted that the time taken to complete the activities was far greater than I had anticipated, and I was feeling some frustration at the slow pace. However, in order for the students to have time to practice their skills and discuss their learning I needed to extend the time that I assigned to math. The pressure to complete specific areas of the

math curriculum in a specified time frame did not quite mesh with the goals of the NCTM standards. However by incorporating exploration, discussion, and reasoning along with skills, the students were gaining a much better understanding of the processes they were learning.

In reviewing the classroom discourse, I noted many areas where students were making connections between various mathematical strands and I could see they were building foundational skills that would assist in learning new mathematical concepts. Allowing students time to practice using and experimenting with mathematical language helped them become more comfortable with the terminology and the meaning behind it. This was the goal of the Circles and Stars booklet, which demonstrated the various ways in which the picture could be described both in words and mathematical symbols. The incidental learning taking place was a reflection of the mathematical culture that was nurtured in the classroom. In addition, further language development was also being fostered during the discussions that took place when the students were creating the multiples charts. New vocabulary was being introduced and word meanings were being analyzed. In reflection, the time spent on the activities provided evidence of learning well beyond the specific outcomes of the mathematics curriculum. However, I thought I now needed to assess the students' individual learning and decide if they were meeting the learning objectives specifically outlined in the curriculum. In order to do this I decided to start the following week with an independent journal activity to examine how each student would respond to an unfamiliar multiplication algorithm. Since the students were not required to commit multiplication facts to memory, I was not interested in their ability to recite facts but in their ability to interpret a question and provide a method of

solving the problem. In other words, I wanted to see if the students had internalized the process they had used in the Circles and Stars booklet.

Assessment. I started the activity by writing the algorithm $6 \times 4 =$ on the board. My instructions were that I wanted the students to show me in pictures, words, or symbols how to find the answer to the question. I chose an algorithm for which I thought most students would not know the answer. Even with that, several students told me they could work it out in their heads. I explained that even though they might know the answer they needed to show me proof that they were right. In doing this I wanted to collect evidence from all the students that they were able to communicate their ideas and explain their thinking. I gave the students about 15 minutes to complete the activity and as they worked I circulated in the classroom observing how different students approached the problem. James, a grade one student approached me and said he didn't know *times*. I reminded him that *times* was just another word for *groups of*, and with this he went back to his desk and began to work. I noticed that other students needed this cue in order to get started. When I checked James' journal later, I saw that he did not answer the question I had put on the board; rather he had picked his own multiplication problems to illustrate and describe. He did not use a multiplication algorithm for his pictures but chose words and the term *groups of* instead. This was interesting to me as it demonstrated he was not yet ready to use the symbolic form but had a good understanding of how the multiplication process worked. He was beyond the expectations for a grade one student but not yet meeting the learning outcomes of the grade two program. However, he was developing a foundational knowledge that he could apply and extend in future learning situations.

All the other students in the class were able to illustrate the algorithm in much the same way as they had illustrated the circles and stars pictures (Appendix I). The explanations generally explained the picture as showing six circles of four stars and counting up the total number of stars. Several students wrote an addition algorithm to demonstrate the repeated addition aspect of the problem. Three of the students went further and demonstrated the commutative property of the multiplication algorithm by providing a picture representing the reversal of the factors and showing the answer to be the same. I was particularly interested in Sara's explanations. Sara was a grade one student whose language ability was well beyond grade level expectations. After she had given proof that 6×4 was the same as 4×6 she went on to give an alternative proof by saying that if we add $4+4$ we get 8 and so the problem is like $8+8+8=24$. This was a very sophisticated way to solve the problem and demonstrated an early awareness of factoring.

In reviewing the student journals I was satisfied that the majority of students were able to demonstrate the meaning of the multiplication algorithm. Listening to the classroom discourse on the tapes I could tell that some students had a tenuous grasp of the concept and still needed guidance and reassurance while they were completing the task. From this it was clear that in order to generalize the process most of the students needed continued practice with real world examples. I continued with *multiplication talk* in the Morning Meetings and the students were becoming more adept at using multiplication algorithms combined with addition to describe the number in the date. This activity provided a limited application of their knowledge, and I knew I had to look for other ways for them to apply and reinforce the skills they were learning. However, I

wanted to introduce the multiplication tables in a more systematic form so that the students could continue to connect the process of repeated addition with the multiplication algorithm. I did this using the items we had listed on the multiples charts.

8. Patterns in Multiples

Referring back to the tricycle problem in the previous lesson, I reviewed the process of figuring out how many tires we would need if I kept adding another tricycle. The students agreed that for each new tricycle they would have to add another three to find the total amount of tires. Using this example, I wrote out the multiplication table for threes on the board starting with 1 tricycle and 3 tires and ending with 11 tricycles and 33 tires. For each example I wrote the multiplication algorithm. Using this as a model, I asked the students to work in pairs and pick an item from the chart that came in twos. Using this item they were to fill in the “Patterns in Multiples” grid that I had handed out. (Appendix J). I used the example of mitts and completed the first few algorithms on the board. Each time I wrote the multiplication algorithm I stressed the language *groups of* and *times* and used them interchangeably so that the weaker students would have the extra reinforcement of seeing the action and hearing the words that described the action. When the students had finished they each had the two times multiplication table written out. The intent was not to memorize the chart but rather reinforce the idea of repeated addition and become familiar with the patterns in the products. This would assist in the recall of the facts later on. From the practice the students already had, some were demonstrating familiarity with the initial set of multiples for three and four. To provide further practice I introduced a patterning activity for students to complete when they had finished their multiplication table. I gave each pair of students a blank hundred chart

(Appendix K) and asked them to systematically shade in all the answers from their multiplication tables starting with two, then four, and so on. Following this the students were to work together to see if they could describe the patterns they had coloured. They would then share the patterns they had found with the whole group later on.

As I circulated throughout the room I had to assist a number of students with the multiplication chart. Even though many of these students could demonstrate in a picture what the multiplication algorithm meant, many students were getting confused with the reverse process. I continued to model the language transitions as I helped these students write out the algorithm. Once this was completed however, the students were very motivated in colouring the product patterns on the hundreds chart. Many asked if they could go ahead and finish the pattern beyond 22 which is the highest product they would have calculated since their multiplication chart only went up to 11×2 . This was my cue to stop the small group activity and switch into whole group sharing. I explained to the students that they could go ahead and finish the pattern only after they could justify which numbers they should colour. Rather than simply continue the pattern I wanted the students to provide reasons why the pattern worked the way it did.

I asked for volunteers to describe the patterns they could see. Even though many of the students said it was “just stripes going down”, I persisted, as I wanted them to practice using descriptive mathematical language. Eventually ideas that were offered included: a skip one pattern, an even numbers pattern, a doubles pattern, and, all the numbers ended in two, four, six, eight and zero. I accepted the pattern of “stripes going down” to introduce the term vertical. Since I was going to repeat this activity with the multiples of three, four and five on the hundreds chart, I wanted to provide some

language that would help the students to accurately describe the patterns they observed. I demonstrated the term *vertical*, then compared it to the term *horizontal* and finally modeled the meaning of the word *diagonal*. I explained to the students that these terms would become useful as they continued to look for patterns in multiples of three, four and five.

When the students finished this activity they had each written out their multiplication tables from the two times to the five times and had also coloured in a hundreds chart showing the products for each set of multiples. As the students practiced describing patterns on the hundreds charts they became more sophisticated in the type of patterns they were able to articulate. Many were using the terms horizontal, vertical, and diagonal with ease and others were noticing stepping patterns both forward and backwards, and up and down. They were familiar with the notion of odd and even from our continued practice of this concept in the Morning Meetings and were able to describe multiples on the chart in these terms. Although the students were noticing visual patterns as they coloured in the multiples on their hundreds charts, I was curious to see if they were able to discern patterns in the digits they were colouring. I circulated around the room listening to the pairs of students talk about their patterns, and as the faster students finished, I started to give them hints about looking for patterns in the digits. This was an opportunity for me to review mathematical language with some of the students, and introduce new ideas that these students could share with their peers during our large group sharing session. Although all the students were engaged in the same patterning activity, some were working at a more sophisticated level than others, using the cues I had given them.

Initially this series of activities was designed to reinforce the idea of multiplication as repeated addition. However, I found that I was able to incorporate many other areas of math and math language into the subsequent discussions. The introduction of the terms vertical, horizontal, and diagonal, was not part of my lesson plan but became an obvious addition to the lexicon once the description of patterns had started. Students who were unfamiliar with the terms previously were now given many opportunities to practice using them during the math class. These terms would later be used in geometry, by which time all the students would have been exposed to them and would have some working knowledge of their meaning in other contexts. By circulating in the classroom and listening to student conversations I was able to provide direction and steer students towards a particular way of thinking. When I listened to Tom and Leo describing the patterns they had found in the multiples of three on the hundreds chart I could see that they were ready to extend their thinking so I explained to them the idea of the digital root of a number. I gave them a cue and then left them to find a pattern in the digital roots. Before long they were telling me the digital root for any multiple of three was three, six or nine. They were inadvertently providing a rule for multiples of three that could be applied in other mathematical contexts. With this realization they quickly set to work checking for patterns in the digital roots of other multiples. Another pair of students, Darcy and Mike were attempting to articulate a pattern for multiples of four. They could see that there was an alternating pattern in the tens and units place but were having difficulty expressing themselves. By demonstrating to them with place value blocks that they could have odd groups of tens and even groups of tens, they eventually developed a rule for their pattern.

These activities took several sessions to complete because students needed time to colour their patterns, discuss their ideas with their partner, and contribute to the class discussions. I noted in my journal that I was concerned about the amount of time I was devoting to the math classes in order to complete a small section of the curriculum. I knew that the activities the students were engaged in were motivating and there was a lot of positive energy in the classroom. The students were also demonstrating progress towards the mathematics curriculum goals. However, I did feel a need to justify the amount of time being allocated to math.

In reviewing the tapes and listening to the discussions taking place it was apparent the many curriculum goals were being met, not only in relation to mathematics but also in language arts and incidentally in science and social studies. The activity of writing out the multiplication tables led to an examination of patterns and the introduction of new vocabulary to describe the patterns. Some students went beyond merely describing visual patterns but started to examine number patterns in the visuals. Thus connections were being made to other strands of mathematics that would not have been accomplished if the students were not given the time to explore and investigate. As the students worked together they were practicing their reasoning skills, making arguments, justifying their positions, and communicating their ideas both orally and in written form in their math journals.

These types of activities are not only valued as part of the NCTM standards, but are also foundational skills in the language arts, science and the social studies curriculum. Providing opportunities in the math class for students to practice these skills enhances their ability to apply the same skills in other areas of the primary curriculum. The

knowledge that the students acquire in any of the curriculum areas only becomes relevant if they can apply that knowledge in meaningful ways. This is the fundamental idea underlying the NCTM standards and underscores the need to view the curriculum in an integrated way. As such, my concern about the time spent on math activities, when viewed from a larger curriculum perspective, was alleviated, and in fact strengthens the argument for greater integration of the primary program.

By focusing on the learning processes the students were using instead of merely thinking about specific learning outcomes required by the mathematics curriculum, time frames became less of an issue. With this in mind, my goal was to provide the students with as many opportunities as possible to practice and use the multiplication skills they had learned. This could be accomplished in two ways. First, I could continue to plan specific activities that would require the application of multiplication skills. These could take place in the regular mathematics class. Secondly, and more importantly, I needed to remain cognizant of events and episodes in the classroom in which students could apply their multiplication skills in meaningful and relevant ways.

One such incident occurred when Eddie brought a medal to school, which he had won in a running event at the local track and field event. During the *show and tell* session Eddie was asked how far he had to run to complete the race. He explained to the students that he had to run around the track three times but wasn't sure how far that was. A discussion ensued and it was determined that the local track was 400 meters around. With this knowledge, I was given my cue to ask if anyone could figure out how far Eddie *times* four...hundred. Very quickly Brady provided the answer twelve hundred. Using the response that I had been perfecting during this unit, I asked how he knew that.

He explained that he knew 3 times 4 was 12 and, because we were talking about hundreds then it would be 12 hundred.

From this exchange Eddie learned how far he ran, I was provided with an opportunity to demonstrate the length of a meter using a meter stick, the students were starting to visualize the length of the running track, and the students were given a lesson by a peer in how to apply multiplication to solve a problem. Not only that, but Brady had demonstrated to me that he had internalized the multiplication process and could apply it to numbers far greater than we had practiced in class. These types of impromptu classroom exchanges, which are student generated, provide the basis for meaningful applications of knowledge and constantly provide opportunities to extend and refine student knowledge. By listening more carefully to students talking and discussing real world issues I was becoming more aware of the opportunities students presented in which I could extend my teaching.

Morning meeting discussion 3. The Morning Meetings continued to provide opportunities for me to integrate math talk into daily conversations. The activity of stating the number in the date in different ways became more sophisticated as the students gained confidence with their knowledge of multiplication. Two weeks after starting the multiplication unit, on the 22nd of the month, some of the responses to the question, “How can we make 22?” included:

(parentheses have been added for clarity)

Zach: 6 plus 6...er...2 times 6...equals 12...plus...10

Suzie: 22 times 1

James: 8 times 2 equals 16...plus...6

Brady: 2 times 11

Jason: 12 plus 12...no 2 times 12... (take away...2)...is 22

Tony: 5 plus ...(2 times 7)... plus... 3

Ian: 5 times 20...is 100...(take away 100)...plus 22

Mike: 3 times 8 is ...23

Teacher: 23? You're close...twenty...

Mike: No! 24...(take away...2) is 22

Tom: 7 times 4 is 28 (plus 1) take away 7

Sara: 5... times... 4...plus... 2 is 22

In order for students to develop these calculations they were using a large visual of a hundreds chart posted on the bulletin board. Not only were students using multiplication algorithms correctly, they were combining these with addition to create the target number. With the help of this visual they were developing fairly complex equations without any formal instruction in this area. As the students came up with various combinations I repeated what they had told me and demonstrated their thinking on the hundreds chart. This provided the stimulus for students to come up with other creative ways to make the number 22.

Sara, the deep thinker from grade one, had a rather complex explanation of how she came up with her answer. She said that with her eyes, she took off the last tally line in each bundle of 5, and made 5 bundles of four, then added the extra 2 making 22. Thus, she demonstrated informally to the class the commutative property of multiplication. Taking Sara's lead, I was able to seize the opportunity and ask the class for examples of other multiplication algorithms that they were aware of where the factors could be reversed and the answer would be the same. As I recorded the class contributions there was a growing awareness and interest in proving the commutative property of multiplication without any formal teaching in this area. Through her ability to communicate her ideas, Sara had stimulated student interest and curiosity, and I was provided with an opportunity to extend the students' learning beyond the planned goals of the unit.

At the end of the sharing session, Tom, who had given a complex equation for arriving at 22, announced that there were 120 hours left before the end of the school year. When I asked how he knew that, he told me that he counted 5 times 24 since there were only 5 days left until the summer holidays. Using this cue, I asked the students if they could tell me a quick way to check Tom's calculations. After some discussion it was determined that we could use the calculator. My intent was to see if the students understood the multiplication process enough in order to use the calculator as a tool to correctly calculate numbers greater than picture drawing would allow. Tom demonstrated how the information could be entered on the calculator in order to get the correct answer.

This interaction again demonstrated the impromptu ways in which math skills can be practiced and incorporated into student discourse. During this brief exchange I was able to reinforce the term *groups of*, refer to hours in a day, review knowledge of time frames, (24 hours in a day), and demonstrate the use of a calculator as a tool to help with more complex calculations. These activities were not pre-planned but resulted from my growing awareness of the connections that could be made between the curriculum strands and how this knowledge can be made relevant to students in their day-to-day interactions. In doing so the students were provided with opportunities to apply their knowledge and extend their thinking.

Both of these previous examples demonstrate the learning potential that can be accessed when the students are provided opportunities to share and communicate their ideas. These ideas emerge from the students' own thinking about problem solving and as such have more meaning than contrived situations created by the teacher. By listening

carefully to the students' explanations and thinking processes I was able to take advantage of these teachable moments and connect their learning with other mathematical concepts. These connections serve to strengthen their understanding, and ultimately allow the students to apply their knowledge in meaningful ways.

9. Book Modeling - Creating a Multiplication Story Book

The final set of activities planned for the multiplication unit were designed to integrate mathematical knowledge into a format that combined story telling and written language. The students by now had many experiences working with the multiplication algorithm and had met the curriculum goals that had been set. They were able to apply their knowledge of the multiplication process in specific problem solving situations and were communicating their thought processes competently. Many students were demonstrating sophisticated thinking beyond what I had intended to achieve, but this was being displayed in situations outside the realm of the mathematics class. In order to extend this mathematical thinking and provide a venue for creative expression I decided on a book modeling activity. Book modeling was a technique I had used previously in the year for language arts projects, and it had proven to be a motivating and stimulating experience for the students. It involved selecting a popular children's picture book with a very structured, repetitive format or textual frame. The teacher reads the book several times to the students so that they can internalize the language frame. The students are then instructed to use the same frame for creating their own story, incorporating their own ideas into the textual frame. All students, with various levels of teacher assistance and input, can complete this activity and the results can be very rewarding as most students are guaranteed success. Students have the opportunity to illustrate their stories

and then read them to the class during *authors' corner* sessions. The resulting stories can be compiled into a class book or made into individual student books that can be published for classroom use. In past experience I found these student books to be very popular in the classroom during reading time, and their repeated use by the students served to reinforce the skills that I was teaching. My feeling was that if I could combine the math skills the students had learned during the multiplication unit with skills in reading, writing, and illustrating, they would develop a deeper understanding of the mathematical processes they were learning. Not only this, they would be developing the idea of mathematics as a form of communication.

I based the modeling activity on the book *Each Orange Had 8 Slices* (Giganti, 1992). The book does not provide a continuous story as such, but rather a series of colorfully and precisely illustrated vignettes that pose problems for the reader to solve. Each page presents a scenario in which sets of items are combined in various multiple forms, followed by a series of questions that can be answered by counting the individual items in the illustration, counting in multiples or using a multiplication algorithm. The first page of the story starts off with:

I was on my way to the playground I saw 3 red flowers. Each flower had 6 petals, each petal had 2 tiny black bugs. How many red flowers were there? How many pretty petals were there? How many tiny bugs in all?
(Giganti, 1992, p.1)

The second scenario uses the same frame posing a different problem to the reader:

I was on my way to school I saw 3 little kids. Each kid rode a tricycle.
Each tricycle had 3 wheels. How many little kids were there? How many

tricycles were there? How many wheels were there in all? (Giganti, 1992, p. 2)

I presented the book to the students at story time rather than in math class, as I wanted to promote the idea that math is an integral part of their daily lives. I was taking my cues from what I had experienced during the Morning Meetings and at other sharing times that had occurred in the preceding days. “Math talk” was starting to take a significant role in the classroom due to my systematic observations and conscious efforts to make mathematical connections during student discourse. As I read each vignette in the storybook and posed the questions, students took turns in demonstrating how they solved the problems. I set up an easel and chalkboard in the story corner so that I could illustrate more abstractly the solutions that the students offered. The book was a very useful tool as it presented the problems in multiple layers that addressed various levels of understanding and problem solving skills. Some students were ready to develop the algorithms immediately, while others were using repeated addition or skip counting. A few students still had to count each item individually to ensure their answers were correct. At the more abstract mathematical level, I was able to demonstrate that the multiplication algorithm, in most cases, could be written in at least two ways and the answer would be the same, thus providing early exposure to much later mathematical concepts in number theory involving factorization and further reinforcing the associative property of multiplication. An example of this was demonstrated in the following exchange:

Teacher: On my way to the to the store I saw 4 trees. Each tree had three bird’s nests. Each bird nest had 2 spotted eggs. How many trees were there? How many bird’s nests were there? How many spotted eggs were there in all?

Tony: 24
 Teacher: How did you know?
 Toy: I counted in 2's
 Teacher: How many 2's were there?
 Tony: 12...It was 12 times 2.
 Teacher: Did anyone do it differently?
 Suzie: I counted by 4's
 Teacher: How?
 Suzie: Well I counted 4 trees and there were 6 in each.
 Brady: It's 6 plus 6 plus 6 plus 6 plus 6...that's 4 times 6
 Zach: I don't know ...it just popped up when I looked at the numbers - 24

Zach was an intuitive student, and despite being in grade one, he was starting to recall many multiplication facts with ease. He was also making some interesting connections between certain products and the combination of factors that create them.

The next step in the process was to model my own version of the story frame and invite the students to provide ideas for our story vignette. Once this was complete the students were then given the task of creating their own story page and problem set, complete with an illustration that would provide the answer to the problems posed. This project took several days to complete and proved to be a challenging task. Creating a story was difficult for the students, but once they had decided on their characters and sketched their illustration, they were able to develop the appropriate sentences to create their own multiplication story. The individual stories were collated to produce a class book modeled on the frame developed by Giganti (1992). The final step in this process was to have each student present his or her story page to the class and give class members the opportunity to solve the multiplication problems they had developed. (Samples of theses stories can be found in Appendix L).

The sharing activity proved to be motivating as each student eagerly awaited his or her turn on the authors chair. As each page was read the students were presenting their

own written work, speaking in front of an audience and challenging their peers to apply their mathematical skills to solve the problems presented. Students were communicating mathematical ideas with one another and practicing reasoning skills in order to make convincing arguments. As a teacher I found this to be rewarding conclusion to the unit of study. There was a high level of student interest and motivation during the activity and the book became a popular addition to the classroom library. As the days passed many students were observed rereading the stories and attempting to solve the multiplication problems. The students had demonstrated ownership of their own learning and were independently motivated to review and practice their skills.

On a final note, I found the subjects the students had chosen to write about and illustrate in their vignettes to be revealing. Many had chosen to include northern BC locales and local wild life, thus reflecting their personal connection with the mathematical principles they were studying, and the internalization of the learning processes.

Summary

This chapter provides a detailed account of how a combined group of grade one and grade two students developed an understanding of the concept of multiplication. Through a series of guided classroom activities, the students were encouraged to explore and discuss their interpretations of the multiplication process. The unit culminated in an integrated language arts and mathematics story writing activity. Thus highlighting the vital connection between numeracy and literacy. Using action research methods of reflection, analysis and further action the chapter also details my personal learning process as the teacher conducting the study. My learning focused on developing the

ability to listen to the students, assess their understanding, and use their cues during exploration and discussion to develop responsive and meaningful instruction. The following chapter provides a summary of this reflective process and identifies major themes that developed as the data was compiled and analyzed.

Chapter Five

Reflections and Conclusion

My original research question was to examine how teaching to the NCTM Curriculum and Professional Standards might enhance teacher and student learning in one elementary mathematics classroom. The Standards were used in conjunction with the regularly mandated BC mathematics curriculum to teach an introductory unit in multiplication to a combined group of grade one and grade two students. In order to answer this research question, I used action research methodology to plan and conduct the investigation. Through the use of ongoing cycles of action, reflection, and further action I was able to develop an enhanced knowledge of my students' learning and at the same time was able to identify and reflect on my own learning as a mathematics teacher at the elementary level. In this chapter I will report on my findings by examining two central research themes that emerged from my analysis. I will use these themes to address each of the four sub questions in turn, highlighting how the themes provided answers to my research questions.

Research Themes

In carrying out the action plan, two key research themes permeated the activities and grew in significance as I observed and interacted with the students. These themes were noted in the Action Plan and Data Collection Matrix (Appendix A), and centre on a) the notion of mathematics as an element of communication, and b) the mathematical connections within related conceptual strands and between mathematics and daily living experiences. These themes became a lens through which I explored my questions relating to the students' understanding of the multiplication process, and how they

demonstrated that understanding. In determining answers to my initial research questions, I was also simultaneously considering how teachers grow in their awareness of their students' understanding. Through this ongoing reflective process I was able to generate answers to my final question concerning responsive and effective instruction.

Student Learning

My first question asked how students develop an understanding of the concept of multiplication using a constructivist approach to learning. The subsequent question asked how the students demonstrate their understanding of the multiplication process. In answering both questions I will refer back to the themes of communication and connections in order to highlight the student learning that was observed in my classroom.

The NCTM Standards promote the goal of developing mathematical literacy for all students using a constructivist approach to learning. Mathematical literacy not only refers to factual knowledge but also includes the intellectual processes that support and verify mathematical knowledge. When teaching to the Standards, the focus in the mathematics class is not necessarily on the content being studied but rather on the processes used in acquiring the content knowledge. These processes are in fact included as content within the standards and are critical skills deemed necessary in developing mathematical literacy. In the context of this study I was initially addressing Standard Seven and the subject of whole number operations. However, in order to present this topic according to the philosophy of the Curriculum Standards, I was required to incorporate the process standards of problem solving, communication, reasoning, and connections (Appendix C). These process standards then became tools through which

students engaged in the learning process. The process standards also provided the framework through which constructivist methods of learning could be developed.

Constructivist philosophy emphasizes the importance of students making connections between previous learning and the effective processing of new information. These connections are made when the students are actively engaged in the learning activities, and when they are given opportunities to interact and communicate their ideas with their peers. As I focused on the process standards and attempted to create an environment to support constructivist learning, it was not surprising that the themes of connections and communication gained increasing prominence; both were fundamental tools necessary for meeting the process based strands.

From my previous experience teaching at this grade level I was aware of some of the basic mathematical skills that were necessary for students to use in developing an understanding of the multiplication process. Through informal practice of these skills during the Morning Meeting I was consciously preparing my students with the foundational skills to which they could connect new information. Prior to the multiplication unit, the students had already been familiar with skip counting in twos, fives, and tens, and were aware of the pattern sequence in these numbers. They had also been exposed to the concept of odd and even numbers, and the idea of doubling numbers in order to help learn addition facts. During the discussion process, I saw clear evidence that the students were applying this prior knowledge in order to make sense of the new information that they were presented. This application of prior knowledge appeared to support the constructivist approach to learning referred to in my initial research question. Students were developing an understanding of the new concept by connecting the ideas to

mathematical information they already knew. Some students were able to make these connections independently while others needed guidance through teacher direction or structured discourse. The result was a greater awareness of the patterning relationships within our mathematical system and the application of this knowledge to the study of multiplication.

The Circles and Stars activity provided an example of how students can be guided through a process of making connections to develop understanding. Starting with the physical activity of rolling the dice, then transferring the information on the dice to a diagram, the students moved from the concrete to a pictorial representation of the information. This information was then expressed mathematically in an addition algorithm that was already familiar to the students. From this algorithm the students were provided with the terms *groups of* and *times* to describe their pictures with words. The final step in this process was to replace the now familiar terms with mathematical symbols. Throughout this process I was conscious of developing the connections between previous mathematical knowledge in addition, while attempting to guide the students in making the connection that multiplication was a process of repeated addition and a shortened way to communicate information. Some of the students made these connections quickly while others needed repeated experiences in order to make the connections and develop their understanding of multiplication.

Further evidence of students' relating prior information to develop new understanding was provided during the exploration of number patterns on the hundred chart. Students' awareness of visual patterns on the chart led to a more complex examination of number patterns, thus connecting number theory and the base ten system

with their developing knowledge of multiples. For students who were ready, I was able to present the concept of digital roots of numbers, which in turn provided the basis for further inquiry and problem solving. As the students explored the concept and presented their ideas about the multiplication process in the group sharing sessions, they were creating the groundwork from which further mathematical connections could be made. The commutative and associative properties of the multiplication algorithm were demonstrated without any formal mention of these ideas. Also, during the literature exploration at the end of the unit the students were introduced visually to the idea of common factors. This visual connection could be expected to lay the foundation for the understanding and application of these principles in subsequent grade levels during the study of fractions and ratios.

Another aspect of the connections theme is the links that can be made between mathematics and daily living experiences, thus providing meaning for the skills the students are learning. The role of the teacher in the Standards-based classroom is to be aware of issues important to the students and to use these as a way to demonstrate how mathematics is important to their lives. Some of this can be planned, such as the chopstick problem, and some opportunities are provided incidentally as students talk about their experiences and share their interests. When Eddie shared his story of success on the running track, I was able to relate the question posed regarding the distance he ran, to the students' knowledge of multiplication. When Leo announced he knew how many hours were left in the school year, he was able to demonstrate to the class how he used multiplication to solve the problem. Both these examples took place during the Morning Meeting and as such incorporated math talk into the daily routine of classroom activities.

As I drew on students' personal experiences and guided them in making mathematical connections, I provided opportunities for the students to learn to apply their knowledge of multiplication in meaningful ways and thus develop a greater understanding of the concept. Again, the connections theme also addresses the initial question regarding the development of student understanding.

The final activity in the multiplication unit was designed to make important cross-curricular connection between numeracy and literacy. Students were not only creating a story, but also developing and presenting the mathematical problem underlying the story. They demonstrated their understanding through pictures and written form using ideas from their personal experiences. In presenting their stories to the class the students were viewed as authors and mathematicians by their peers, and the class publication of their work became a powerful expression of their success in articulating mathematical concepts. This final activity not only served to consolidate students' knowledge of the multiplication process but also highlighted the theme of communication, which relates to my second question, about how students demonstrate their understanding of the multiplication process. In creating their individual stories the students were not only demonstrating their depth of knowledge, but at the same time they were communicating their thinking and ideas to their peers.

According to the NCTM Standards, effective communication of mathematical ideas is central to the notion of developing mathematical literacy (2000). In the Standards based classroom, ongoing communication among the students, and between the teacher and individual students is the key to developing mathematical understanding. The communication forum is focused and purposive, and orchestrated by the teacher so

that students can demonstrate their thinking, and at the same time refine their knowledge as they attempt to articulate their ideas. It is through this process that teachers can become aware of their students' level of understanding.

Communication in the context of this study encompasses both oral and written forms of expression, and I had intended to nurture both aspects in the classroom. I started the unit by introducing the math journal. Although the students were familiar with keeping personal journals, the idea of writing about math or illustrating math ideas was quite foreign to them. The purpose of the journal was twofold. First I wanted the students to develop the idea that math learning and knowledge could be represented in many ways: in written form, in diagrams, or in pictures. Secondly, as the students became more adept at representing their thinking in their journals, the journals could be used for evaluation purposes in conjunction with other assessment procedures in the classroom. I wanted the students to be able to think and reflect on their learning and use a variety of avenues to express their mathematical knowledge. The initial attempts at using the journals were not too successful as the students had a very narrow concept of what a journal should be. They spent more time thinking about correct spellings and what the "right" answer should be rather than focusing on their mathematical thinking. I realized that in order for them to be successful they needed some structures to follow just like the frame sentence structures used to teach sentence construction in language arts. However, I did not want the focus of the journal to be just words. The solution to this dilemma came with the Circles and Stars activity. Working through this activity students became aware of the various ways mathematical knowledge could be represented, starting with the pictorial representation, sentences, addition algorithms, and finally the

symbolic representation of the multiplication algorithm. Once the students had been given these methods to work with, the quality of the journals improved and the students were able to spend more time reflecting on their mathematical knowledge. Students realized that if they drew a picture of their understanding it became much easier to describe their thinking processes. Students were not limited to the structures in the Circles and Stars booklet, and as the unit progressed they were exposed to other forms of mathematical representation, which also served to extend their literacy skills. The creation of class lists and charts, tables (patterns in multiples), designs (patterns on the hundred chart), all demonstrated alternative ways of displaying and interpreting information.

As the students began to internalize and use these structures independently in their journals and in their group work, they were able to demonstrate their thinking and understanding of the multiplication process in a variety of ways. This allowed me, as the teacher, to gain greater insight into their thinking processes. In accepting and encouraging alternative forms of expression I was able to better understand the cognitive processes of my students and respond appropriately to their level of understanding.

Another important aspect of communication in the Standards-based classroom is the emphasis placed on social interaction and student discourse. The constructivist approach to learning supports the notion that knowledge is created through personal interactions, exchanges of ideas, and negotiation of meaning. In a constructivist classroom the teacher is regarded more as the facilitator rather than the centre of knowledge. Knowledge is created when students engage together in meaningful activities, identifying problems, sharing ideas, and cooperating to find solutions. By

engaging in these activities the students are helping each other make the connections that aid in developing the understanding that was referred to in question one.

Incorporating discourse in the mathematics class was a key element in planning the multiplication unit. I developed a selection of activities that would require students to work in small groups, and then report their findings back to the large group. The benefit of this approach was that students were given responsibility for their learning and had to take an active role in decision-making. The strategy also provided me with the opportunity to observe students working together, assess their progress on an individual level, and provide specific assistance to those students requiring extra help. Further, small group work and large group reporting created an opportunity to attend to the needs of more advanced students by providing additional activities to challenge their thinking. As a result, I was able to individualize student learning. By removing myself as the central focus in the learning process I was able to spend more time listening to my students and I became more aware of their level of thinking. As a result I could individualize my instruction to meet the different needs of my students.

By constantly changing the composition of the small groups I was able to encourage greater discourse and the sharing of ideas. The sessions generally ended with a large group reflection and sharing time, and it was during these periods that students could share their knowledge with the whole group. Their ideas were collated on class charts that were posted in the classroom for the duration of the unit. Recording and displaying student thinking served to validate new ideas and provided modeling for all students. As the class revisited and reviewed them, these charts and posters became powerful tools in the learning process. They not only provided modeling for the

expression of ideas but also provided opportunities to reflect back on past activities and make the vital connections that would develop understanding.

The sharing of ideas in both the large group and small group situations addressed the third Standard, which emphasizes the process of reasoning in mathematics. My constant refrain during sharing times became, “How do you know that?” This question challenged the students to justify their answers and solution processes, and in doing so, provided student peers with further modeling and examples of alternative approaches to the problem solving. When Sara explained her creative way of thinking about the tallies and producing the 5×4 algorithm instead of the obvious 4×5 , she was able to demonstrate and explain her thinking, and at the same time inadvertently provided proof for the commutative property of multiplication. In this scenario Sara was being challenged to articulate her thinking and demonstrate her knowledge. She was modeling information and providing opportunities for her peers to make connections that would assist in their developing understanding. At the same time, by acknowledging her thinking processes, I was able to initiate an informal discussion on the commutative property of multiplication, thus providing the foundation for future connections. At the end of this session Sara had demonstrated her knowledge to me and I was able to take her lead and respond by introducing further concepts to challenge and extend student thinking. Through careful listening to what my students were saying and recognizing the opportunities to make connections I was addressing the issues raised in my research questions concerning teacher learning and at the same time I was developing student understanding.

The ability to reason and explain positions is not only important as a way to make sense of mathematics, but it is also a fundamental skill that is applied to many areas of

the curriculum. I believe that students who develop and refine those skills in the mathematics classroom will be able to transfer the skills and apply them in many areas of scientific enquiry or in solving day-to-day social problems. Thus teaching to the Standards in the mathematics class provides opportunities to enhance the general processes of student learning.

Through an examination of these classroom scenarios it is interesting to note the reciprocal relationship between the themes of communication and connections that became apparent. This relationship also underscores the nature of the constructivist approach and addresses the research questions of how students develop and demonstrate their knowledge. Through communication in its various forms the children displayed and demonstrated their knowledge. In sharing and discussing that knowledge with their peers the students were assisting each other in articulating their ideas, and making the connections that will eventually help them deepen their understanding. As new learning emerged it was presented, discussed, applied, and analyzed so that further connections could be made. The teacher's role in this cycle of knowledge development is to listen and guide so that the critical connections can be made. However, as the teacher listens and guides she is also making connections between her knowledge of students' cognitive processes, her knowledge of the mathematical content she is teaching, and the social constructs that need to be created in the classroom that allow for effective communication. As the teacher becomes conscious of these connections she is able to apply this knowledge in order to plan for responsive and effective instruction.

The development of teacher knowledge and planning for effective instruction addresses my final two research questions, and highlights the fundamental difference

between using a Standards-based approach and the more traditional teacher and text centered method.

Teacher Learning

Using the Standards as a guide for planning this unit of study forced me to examine and change the way information would be presented to the students. The specific learning outcomes for grade two students are succinctly laid out in the BC Mathematics Curriculum (1995) and focus on specific skills to be demonstrated by the students. For example, the grade two learning outcome for multiplication states, “students will explore and demonstrate the process of multiplication up to 50 using manipulatives, diagrams and symbols”. Although student understanding is implied through the various modes of representing knowledge, the general teacher orientation would be to guide the students in the demonstration of a specific skill. The Standards on the other hand, require the teacher to focus on the processes that children use to construct their own understanding: by using problem solving strategies, by relating mathematics ideas to their own lives, and by representing their understanding in ways that make sense to them.

With the Standards approach the emphasis in planning moves from the “what” to the “how” of learning: how is it that as teachers we can best facilitate opportunities for learning through the use of processes of mathematical learning? It is the practice of becoming mathematically literate that is emphasized. By focusing on the process of student learning and engaging in cycles of personal reflection, which also required me to consider the contexts of successful mathematics learning, I shifted my thinking from models *of planning* to pedagogies *for learning*.

The data matrix (Appendix A) was particularly helpful in guiding my shift in focus from planning to learning. The initial framework began as a series of action steps that would traditionally have been the basis for a unit plan. However, the action steps were heavily influenced by the curriculum standards that I wanted to emphasize. After examining the curriculum standards, I was able to design class activities that would promote the mathematical processes, and then plan how these processes would be incorporated throughout the unit. This information, and my eventual goal of meeting the BC learning outcomes, were laid out before the study began. The changes in the teaching process occurred as I collected the data, analyzed it, and reflected on its meaning in the context of the classroom activities. The tapes, journals, and student work samples all provided information on student thinking, and I found that I started to focus more on how the students were learning. I began using this knowledge to adjust the planned action steps. In reviewing the collection of data over a period of time, I was able to identify the themes of connections and communications that became significant in the study. I could then focus on promoting these themes in subsequent activities as they provided the basis for developing student knowledge.

The circle and stars game is a good illustration of the shift in my thinking from planning for instruction to pedagogies of teaching. In the past, my sequence of learning activities would have begun with the concrete and then moved to symbolic representations of these in a staged process. For example, asking the students to create groups of objects and count them would have been followed by direct instruction in the multiplication algorithm as a substitute for counting. The circles and stars game provides a different approach because it demanded that students make connections between the

verbal, the visual, the concrete, and the pictorial in a process that required their ongoing engagement in making connections between the different forms of mathematical representations for the operation. Rather than rote completing worksheets that followed a pattern of algorithmic development, the circles and stars game demanded that students be able to describe, name, and illustrate the mathematical ideas represented in multiplication.

The shift in teacher focus from *what* to the *how* is further illustrated in the data matrix as new action steps were incorporated into the overall plan as the unit progressed. As my awareness of how students were learning increased I was able to guide students in making connections between their mathematical knowledge and how that knowledge could be applied in their daily lives. The Morning Meeting, although not originally part of the math unit, started to become a valuable avenue for students to express their mathematical learning. Through a conscious effort on my part to include multiplication concepts in our daily opening activities and discourse, the students were given a chance outside the regular math class to display their knowledge and provide me with valuable insights as to how they were able to apply their emerging skills. It was also a venue for students to share their stories, and as I listened I became more vigilant in finding ways to connect students' mathematical knowledge to events in their lives, thus reinforcing the idea that mathematics is a relevant and important element of their daily experiences.

The value of discourse in enhancing student learning was also demonstrated during the, "What comes in groups of...?" activity. Students discussed and recorded their ideas amongst themselves, and were then given the task of continuing the activity at home with family members. The result was the construction of large charts recording

many familiar and unfamiliar items on the theme of multiples. In reviewing these items the students had an opportunity to explain and justify their contributions, and at the same time introduce new vocabulary and ideas to the class. The activity also allowed me to highlight certain elements of word structures (“tri”, “quad”) that could be applied in further literacy activities. In presenting the challenge of finding patterns on the hundred chart, I was able to introduce descriptive terms such as horizontal, vertical, and diagonal that were later used by the students to describe the patterns both orally and in their journals. Quite often the language used in unique and novel situations appeared to have a particular resonance with young students, and their ability to remember the terms and apply them was elevated. Through this communication process the students provided me with cues that I could use to make the necessary connections to develop understanding. This discussion was not planned in the action steps, but emerged as I responded to the information my students provided.

Evaluation of student performance became an important priority over the course of the study, as it was a critical means by which I could assess learning in the day-to-day activities, and adapt my plans to develop responsive instruction. Increased emphasis on evaluation to inform instruction was not just an important element of this study as a formal research project, but represented a necessary element to teaching in a constructivist-learning environment. Van De Walle (2001) refers to this as formative evaluation, which he believes should be a continuous process that informs practice. Traditional concepts of evaluation are more summative in nature and are regarded more as an event at a specific time rather than an ongoing activity by the teacher. Through the

process of collecting and analyzing data during this study I was conducting formative evaluations not only on my students but also on my own pedagogical learning.

The variety of activities, flexibility in groupings, and discussion sessions all provided me with new and alternative ways to assess my students' learning. Focusing on what the students were saying, and becoming more aware of the knowledge that they were displaying in various forms, allowed me to continually assess their learning in different ways, and adapt my teaching to accommodate their needs and interests. The use of the math journal, student made booklets, discourse formats, narrative sessions all represent the shift I was making from planning for skill development to pedagogies and practices that supported student learning. This form of reflective practice was a necessary step in developing my own knowledge and understanding of the students' cognitive processes, and assisted me in formulating questions and hypotheses regarding the nature and characteristics of student learning. Communicating with, and listening to students' explanations and stories provided cues to connect their learning with their previous experiences and thus provided the framework or scaffolding for subsequent learning.

The data matrix not only served as a framework for planning and analyzing student learning, but it was also from this analysis that I was able to reflect on my pedagogical knowledge and challenge myself to become a better mathematics teacher. My final question asks how teachers can use their knowledge of student understanding to plan for responsive and effective instruction when teaching the concept of multiplication. This question specifically addresses NCTM Professional Standard Six, which states,

“the teacher of mathematics should engage in ongoing analysis of teaching and learning”.

The implication is that the learning process for the student and the teacher is ongoing and that each teaching and learning situation is unique to the set of individuals involved.

Planning for Effective and Responsive Instruction

The experience I gained teaching the multiplication unit reaffirmed my commitment to the constructivist approach to teaching mathematics at elementary school level. As the unit progressed and I became more attuned and responsive to the students ideas, my awareness of the connections between ideas heightened. I found that I was training myself to become a vigilant listener to student discourse, and was continually assessing how I could use that discourse to further the students’ knowledge and understanding. This notion of careful “listening to students” and making decisions during the midst of instruction is a fundamental teaching skill highlighted in the Professional Standards (1991). It is one that continues to evolve with practice. Sherin and Van Es (2003) refer to this listening skill as “learning to notice”, and distinguish it from other forms of professional development which generally focus on “learning to do”. My skill at “learning to notice” continued to evolve as I used the reflective process. Initially the reflections occurred after the classroom episodes had taken place, but as the unit progressed I found that I was using the reflective process during the classroom episodes and adjusting my discussion format to adapt to needs and experiences of my students.

The action research method of enquiry that I used for this project allowed me to train myself in this “learning to notice” process and proved to be a very instructive form of inquiry. The vignettes of student learning presented in this study support the belief

system necessary to implement a Standards-based constructivist view of mathematical learning outlined by Nelson (1997). Nelson identified four key beliefs that teachers need to adopt if they are to make substantial shifts towards a Standards-based classroom: a) that teachers need to view students as learners who are intellectually generative, b) instruction can be based on the development of student thinking rather than relying predominantly on text-based instruction, c) that the text is not the focus for intellectual authority but rather, authority for learning is negotiated by the teachers and students through discussion and debate generated in the classroom, and d) teachers and students can use the mathematical modes of reasoning to generate and validate mathematical knowledge. As this study has illustrated, each of these has become fundamental elements of my changing approach towards planning for learning rather than planning for instruction.

As I reflect on my practice I am continually learning from my students by listening to their stories and responding to their cues. However, I am also becoming more cognizant of the knowledge and supports necessary to sustain this change process. In my literature review I outlined the three perspectives on teacher change that have been articulated in the literature: the psychological perspective, the sociological perspective, and the knowledge perspective. These perspectives provide a framework for analyzing my own learning in relation to this research project.

The Psychological Perspective

One of my main goals in this research project was to focus on student thinking and use this as a guide to determine my action steps. This was a conscious attempt to apply the principles of constructivist learning in the classroom. In order for this

application to be successful it was necessary for the students to have many opportunities to explore concepts and share their knowledge with their peers. My ability to listen to what the students were saying and respond effectively was limited to my own experiences with the subject matter and the literature available concerning students' mathematical reasoning in relation to teaching the multiplication concept. The work being carried out in the CGI project (Carpenter et al. 1999) provided some models of student thinking; however the collection of data was limited. Without these models teachers' experiences are limited to the specific events in isolated classrooms. As my study progressed I realized that my interpretation of the data I was collecting would have been enhanced and informed if I had access to the work and experiences of other teachers engaged in similar tasks.

However, as this unit evolved, and I reflected on the class discussions, I was becoming more aware of the knowledge cues the students were providing. With this information I was able to lead the students towards making connections between the ideas they were exploring in the mathematics class and the personal stories that they were sharing. The idea of "learning to notice" is a fundamental aspect of the psychological perspective on professional teacher development, and also reflects a constructivist approach to professional learning. Through the classroom experiences illustrated in this study I was able to construct my own learning environment and test my knowledge of student cognitive processes as I listened and interacted with my students.

The Sociological Perspective

The effective transmission of ideas within the constructivist classroom is dependent on the development of social structures that allow students to express

themselves effectively. My role as the teacher in this project was to develop these social learning structures. The social structures I used were not developed solely in relation to this project, but were the result of a continuous process of reflection and refinement throughout the year. As this project progressed I found that I was continually reflecting on the classroom structures that I had employed to present the math activities. My concerns centered on issues such as: the effectiveness of small group activities as opposed to large group activities, providing opportunities for all students to express themselves, providing effective listening and speaking forums for students, maintaining a balance between *hands on* activities and discussion, and maintaining the curriculum goals while allowing students some direction in the learning process. I now understand these social management issues to have a direct impact on the effectiveness of a constructivist approach to learning and need to be addressed continually to support communication and the sharing of ideas within the classroom. In order to plan for responsive and effective instruction, I believe these social structures need to be supported and nurtured so that all individuals within the group have equal opportunities to contribute to the learning environment.

The sociological perspective on teacher learning emphasizes the key role teachers play in developing the classroom social structures that support constructivist learning. Evidence from this study provides examples of these social structures supporting learning. These structures should be dynamic and constantly evolving. Through reflective practice teachers can examine their classroom structures and continuously modify them in order to meet the individual learning needs in the classroom.

The Knowledge Perspective

The theme of *connections* refers to two ideas in the context of this study; connections between the mathematical concepts and personal experience, and the connections between mathematical ideas. In order for the teacher to demonstrate the mathematical connections that will develop understanding, she must have a clear picture of how each concept relates to the total curriculum. My previous experience of teaching mathematics at a variety of grade levels provided me with some insight into how the mathematical skills are developed. As I learned to pay closer attention to what the students were saying and the mathematical knowledge they were expressing, I was becoming more aware of the mathematical connections I could draw on during the discussion. Also, I found that the discussions led to the informal introduction of concepts that would not be introduced until later grades. By informally exploring these concepts the students were starting to develop the vital connections that would provide the basis for new knowledge.

The knowledge perspective emphasizes the need for teachers to examine their own mathematical knowledge and become more informed in the area of mathematical content. In so doing they are better able to guide students in making the mathematical connections that develop understanding. It requires that teachers not only be vigilant listeners to their students, but also questioners regarding the limits of their own knowledge.

Clearly these three perspectives on teacher change do not operate in isolation from one another. The free flow of ideas and information in the classroom requires the

development of social learning structures that promote communication. The teacher's ability to guide students to make the learning connections requires that she have a sound knowledge of the subject matter, and the ability to highlight its relevance for the students. The vigilant teacher is constantly listening to and interacting with her students in order to develop insight into their cognitive processes and create effective learning environments.

Action Research as a Vehicle For Change

Research into teacher change recognizes the significant role of teacher generated professional development in the change process (Fullan & Hargreaves, 1992; Nelson, 1997; Wells, 1994). Teacher initiated action research can provide a vehicle through which change can be accomplished. Reflection on the part of the teacher researcher leads to the development of new knowledge, which in turn affects further actions. By consciously reviewing and analyzing the classroom activities I became more aware of the teaching and learning possibilities in my own classroom. Going through the action research process has trained me to be more critical of my own teaching, and more cognizant of the daily events in the classroom that can lead to meaningful interactions and the application of knowledge. This new awareness is evident, not only for mathematics education but applies throughout the curriculum. Becoming more aware of how the students are thinking, and then using those cues to plan instruction has helped me to create more responsive and effective learning environments.

Limitations of the Study

This research presents a small snapshot of a series of activities in a primary classroom that focused on the processes used to teach a set of learning objectives in mathematics. As such, the information gathered is quite specific to a particular time,

place, and a unique group of individuals. The approach used to reach the learning goals, namely my interpretation of the NCTM Curriculum Standards, is a specific set of principles on which the mathematics instruction is based. There is no prescribed method for implementation of the standards, and successful implementation is dependent on the skills and knowledge of individual teachers and their ability to adapt to a new way of thinking about mathematics teaching. The decisions made during the study were based on my particular interpretation of events and were limited by my knowledge of student cognitive processes and my understanding of the mathematical content involved. The constructivist approach used in the lesson format necessitated that the students be given some control over their own learning. As such, their unique interests and needs were addressed throughout the process and this affected content of the lessons. The result was that the learning outcomes of many of the lessons could not be predicted or replicated. However, the study provided some clues on the approaches that may be used with students in order to develop their mathematical thinking and ability.

Implications For Practice and Further Research

The results of the research indicate that the adoption of the NCTM Standards in a primary classroom enhanced the learning environment and provided opportunities for the students to deepen their knowledge and understanding of mathematical processes. Potential future studies could look at how to measure the success of student learning in a constructivist learning environment. The data matrix could be expanded to include both formative and summative evaluation as part of the action plan. Further research in this area might take the form of longitudinal studies that measure and compare the

mathematical learning outcomes of students in constructivist learning environments and those taught based on the transmission model.

Evidence was presented to support the observation that my own teacher knowledge was developed and enhanced through the process of analysis. The knowledge I gained through the study was useful for my own purposes and provided me with valuable insights in how to integrate mathematics teaching into the total curriculum. However, the study took place in an individual classroom, and as a researcher, I was isolated from other professionals in the field. The information gained is therefore only a small piece of evidence that can be added to the growing body of knowledge concerning elementary mathematics education. In order for change to occur, teachers working in the field need to be able to share the knowledge they are gaining in the classroom with other colleagues and be given the opportunity to explore alternative methods of instruction. For change to occur there needs to be a collaborative effort to work together and compile the collective knowledge of practicing teachers. In much the same way that we want our students to collaborate and share ideas in order to create new knowledge, teachers need to be engaged in the same process. This study has only lasting value if it is shared with colleagues for the purposes of developing a larger body of knowledge about mathematics instruction.

Collaborative action research would be the next step in this process of teacher development. This method of research and professional development involves teachers working together in planning a series of specific actions to take place in their individual classrooms. Through meetings, sharing their results, analyzing others' interpretation of events, groups of teachers can gain greater understanding of the learning processes they

are developing in their mathematics classroom, both in terms of teacher learning and student learning. They can then apply this accumulated knowledge in their particular learning environments to improve the quality of instruction, and ultimately enhance the learning potential for students.

Conclusion

The content of this study represents the continuation of a personal journey of professional development and growth in the area of mathematics teaching at the primary level. My interest in using alternative methods for teaching mathematics began in the early 1990s when I served as a teacher representative on the Manitoba Mathematics Curriculum Committee (K-4). A recent return to teaching at the early primary level and the new initiatives implemented by Prince George School District 57 in mathematics education rekindled my interest in teaching methodologies. My decision to pursue graduate studies at the University of Northern British Columbia provided me with the opportunity to examine my own professional values and beliefs. In particular, it allowed me to continue to explore my interest in mathematics education.

Although my research question is broad in scope and refers to general NCTM principles of teaching mathematics at the elementary level, I attempted to answer the question by conducting an in-depth examination of both student and teacher learning while implementing a unit of study on the concept of multiplication. It is a story of my personal interpretation of the Standards and how I learned, through personal experience, to apply them in my classroom.

The literature review presented in this document examined the evolution of the mathematics reform movement and highlighted the principles of the NCTM standards. It

also explored the notion of teacher change in mathematics education and alternative approaches to teacher professional development. Using an action research approach with its cycles of action, reflection, and further action I was able to examine and reflect on my experiences in implementing the NCTM Standards in mathematics education. The design of the research used multiple methods of data collection: audio taped recordings, student work samples, group work charts, and a personal teacher journal. These various sources served to ensure trustworthiness of the data. The evolving action plan and accumulated data were organized for analysis using a data matrix. The matrix allowed me to systematically review the teaching and learning process and analyze the discourse transpiring in the classroom. The multiple sources of data for each action step provided various perspectives for analyzing the data.

Through a process of continuous reflection on daily classroom interactions, I became more cognizant of the students' thinking and learning processes. By focusing on students' expressions of understanding and connecting their knowledge with their experiences I was "learning to listen" and structuring my teaching to the needs of my students. As a result, my teaching became more responsive to the individual students in the group and continues to evolve as I train myself to be a more effective communicator in the classroom.

The results of the study demonstrate the intricate role that discourse and communication play in Standards-based mathematics classroom. It is a story of one teacher's adaptation to teach mathematics for understanding, and as such provides no definitive answers in how the NCTM Standards should be implemented. However, by going through this research process and documenting the events that occurred, the study

provides other teachers with information to better understand and support the type of teacher change that is crucial to the reform agenda in mathematics education. The information gathered in this research will be of particular use to me in my role as a Math Mentor for the Prince George School District. My task as part of this group will be to support and encourage teachers who are attempting to introduce a constructivist, Standards based mathematics program.

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Appendix A

Figure 1. Action Plan and Data Collection Matrix

ACTION STEPS	DATA COLLECTION AND ANALYSIS
<p>AS 1. WHAT IS MULTIPLICATION?</p> <ul style="list-style-type: none"> • elicit student information • class discussion • introduction to journal writing • journal assignment <p>Curriculum Standards 2, 3, 6, 13</p>	<p>Tape 1A - class discussion</p> <p>Student Journals - individual accounts of mathematical knowledge</p> <p>Class Chart - student generated ideas</p> <p>Teacher Journal - after class notes and general impressions</p>
<p>THEMES FROM ANALYSIS</p> <ul style="list-style-type: none"> • communication • oral / written • using language versus knowledge of language 	
<p>AS 2. CHOPSTICK PROBLEM</p> <ul style="list-style-type: none"> • teacher introduction to the problem • teacher / class discussion and problem solving activity • small group activity problem solving • groups share solutions to problem • individuals explain the solution in their journals <p>Curriculum Standards 1, 2, 3, 4, 6, 7, 13</p>	<p>Tape 1A – 1B - class discussion</p> <p>Student Journals - individual accounts of how the chopstick problem was solved</p> <p>Teacher Journal - impressions of the class discussion, small group work, solutions generated</p>
<p>THEMES FROM ANALYSIS</p> <ul style="list-style-type: none"> • oral communication • variety of groupings lg. ‘v’ small • listening skills • reasoning • written expression of ideas /journal 	

Figure 1. Action Plan and Data Collection Matrix (Continued)

ACTION STEPS	DATA COLLECTION AND ANALYSIS
<p>AS 3. OTHER THINGS THAT COME IN 2'S (LIKE CHOPSTICKS)?</p> <ul style="list-style-type: none"> • teacher led group discussion re objects in 2's • teacher records answers on class chart • homework assignment – brainstorm with family to generate a list of items that usually come in 2's <p>Curriculum Standards 2, 4, 6, 13</p> <p>THEMES FROM ANALYSIS</p> <ul style="list-style-type: none"> • real world connections • vocabulary development • brainstorming / whole group sharing, listening, taking turns • home /school connection 	<p>Tape 1B</p> <ul style="list-style-type: none"> - class discussion <p>Class Chart</p> <ul style="list-style-type: none"> - student generated ideas <p>Teacher Journal</p> <ul style="list-style-type: none"> - after class notes and general impressions
<p>AS 4. WHAT COMES IN GROUPS OF 2's, 3's, 4's¹</p> <ul style="list-style-type: none"> • <i>review of previous class and chopstick problem</i> • <i>review of possible solutions and difficulties finding solutions</i> • students share lists of 2's generated for homework • teacher reads "What Comes in 2's, 3', 4's?" • students work in pairs to generate lists of objects usually found in groups of 2's, 3's, 4's • student pairs share lists with whole group and teacher records on a class chart • whole class generated chart of objects in 5's and 6's • homework to generate family lists of groups <p>Curriculum Standards 2, 13</p> <p>THEMES FROM ANALYSIS</p> <ul style="list-style-type: none"> • language development • real world connections • literature connection 	<p>Tape 2A</p> <ul style="list-style-type: none"> • class discussion and review <p>Class Charts</p> <ul style="list-style-type: none"> • student generated ideas for groups of 2's • class generated lists for things that come in 3's, 4's 5's, 6's <p>Teacher Journal</p> <ul style="list-style-type: none"> • after class notes and general impressions of class discussions

¹ The use of italics indicates changes and revisions to the original action plan.

Figure 1. Action Plan and Data Collection Matrix (Continued)

ACTION STEPS	DATA COLLECTION AND ANALYSIS
<p>AS 5. CIRCLES AND STARS GAME</p> <ul style="list-style-type: none"> • students work in pairs each rolling the dice twice. The first roll indicates the number of circles to draw. The second roll indicates the number of stars to put in each circle. Students record the total for each picture using repeated addition i.e. $3+3+3+3 = 12$ • students continue until all the pages in their booklet are filled (10) • students who finish early use calculators to add up total number of stars drawn on all 10 pages • students identify highest and lowest totals in the class <p>Curriculum Standards 1, 3, 6, 7, 8</p>	<p>Tape 3A</p> <ul style="list-style-type: none"> • teacher instructions, student responses, student interactions during the dice game <p>Student Created Booklets</p> <ul style="list-style-type: none"> • pictorial representation of dice generated groups • symbolic representation of pictures using repeated addition <p>Teacher Journal</p> <ul style="list-style-type: none"> • after class notes and general impressions of class discussions
<p>THEMES FROM ANALYSIS</p> <ul style="list-style-type: none"> • need models of processes / repetition • language needs to be repeated and modeled with action • pictorial to words then add sentence 	
<p><i>Morning Meeting</i></p> <ul style="list-style-type: none"> • <i>incorporating mathematical knowledge (multiplication) with other classroom activities</i> • <i>whole group review and chalk board demonstration of Circles and Stars activity stressing “groups of”</i> <p>Curriculum Standards 1, 2, 3, 4, 13</p>	<p>Tape 3A – 3B</p> <ul style="list-style-type: none"> • class discussion <p>Teacher Journal</p> <ul style="list-style-type: none"> • after class notes and general impressions of class discussions
<p>THEMES FROM ANALYSIS</p> <ul style="list-style-type: none"> • connections to real world • problem solving • making math connections • applying / reasoning 	

Figure 1. Action Plan and Data Collection Matrix (Continued)

ACTION STEPS	DATA COLLECTION AND ANALYSIS
<p>AS 6. MOVING FROM THE PICTORIAL TO THE SYMBOLIC</p> <ul style="list-style-type: none"> • students describe each circle and stars picture in words i.e.six groups of three equals eighteen • repeat the sentence using numerals and the word “times” instead of “groups of” i.e.4 times 3 equals 12 • repeat the sentence using symbols i.e. $4 \times 3 = 12$ • students work in pairs checking each other’s work <p>Curriculum Standards 2, 3, 4, 6, 7, 8</p>	<p>Tape 4A – 4B</p> <ul style="list-style-type: none"> - class discussion <p>Student Created Booklets</p> <ul style="list-style-type: none"> • replacing word descriptions with mathematical symbols <p>Teacher Journal</p> <ul style="list-style-type: none"> • after class notes and general impressions of class discussions
<p>THEMES FROM ANALYSIS</p> <ul style="list-style-type: none"> • scaffolding for students, various levels of support • connect language and actions 	
<p><i>. Morning Meeting</i></p> <ul style="list-style-type: none"> • <i>incorporating mathematical knowledge (multiplication) with other classroom activities</i> • <i>whole group oral review of concrete to the symbolic</i> • <i>assessment activity “What I Know about Times”</i> <p>AS 7. REVIEW OF CLASS CHARTS - groups of 3’s, 4’s, 5’s, 6’s</p> <ul style="list-style-type: none"> • add to charts using family generated lists • whole class problem oral solving using groups of tricycles and number of wheels 	<p>Tape 4B, 5A, 5B</p> <ul style="list-style-type: none"> • class discussion <p>Student Journals</p> <ul style="list-style-type: none"> • independent writing assignment <p>Class Charts</p> <ul style="list-style-type: none"> • student generated ideas for groups of 3’s, 4’s, 5’s, 6’s <p>Teacher Journal</p> <ul style="list-style-type: none"> • after class notes and general impressions of class discussions
<p>THEMES FROM ANALYSIS</p> <ul style="list-style-type: none"> • connections between classroom activities, • constant review and modeling, • vocabulary, model of problem solving 	

Figure 1. Action Plan and Data Collection Matrix (Continued)

ACTION STEPS	DATA COLLECTION AND ANALYSIS
<p>AS 8. PATTERNS IN MULTIPLES</p> <ul style="list-style-type: none"> • <i>review assignment in journal – explain in words or pictures 6x4</i> • in pairs, students choose an object that comes in 2's and fill in a chart demonstrating the number in 1 set, 2 sets, etc. up to 11 sets, then colour in the answers on 100s chart • in pairs students identify patterns on the 100's chart • whole group sharing and discussion of patterns • repeat of previous activity using 3's • repeat of previous activity using 4's <p>Curriculum Standards 2, 3, 4, 6, 7, 8, 13</p>	<p>Tape 6B</p> <ul style="list-style-type: none"> - class discussion <p>Student Journals</p> <ul style="list-style-type: none"> • individual review assignment • descriptions of patterns on 100's chart <p>Student Charts</p> <p>Teacher Journal</p> <ul style="list-style-type: none"> • after class notes and general impressions of class discussions
<p>THEMES FROM ANALYSIS</p> <ul style="list-style-type: none"> • journals for communication, assessment • math connections, patterns, place value, number theory • small group, large group sharing, communication, reason, justify 	
<p><i>Morning Meeting</i></p> <ul style="list-style-type: none"> • <i>incorporating mathematical knowledge (multiplication) with other classroom activities</i> • repeat of previous activity using 5's • assessment activity - What Does 7x3 mean? <p>Curriculum Standards 2, 3, 4, 13</p>	<p>Tape 7A – 7B, 8A</p> <ul style="list-style-type: none"> • class discussion <p>Student Journals</p> <ul style="list-style-type: none"> • individual accounts of how the chopstick problem was solved <p>Teacher Journal</p> <ul style="list-style-type: none"> • impressions of the class discussion, small group work, solutions generated
<p>THEMES FROM ANALYSIS</p> <ul style="list-style-type: none"> • connections between classroom activities • application and oral reasoning • journal writing for assessment 	

Figure 1. Action Plan and Data Collection Matrix (Continued)

ACTION STEPS	DATA COLLECTION AND ANALYSIS
<p><i>Morning Meeting</i></p> <ul style="list-style-type: none"> • <i>incorporating mathematical knowledge (multiplication) with other classroom activities</i> <p>AS 9. BOOK SHARING “EACH ORANGE HAD 8 SLICES”</p> <ul style="list-style-type: none"> • teacher led problem solving using text and pictures • book modeling activity where students individually create a story page and problem set based on the story frame in “Each Orange Had 8 Slices”. Teacher collates to create a class book of story problems. <p>Curriculum Standards 1, 2, 3, 4, 7, 8</p>	<p>Tape 8A - 8B</p> <ul style="list-style-type: none"> • class discussion during opening activities • class discussion and problem solving activities during book sharing <p>Student Work Samples</p> <ul style="list-style-type: none"> • story page and illustration <p>Teacher Journal</p> <ul style="list-style-type: none"> • after class notes and general impressions
<p>THEMES FROM ANALYSIS</p> <ul style="list-style-type: none"> • literature connection • real world connections • visuals to help with problem solving, process modeled by students and teacher, explaining thinking 	
<p><i>Morning Meeting</i></p> <ul style="list-style-type: none"> • <i>incorporating mathematical knowledge (multiplication) with other classroom activities</i> <p>AS 10. AUTHORS CORNER – students read their story page to the class. Teacher leads the class in problem solving with each story</p> <p>Curriculum Standards 1, 2, 3, 4, 6, 7, 8, 13</p>	<p>Tape 9A - 9B</p> <ul style="list-style-type: none"> • class discussion during opening activities • class discussion and problem solving during student book sharing <p>Student Work Samples</p> <ul style="list-style-type: none"> • story page and illustration <p>Teacher Journal</p> <ul style="list-style-type: none"> • after class notes and general impressions of class discussions
<p>THEMES FROM ANALYSIS</p> <ul style="list-style-type: none"> • literature connection • students teaching students, making math relevant and useful • student created problem and solutions 	

Appendix B

Appendix B

Learning Outcomes (BC Mathematics K-7 Integrated Resource Package 1995)

Topic	Learning Outcomes
Number (Number Concepts)	Grade 1 <ul style="list-style-type: none"> • count orally by 1s, 2s, 5s, 10s to 100
	Grade 2 <ul style="list-style-type: none"> • skip count forward and backward by 2s, 5s, 10s, 25s, 100s, 1000s using starting points that are multiples • recognize and explain if a number is divisible by 2, 5, 10
Number Operations	Grade 1 N/A
	Grade 2 <ul style="list-style-type: none"> • explore and demonstrate the process of multiplication up to 50 using manipulatives diagrams and symbols
Patterns And Relationships	Grade 1 <ul style="list-style-type: none"> • identify, reproduce, extend create and compare patterns using actions, manipulatives, diagrams and spoken terms • recognize patterns in the environment
	Grade 2 <ul style="list-style-type: none"> • identify, create, and describe number and non number patterns • explain the rule for a pattern and make predictions based on patterns using models and objects

Appendix C

NCTM Curriculum Standards for Grades K-4

Standard 1: Mathematics as Problem Solving

In grades K-4, the study of mathematics should emphasize problem solving so that the students can-

- use problem-solving approaches to investigate and understand mathematical content;
- formulate problems from everyday and mathematical situations;
- develop and apply strategies to solve a wide variety of problems;
- verify and interpret results with respect to the original problem ;
- acquire confidence in using mathematics meaningfully;

Standard 2: Mathematics as Communication

In grades K-4, the study of mathematics should include numerous opportunities for communication so that students can-

- relate physical materials, pictures and diagrams to mathematical ideas;
- reflect on and clarify their thinking about mathematical ideas and situations;
- relate their everyday language to mathematical language and symbols;
- realize that representing, discussing, reading writing and listening to mathematics are a vital part of learning and using mathematics;

Standard 3: Mathematics as Reasoning

In grades K-4, the study of mathematics should emphasize reasoning so that students can-

- draw logical conclusions about mathematics
- use models, known facts, properties, and relationships to explain their thinking;
- justify their answers and solution processes;
- use patterns and relationships to analyze mathematical situations;
- believe that mathematics makes sense;

Standard 4: Mathematical Connections

In grades K-4, the study of mathematics should include opportunities to make connections so that students can-

- link conceptual and procedural knowledge;
- relate various representations of concepts or procedures to one another;
- recognize relationships among different topics in mathematics;
- use mathematics in other curriculum areas;
- use mathematics in their daily lives;

Standard 5: Estimation

In grades K-4, the curriculum should include estimation so students can-

- explore estimation strategies;

- recognize when an estimate is appropriate;
- determine the reasonableness of results;
- apply estimation in working with quantities, measurement, computation, and problem solving;

Standard 6: Number Sense and Numeration

In grades K-4, the mathematics curriculum should include whole number concepts and skills so that students can-

- construct number meanings through real-world experiences and the use of physical materials;
- understand our numeration system by relating counting, grouping, and place-value concepts;
- develop number sense
- interpret the multiple uses of number encountered in the real world;

Standard 7: Concepts of Whole Number Operations

In grades K-4, the mathematics curriculum should include concepts of addition, subtraction, multiplication, and division of whole numbers so that the students can-

- develop meaning for the operations by modeling and discussing a rich variety of problem situations
- relate the mathematical language and symbolism of operations to problem situations and informal language
- recognize that a wide variety of problem structures can be represented by a single operation
- develop operation sense

Standard 8: Whole Number Computation

In grades K-4, the mathematics curriculum should develop whole number computation so that students can-

- model, explain, and develop reasonable proficiency with basic facts and algorithms
- use a variety of mental computation and estimation techniques
- use calculator in appropriate computational situations
- select and use computational techniques appropriate to specific problems and determine whether the results are reasonable

Standard 9: Geometry and Spatial Sense

In grades K-4, the mathematics curriculum should include two- and three-dimensional geometry so that students can-

- describe, model, draw, and classify shapes

- investigate and predict the results of combining, subdividing, and changing shapes;
- develop spatial sense;
- relate geometric ideas to number and measurement ideas;
- recognize and appreciate geometry in their world.

Standard 10: Measurement

In grades K-4, the mathematics curriculum should include measurement so that the students can-

- understand the attributes of length, capacity, weight, area, volume, time temperature, and angle;
- develop the process of measuring concepts related to units of measurement;
- make and use estimates of measurement.

Standard 11: Statistics and Probability

In grades K-4, the mathematics curriculum should include experiences with data analysis and probability so that students can-

- collect, organize, and describe data;
- construct, read, and interpret displays of data;
- formulate and solve problems that involve collecting and analyzing data;
- explore concepts of chance.

Standard 12: Fractions and Decimals

In grades K-4, the mathematics curriculum should include fractions and decimals so that the students can-

- develop concepts of fractions, mixed numbers, and decimals;
- develop number sense for fractions and decimals;
- use models to relate fractions to decimals and to find equivalent fractions;
- use models to explore operations on fractions and decimals;
- apply fractions and decimals to problem solving situations.

Standard 13: Patterns and Relationships

In grades K-4, the mathematics curriculum should include the study of patterns and relationships so that students can-

- recognize, describe, extend, and create a wide variety of patterns;
- represent and describe mathematical relationships;
- explore the use of variables and open sentences to express relationships;

NCTM Professional Standards for Teaching Mathematics

Standard 1: Worthwhile Mathematical Tasks

The teacher of mathematics should pose tasks that are based on –

- sound and significant mathematics;
 - knowledge of students' understandings, interests, and experiences;
 - knowledge of the range of ways that diverse students learn mathematics and that;
 - engage students' intellect;
 - develop students understandings, and skills;
 - stimulate students to make connections and develop a coherent framework for mathematical ideas;
 - call for problem formulation, problem solving, and mathematical reasoning;
 - promote communication about mathematics;
 - represent mathematics as an ongoing human activity;
 - display sensitivity to, and draw on, students' diverse background experiences and dispositions;
- promote the development of all students' dispositions to do mathematics.

Standard 2: The Teachers Role in Discourse

The teacher of mathematics should orchestrate discourse by-

- posing questions and tasks that elicit, engage, and challenge each students thinking;
- listen carefully to students ideas;
- asking students to clarify and justify their ideas orally and in writing;
- decide what to pursue in depth from among the ideas that students bring up in discussion;
- decide when and how to attach mathematical notation and language to student ideas;
- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
- monitoring students' participation in discussions and deciding when and how to encourage each student to participate.

Standard 3: Students' Role in Discourse

The teacher of mathematics should promote classroom discourse in which students –

- listen to, respond to and question the teacher and one and other;
- listen to, respond to, and question the teacher and one and other;
- use a variety of tools to reason, make connections, solve problems, and communicate;
- initiate problems and questions;
- make conjectures and present solutions;
- explore examples and counter examples to investigate a conjecture;

- try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers;
- rely on mathematical evidence and argument to determine validity.

Standard 4: Tools for Enhancing Discourse

The teacher of mathematics, in order to enhance discourse, should encourage and accept the use of –

- computers, calculators and other technology;
- concrete materials used as models;
- pictures, diagrams, tables, and graphs;
- invented and conventional terms and symbols;
- metaphors, analogies, and stories;
- written hypotheses, explanations, and arguments;
- oral presentations, and dramatizations.

Standard 5: Learning Environment

The teacher of mathematics should create a learning environment that fosters the development of each student's mathematical power by –

- providing the structure and the time necessary to explore sound mathematics and grapple with significant ideas and problems;
 - using the physical space and materials in ways that facilitate students' learning of mathematics;
 - providing a context that encourages the development of mathematical skill and proficiency;
 - respecting and valuing students' ideas, ways of thinking, and mathematical dispositions;
- and by consistently expecting and encouraging students to –
- work independently or collaboratively to make sense of mathematics;
 - take intellectual risks by raising questions and formulating conjectures;
 - display a sense of mathematical competence by validating and supporting ideas with mathematical argument.

Standard 6: Analysis of Teaching and Learning

The teacher of mathematics should engage in ongoing analysis of teaching and learning by –

- observing, listening to, gathering other information about students to assess what they are learning
 - examining effects of tasks, discourse and learning environment on students' mathematical knowledge, skills, and dispositions;
- in order to –
- ensure that every student is learning sound and significant mathematics, and is developing a positive disposition towards mathematics;
 - challenge and extend students' ideas;

- adapt or change activities while teaching;
- make plans, both short- and long-range;
- describe and comment on each students' learning to parents and administrator, as well as to the students themselves.

Appendix D



SCHOOL DISTRICT NO. 57 (PRINCE GEORGE)

1894 Ninth Avenue, Prince George, B.C. V2M 1L7

Phone: (250) 561-6800 • Fax (250) 561-6801
www.sd57.bc.ca

May 10, 2004

Jo Kerrigan
C/O College Heights Elementary School
Prince George, BC

Dear Ms. Kerrigan:

This letter is to confirm the discussion at our meeting on May 7, 2004 regarding your request to obtain access to schools in the Prince George School District for the purpose of educational research. As we discussed, the school district recognizes the integral part that research plays in education. We support the research sponsored by our local tertiary institutes as a priority. Your project on Teaching Mathematics for Understanding will provide interesting information for your school and for our district.

This letter's purpose is to indicate that you have district approval to proceed with your project. ***"District approval" allows the researcher to approach principals and subsequently teachers to request their permission to conduct research in their school/classroom.*** Researchers must understand that circumstances may be difficult and school administrators have the final decision. Your next step will be to contact your principal to set up a meeting to discuss your project and obtain her permission to undertake the project in your school. A copy of this letter has been forwarded to Ms. Madeleine Crandell, Principal, College Heights Elementary School.

If you have any questions, please do not hesitate to call me. Good luck with your project. I look forward to receiving a copy of the final report.

Sincerely,

Bonnie Chappell
Director, School Services

BC/hg



CC: M. Crandell
Principal, College Heights Elementary School

Appendix E



Wednesday, April 28, 2004 10:43:11 AM

Message

From:  Madeleine Crandell
Subject: Re: Research Project
To:  Jo Kerrigan

Jo Kerrigan writes:
MEMO: April 27, 2004

To: Madeleine Crandell, Principal, Ecole College Heights Elementary

Hi Madeleine,

Attached is the letter would like to send out to the parents of my students explaining about my research project. At the bottom of the letter there is a consent form to be signed and returned to me. Is this letter sufficient for your purposes? Also, would you like copies of the permission forms when they are returned? I am also submitting a copy of this letter and consent form to Bonnie Chappell as part of the requirements for school district approval.

Thank you for your support in this project.

Josephine Kerrigan

Yes the letter is fine. Very well written. Good luck in your research

Madeleine

Appendix F

June 3, 2004

Dear Parents,

I am writing this letter as a personal request to use student information outside the regular classroom and school district jurisdiction.

As some of you know, over the last few years I have been working towards my Masters degree in Education. In order to complete my studies I would like to conduct a research project analyzing an area of my teaching in the classroom. The form of analysis I wish to use is known as action research and it is intended to promote self-reflection in order to improve teaching and learning. It requires the teacher to keep a detailed journal and possibly taped audio recordings of classroom teaching and learning events so that the episodes can be analyzed and evaluated. Information gathered is used to plan and develop further learning situations. This is not much different from what generally happens in the regular classroom. However, with this project, I will be using the data collected in the classroom to write a research paper, and the information will be shared with my thesis committee at the University of Northern British Columbia.

The purpose of the analysis is to focus on the positive learning situations that are created in the classroom as the teacher and students interact during learning situations. It is not intended to focus on any particular student, or on individual student achievement. However, confidentiality is of utmost importance in classroom research, so in order to address this issue all students mentioned in the study will be given pseudonyms and real names will not be used. Any taped audio recordings will be used for my own record keeping and transcription. They will not be shared with any other individual and will be erased by me after analysis. In my thesis discussion I may want to provide samples of student work. Again, no individual student will be identified and pseudonyms will be assigned.

The area I wish to focus on is the introduction of the concept of multiplication. This is part of the Grade 2 curriculum and is also developed informally in the Grade 1 curriculum through number patterns and counting in multiples. School District 57 is currently re-evaluating district math programs and is concerned that students develop a better understanding of math concepts early, rather than learning processes and procedures by rote. This research project aims to address the district's concerns and I hope it provides the students with learning opportunities to promote greater understanding of the multiplication process.

If you have any questions regarding this research project please feel free to phone me at College Heights Elementary 964-4408 or at home 964-2979. If you have any further concerns or complaints regarding this project you may contact Dr. Max Blouw at the Office of Research and Graduate Studies at UNBC.

Thank you for your assistance in this project.

Josephine Kerrigan

By signing this form I acknowledge that have read the letter of explanation and I consent to my child's participation in the research project.

Child's Name

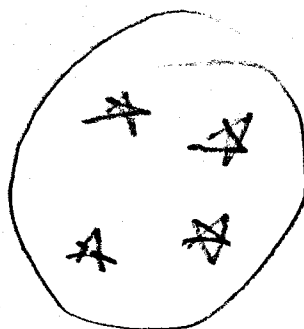
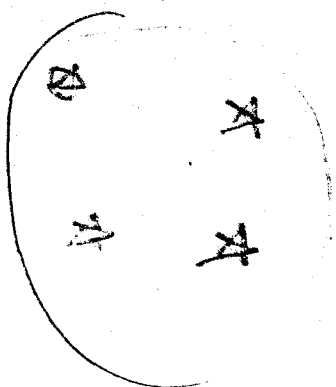
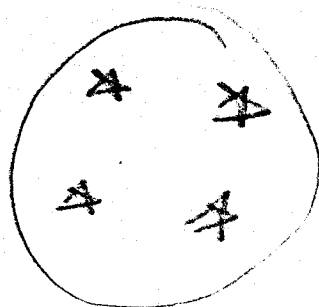
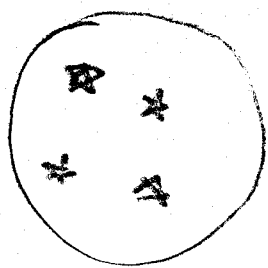
Parent or Guardian Signature

Date

Appendix G

Blank handwriting practice lines consisting of solid top and bottom lines with a dashed midline for letter height guidance. The page contains ten sets of these three-line guides.

Appendix H

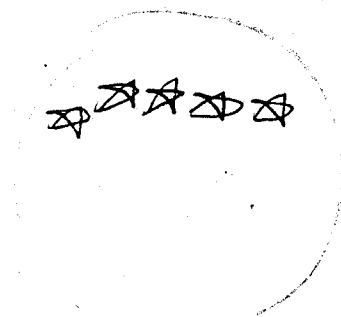
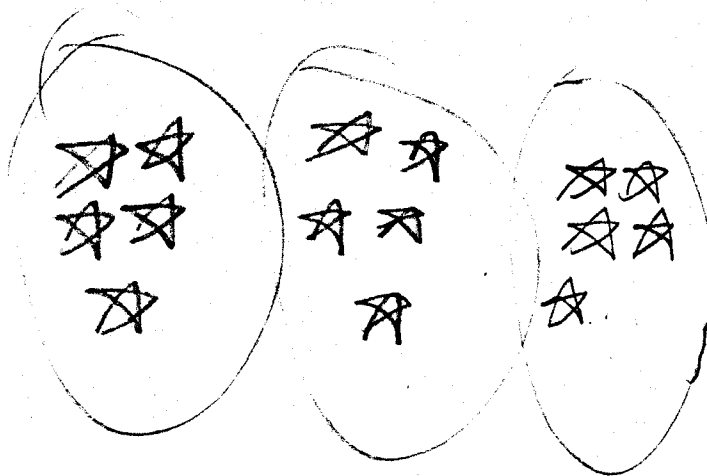


$$4 + 4 + 4 + 4 = 16 \quad \checkmark$$

4 groups of 4 equals 16

4 times 4 equals 16.

$$4 \times 4 = 16$$



$$5 + 5 + 5 + 5 = 20 \quad \checkmark$$

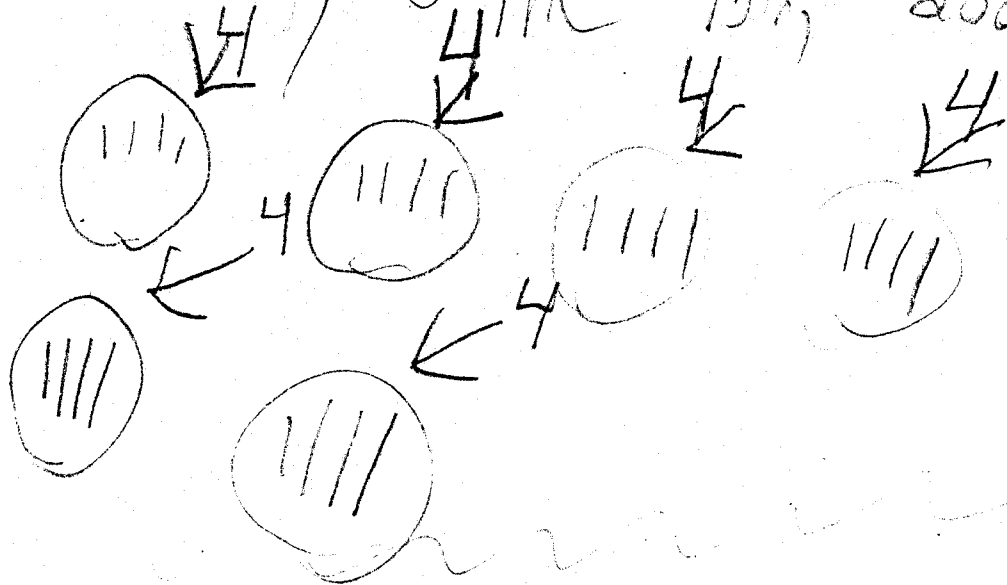
4 groups of 5 equals 20

4 times 5 equals 20

$$4 \times 5 = 20$$

Appendix I

Thursday, June 15th, 2004



$$4 + 4 + 4 + 4 + 4 + 4 = 24$$
$$6 \times 4 = 24$$

Tell me all you know about 6×4
 6×4 is six groups of
4 equals 24 because
you have 6 circles
and you put 4 dots
in each one then you
will get the answer

Appendix J

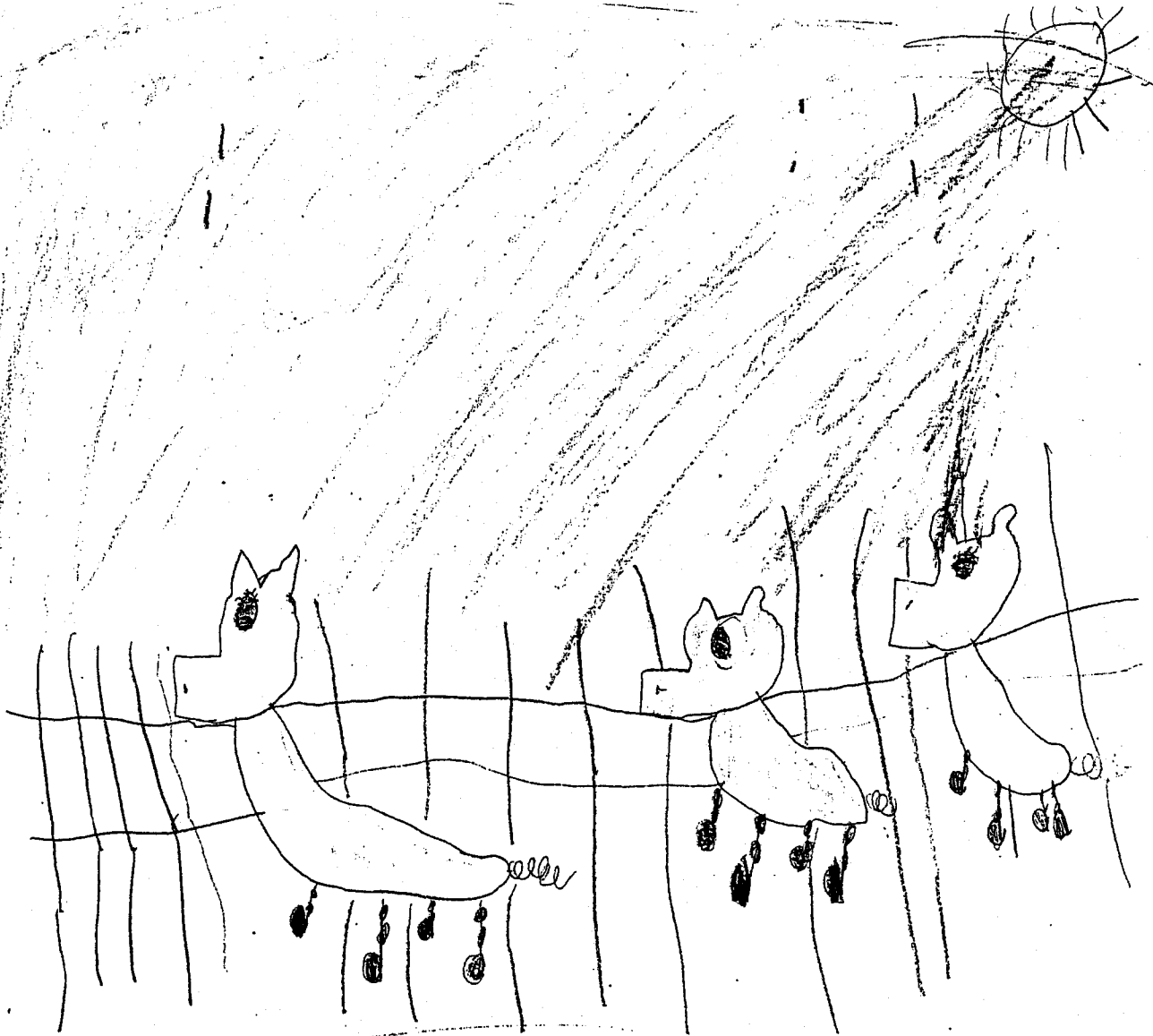
Patterns In Multiples

triangle	sides	
1	3	$1 \times 3 = 3$
2	6	$2 \times 3 = 6$
3	9	$3 \times 3 = 9$
4	12	$4 \times 3 = 12$
5	15	$5 \times 3 = 15$
6	18	$6 \times 3 = 18$
7	21	$7 \times 3 = 21$
8	24	$8 \times 3 = 24$
9	27	$9 \times 3 = 27$
10	30	$10 \times 3 = 30$
11	33	$11 \times 3 = 33$

Appendix K

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Appendix L



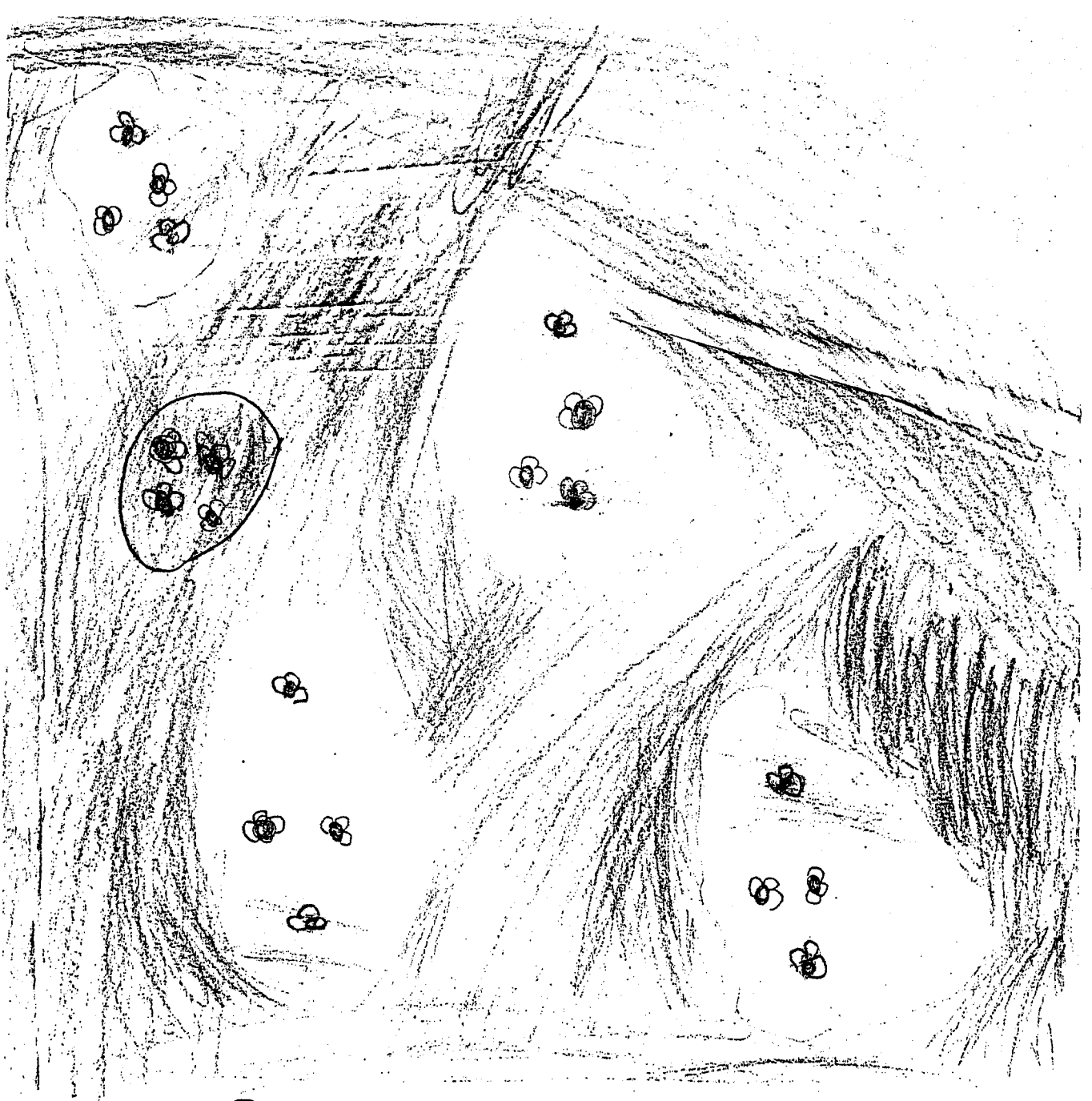
On my way to grandma's
house I saw three
little pigs. Each pig
had 4 legs. Each pig had
spots. How many pigs?
How many legs? How many
spots?

by



On my way to Green
Lake. I saw 4 bears.
Each bear had 4 feet.
Each foot had 3
claws.

How many bears?
How many feet?
How many claws?



On my way to the Park
I saw five rocks. They each
have four dots on them.
Each dot had three bumps.
How many rocks?
How many dots?
How many bumps?
by _____

Appendix M